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Doppler effect, speed of the light, stellar velocity and distance

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Abstract: Calculating the distance of a star from Earth using stellar parallax makes sense only in two cases. In the first case, this is possible if the object we are observing is stationary in relation to the Sun and in the second case if the measurements are made simultaneously, but from two different locations. We will consider the case when the star moves uniformly with respect to the Sun. By measuring the Doppler effect at three different points, we will be able to determine the speed of light emitted from the star in the direction of the observer, the velocities at which the star and Sun move regarding the referential coordinate system and finally we will calculate the distance between the star and the observer.

Keywords: *Doppler effect, speed of light, stellar velocity, stellar distance*

1. Introduction

Suppose that the sender S and receiver A of the signal move uniformly at velocities $\mathbf{u}$ and $\mathbf{v}$, respectively regarding to the coordinate system (K).

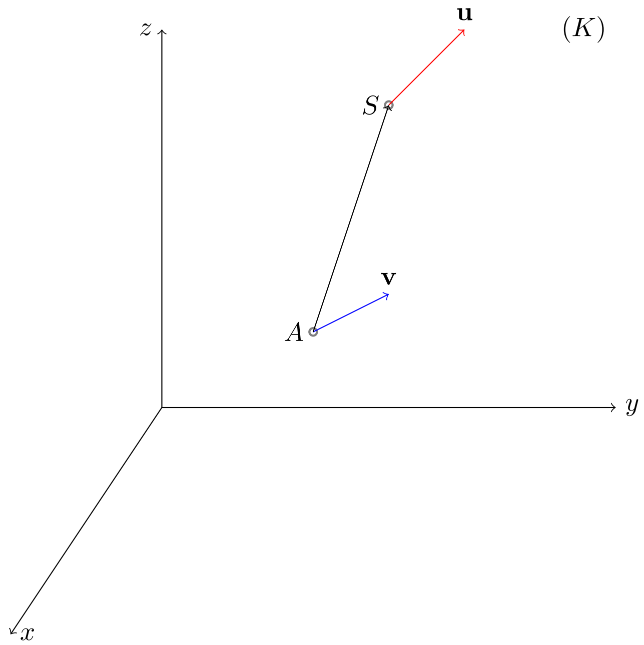

Figure 1: The sender S and the observer A move uniformly with respect to the coordinate system (K)

Referring to Fig[1] we have that:

$$\mathbf{a} = [a_x, a_y, a_z] = \frac{\mathbf{AS}}{||\mathbf{AS}||} \tag{1}$$

$$\mathbf{u} = [u_x, u_y, u_z] \tag{2}$$

$$\mathbf{v} = [v_x, v_y, v_z] \tag{3}$$

$$u_r = \mathbf{a} \cdot \mathbf{u} = a_x\, u_x + a_y\, u_y + a_z\, u_z \tag{4}$$

$$v_r = \mathbf{a} \cdot \mathbf{v} = a_x\, v_x + a_y\, v_y + a_z\, v_z \tag{5}$$

The positive direction is directed from the observer toward the sender.

If $||\mathbf{v}||< c$ and $||\mathbf{u}||< c$, then Doppler equation [1] can be written as it follows:

$$f' = \frac{c + v_r}{c + u_r}\, f = \frac{c + \mathbf{a} \cdot \mathbf{v}}{c + \mathbf{a} \cdot \mathbf{u}}\, f \tag{6}$$

where

f - the frequency of the emitted light
f' - the frequency of the observed light
c - the speed of light

From the equation (6) it follows that:

$$c + \mathbf{a} \cdot \mathbf{u} = \frac{f}{f'}\,(c + \mathbf{a} \cdot \mathbf{v}) \tag{7}$$

$$\mathbf{a} \cdot \mathbf{u} = \frac{f - f'}{f'}\, c + \frac{f}{f'}\, \mathbf{a} \cdot \mathbf{v} \tag{8}$$

$$a_x\, u_x + a_y\, u_y + a_z\, u_z = \frac{f - f'}{f'}\, c + \frac{f}{f'}\, \mathbf{a} \cdot \mathbf{v} \tag{9}$$

2. The heliocentric coordinate system as a referential coordinate system

Suppose that the observed star Z is moving with a uniform, rectilinear space motion noted by $\mathbf{u}$ regarding the coordinate system (K) Fig[2]. We will assume that the observed object belongs to the Milky Way Galaxy. Let us denote by τ_A the time when the signal was sent from the point noted by Z_1 and by t_A the time when the signal is registered at point noted by A. We assume that τ_A and t_A are expressed in the same units of time. The unit vector of the direction AZ_1 is denoted by $\hat{\mathbf{a}}$. In an analogous way, we will define triples $(\tau_B, t_B, \hat{\mathbf{b}})$ and $(\tau_C, t_C, \hat{\mathbf{c}})$ for pairs of points (B, Z_2) and (C, Z_3), respectively.

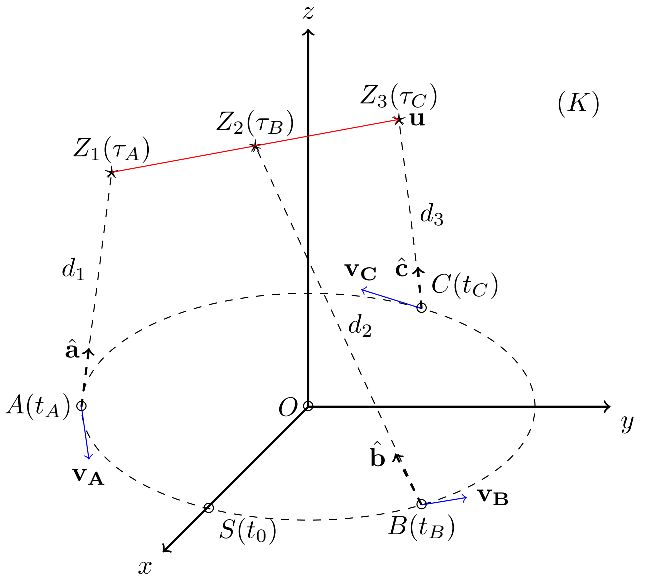

Figure 2: Star Z moves uniformly regarding the sun.

Coordinate system (K) is heliocentric ecliptic coordinate system. Coordinate axes are determined in accordance with the $ICRS$ standard. Referring to Fig[2] we have that:

$$d_1 = ||\mathbf{AZ}_1|| \tag{10}$$

$$d_2 = ||\mathbf{BZ}_2|| \tag{11}$$

$$d_3 = ||\mathbf{CZ}_3|| \tag{12}$$

$$\mathbf{A} = \mathbf{OA} = [A_x, A_y, A_z] \tag{13}$$

$$\mathbf{B} = \mathbf{OB} = [B_x, B_y, B_z] \tag{14}$$

$$\mathbf{C} = \mathbf{OC} = [C_x, C_y, C_z] \tag{15}$$

$$\hat{\mathbf{a}} = [a_x, a_y, a_z] = \frac{\mathbf{AZ}_1}{||\mathbf{AZ}_1||} \tag{16}$$

$$\hat{\mathbf{b}} = [b_x, b_y, b_z] = \frac{\mathbf{BZ}_2}{||\mathbf{BZ}_2||} \tag{17}$$

$$\hat{\mathbf{c}} = [c_x, c_y, c_z] = \frac{\mathbf{CZ}_3}{||\mathbf{CZ}_3||} \tag{18}$$

3. The coordinate system that is stationary with respect to the Galactic Center

By definition, the origin of the $ICRS$ coordinate system is at the *barycenter* of the Solar System. This means that in our calculations the velocity, denoted by $\mathbf{V}$, at which the solar system is moving relative to the Galactic Center has been omitted. In order to include the velocity $\mathbf{V}$ in our calculations, we will assume that the origin of the referential coordinate system, denoted by (K'), is stationary in relation to the Galactic Center. At one moment, which we can mark as some initial time T_0, the coordinate system (K') is identical to (K), after which (K') remains fixed in relation to the Galactic Center, while (K) together with the Solar system, continues to move at velocity $\mathbf{V}$ in relation to the to the Galactic Center. We do not yet know the true value of the $\mathbf{V}$, but we can take any $\mathbf{V}$ so that the inequality (20) holds:

$$\mathbf{V} = [V_x, V_y, V_z] \tag{19}$$

$$||\mathbf{V}|| < M \tag{20}$$

where M is some previously given positive number

Let's us denote with A', B' and C' the points from the coordinate system (K') that correspond to the points A, B and C from the coordinate system (K).

$$\mathbf{A}' = [A'_x, A'_y, A'_z] = \mathbf{A} + t_A\, \mathbf{V} \tag{21}$$

$$\mathbf{B}' = [B'_x, B'_y, B'_z] = \mathbf{B} + t_B\, \mathbf{V} \tag{22}$$

$$\mathbf{C}' = [C'_x, C'_y, C'_z] = \mathbf{C} + t_C\, \mathbf{V} \tag{23}$$

Let us denote by $\mathbf{v}'_A$, $\mathbf{v}'_B$ and $\mathbf{v}'_C$ the velocities with which the points A', B' and C' move in relation to the coordinate system (K').

$$\mathbf{v}'_A = \mathbf{v}_A + \mathbf{V} \tag{24}$$

$$\mathbf{v}'_B = \mathbf{v}_B + \mathbf{V} \tag{25}$$

$$\mathbf{v}'_C = \mathbf{v}_C + \mathbf{V} \tag{26}$$

Let f denotes the frequency of the emitted light at points Z_1, Z_2 and Z_3 respectively. Let f_A, f_B and f_A denote the frequencies of the observed light at points A', B' and C' respectively. Let $\mathbf{v'_A}$, $\mathbf{v'_B}$ and $\mathbf{v'_C}$ denote the velocities of the observed light at points A', B' and C' respectively regarding to the coordinate system (K').
From (9) it follows that:

$$a_x\, u_x + a_y\, u_y + a_z\, u_z = \frac{f - f_A}{f_A}\, c + \frac{f}{f_A}\, \hat{\mathbf{a}} \cdot \mathbf{v'_A} \tag{27}$$

$$b_x\, u_x + b_y\, u_y + b_z\, u_z = \frac{f - f_B}{f_B}\, c + \frac{f}{f_B}\, \hat{\mathbf{b}} \cdot \mathbf{v'_B} \tag{28}$$

$$c_x\, u_x + c_y\, u_y + c_z\, u_z = \frac{f - f_C}{f_C}\, c + \frac{f}{f_C}\, \hat{\mathbf{c}} \cdot \mathbf{v'_C} \tag{29}$$

We have obtained a linear system of three equations with three unknowns u_x, u_y and u_z.

$$U = \begin{vmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ a_z & b_z & c_z \end{vmatrix} \tag{30}$$

$$U_x = c\, U'_x + U''_x = \begin{vmatrix} \frac{f-f_A}{f_A}\, c + \frac{f}{f_A}\, \hat{\mathbf{a}} \cdot \mathbf{v'_A} & b_x & c_x \\ \frac{f-f_B}{f_B}\, c + \frac{f}{f_B}\, \hat{\mathbf{b}} \cdot \mathbf{v'_B} & b_y & c_y \\ \frac{f-f_C}{f_C}\, c + \frac{f}{f_C}\, \hat{\mathbf{c}} \cdot \mathbf{v'_C} & b_z & c_z \end{vmatrix} = c \begin{vmatrix} \frac{f-f_A}{f_A} & b_x & c_x \\ \frac{f-f_B}{f_B} & b_y & c_y \\ \frac{f-f_C}{f_C} & b_z & c_z \end{vmatrix} + \begin{vmatrix} \frac{f}{f_A}\, \hat{\mathbf{a}} \cdot \mathbf{v'_A} & b_x & c_x \\ \frac{f}{f_B}\, \hat{\mathbf{b}} \cdot \mathbf{v'_B} & b_y & c_y \\ \frac{f}{f_C}\, \hat{\mathbf{c}} \cdot \mathbf{v'_C} & b_z & c_z \end{vmatrix} \tag{31}$$

$$U_y = c\,U'_y + U''_y = \begin{vmatrix} a_x & \frac{f-f_A}{f_A}\,c + \dfrac{f}{f_A}\,\hat{\mathbf{a}}\cdot\mathbf{v}'_\mathbf{A} & c_x \\ a_y & \frac{f-f_B}{f_B}\,c + \dfrac{f}{f_B}\,\hat{\mathbf{b}}\cdot\mathbf{v}'_\mathbf{B} & c_y \\ a_z & \frac{f-f_C}{f_C}\,c + \dfrac{f}{f_C}\,\hat{\mathbf{c}}\cdot\mathbf{v}'_\mathbf{C} & c_z \end{vmatrix} = c\begin{vmatrix} a_x & \frac{f-f_A}{f_A} & c_x \\ a_y & \frac{f-f_B}{f_B} & c_y \\ a_z & \frac{f-f_C}{f_C} & c_z \end{vmatrix} + \begin{vmatrix} a_x & \dfrac{f}{f_A}\,\hat{\mathbf{a}}\cdot\mathbf{v}'_\mathbf{A} & c_x \\ a_y & \dfrac{f}{f_B}\,\hat{\mathbf{b}}\cdot\mathbf{v}'_\mathbf{B} & c_y \\ a_z & \dfrac{f}{f_C}\,\hat{\mathbf{c}}\cdot\mathbf{v}'_\mathbf{C} & c_z \end{vmatrix} \tag{32}$$

$$U_z = c\,U'_z + U''_z = \begin{vmatrix} a_x & b_x & \frac{f-f_A}{f_A}\,c + \dfrac{f}{f_A}\,\hat{\mathbf{a}}\cdot\mathbf{v}'_\mathbf{A} \\ a_y & b_y & \frac{f-f_B}{f_B}\,c + \dfrac{f}{f_B}\,\hat{\mathbf{b}}\cdot\mathbf{v}'_\mathbf{B} \\ a_z & b_z & \frac{f-f_C}{f_C}\,c + \dfrac{f}{f_C}\,\hat{\mathbf{c}}\cdot\mathbf{v}'_\mathbf{C} \end{vmatrix} = c\begin{vmatrix} a_x & b_x & \frac{f-f_A}{f_A} \\ a_y & b_y & \frac{f-f_B}{f_B} \\ a_z & b_z & \frac{f-f_C}{f_C} \end{vmatrix} + \begin{vmatrix} a_x & b_x & \dfrac{f}{f_A}\,\hat{\mathbf{a}}\cdot\mathbf{v}'_\mathbf{A} \\ a_y & b_y & \dfrac{f}{f_B}\,\hat{\mathbf{b}}\cdot\mathbf{v}'_\mathbf{B} \\ a_z & b_z & \dfrac{f}{f_C}\,\hat{\mathbf{c}}\cdot\mathbf{v}'_\mathbf{C} \end{vmatrix} \tag{33}$$

$$u_x = \frac{U_x}{U} = \frac{c\,U'_x + U''_x}{U} \tag{34}$$

$$u_y = \frac{U_y}{U} = \frac{c\,U'_y + U''_y}{U} \tag{35}$$

$$u_z = \frac{U_z}{U} = \frac{c\,U'_z + U''_z}{U} \tag{36}$$

In order to calculate the three unknowns u_x, u_y and u_z, we first need to determine the speed of light c. We have the following equations:

$$t_{AB} = t_B - t_A \tag{37}$$

$$\mathbf{A}'\mathbf{B}' = [B'_x - A'_x, B'_y - A'_y, B'_z - A'_z] \tag{38}$$

$$\mathbf{A}'\mathbf{B}' = \mathbf{AB} + t_{AB}\,\mathbf{V} = [(B_x - A_x) + t_{AB}\,V_x, (B_y - A_y) + t_{AB}\,V_y, (B_z - A_z) + t_{AB}\,V_z] \tag{39}$$

$$\mathbf{Z}_1\mathbf{Z}_2 = -d_1\,\hat{\mathbf{a}} + \mathbf{A}'\mathbf{B}' + d_2\,\hat{\mathbf{b}} \tag{40}$$

$$\Delta\,\tau = \tau_B - \tau_A \tag{41}$$

$$\mathbf{Z}_1\mathbf{Z}_2 = \Delta\,\tau\,\mathbf{u} \tag{42}$$

$$d_1\,\hat{\mathbf{a}} - d_2\,\hat{\mathbf{b}} + \Delta\,\tau\,\mathbf{u} = \mathbf{A}'\mathbf{B}' \tag{43}$$

$$d_1\,a_x - d_2\,b_x + \Delta\,\tau\,u_x = B'_x - A'_x \tag{44}$$

$$d_1\,a_y - d_2\,b_y + \Delta\,\tau\,u_y = B'_y - A'_y \tag{45}$$

$$d_1\,a_z - d_2\,b_z + \Delta\,\tau\,u_z = B'_z - A'_z \tag{46}$$

We have obtained a linear system of three equations with three unknowns d_1, d_2 and τ.

$$abD = (c)\,abD' + abD'' = \begin{vmatrix} a_x & -b_x & u_x \\ a_y & -b_y & u_y \\ a_z & -b_z & u_z \end{vmatrix} = \begin{vmatrix} a_x & -b_x & \frac{c\,U'_x+U''_x}{U} \\ a_y & -b_y & \frac{c\,U'_y+U''_y}{U} \\ a_z & -b_z & \frac{c\,U'_z+U''_z}{U} \end{vmatrix} = c\begin{vmatrix} a_x & -b_x & \frac{U'_x}{U} \\ a_y & -b_y & \frac{U'_y}{U} \\ a_z & -b_z & \frac{U'_z}{U} \end{vmatrix} + \begin{vmatrix} a_x & -b_x & \frac{U''_x}{U} \\ a_y & -b_y & \frac{U''_y}{U} \\ a_z & -b_z & \frac{U''_z}{U} \end{vmatrix} \tag{47}$$

$$abD_1 = (c)\,abD'_1 + abD''_1 = \begin{vmatrix} B'_x - A'_x & -b_x & u_x \\ B'_y - A'_y & -b_y & u_y \\ B'_z - A'_z & -b_z & u_z \end{vmatrix} = c\begin{vmatrix} B'_x - A'_x & -b_x & \frac{U'_x}{U} \\ B'_y - A'_y & -b_y & \frac{U'_y}{U} \\ B'_z - A'_z & -b_z & \frac{U'_z}{U} \end{vmatrix} + \begin{vmatrix} B'_x - A'_x & -b_x & \frac{U''_x}{U} \\ B'_y - A'_y & -b_y & \frac{U''_y}{U} \\ B'_z - A'_z & -b_z & \frac{U''_z}{U} \end{vmatrix} \tag{48}$$

$$AB\_d_1 = \frac{(c)\, abD_1' + abD_1''}{(c)\, abD' + abD''} \tag{49}$$

$$abD_2 = (c)\, abD_2' + abD_2'' = \begin{vmatrix} a_x & B_x' - A_x' & u_x \\ a_y & B_y' - A_y' & u_y \\ a_z & B_z' - A_z' & u_z \end{vmatrix} = c \begin{vmatrix} a_x & B_x' - A_x' & \frac{U_x'}{U} \\ a_y & B_y' - A_y' & \frac{U_y'}{U} \\ a_z & B_z' - A_z' & \frac{U_z'}{U} \end{vmatrix} + \begin{vmatrix} a_x & B_x' - A_x' & \frac{U_x''}{U} \\ a_y & B_y' - A_y' & \frac{U_y''}{U} \\ a_z & B_z' - A_z' & \frac{U_z''}{U} \end{vmatrix} \tag{50}$$

$$AB\_d_2 = \frac{(c)\, abD_2' + abD_2''}{(c)\, abD' + abD''} \tag{51}$$

$$abD_\tau = \begin{vmatrix} ax & -b_x & B_x' - A_x' \\ ay & -b_y & B_y' - A_y' \\ az & -b_z & B_z' - A_z' \end{vmatrix} \tag{52}$$

$$\Delta\tau_{ab} = \frac{abD_\tau}{(c)\, abD' + abD''} \tag{53}$$

In a similar way for the direction $\mathbf{A}'\mathbf{C}'$ we will get the following equations:

$$d_1\, a_x - d_3\, c_x + \Delta\, \tau\, u_x = C_x' - A_x' \tag{54}$$

$$d_1\, a_y - d_3\, c_y + \Delta\, \tau\, u_y = C_y' - A_y' \tag{55}$$

$$d_1\, a_z - d_3\, c_z + \Delta\, \tau\, u_z = C_z' - A_z' \tag{56}$$

$$acD = (c)\, acD' + acD'' = \begin{vmatrix} a_x & -c_x & u_x \\ a_y & -c_y & u_y \\ a_z & -c_z & u_z \end{vmatrix} = \begin{vmatrix} a_x & -c_x & \frac{c\, U_x' + U_x''}{U} \\ a_y & -c_y & \frac{c\, U_y' + U_y''}{U} \\ a_z & -c_z & \frac{c\, U_z' + U_z''}{U} \end{vmatrix} = c \begin{vmatrix} a_x & -c_x & \frac{U_x'}{U} \\ a_y & -c_y & \frac{U_y'}{U} \\ a_z & -c_z & \frac{U_z'}{U} \end{vmatrix} + \begin{vmatrix} a_x & -c_x & \frac{U_x''}{U} \\ a_y & -c_y & \frac{U_y''}{U} \\ a_z & -c_z & \frac{U_z''}{U} \end{vmatrix} \tag{57}$$

$$acD_1 = (c)\, acD_1' + acD_1'' = \begin{vmatrix} C_x' - A_x' & -c_x & u_x \\ C_y' - A_y' & -c_y & u_y \\ C_z' - A_z' & -c_z & u_z \end{vmatrix} = c \begin{vmatrix} C_x' - A_x' & -c_x & \frac{U_x'}{U} \\ C_y' - A_y' & -c_y & \frac{U_y'}{U} \\ C_z' - A_z' & -c_z & \frac{U_z'}{U} \end{vmatrix} + \begin{vmatrix} C_x' - A_x' & -c_x & \frac{U_x''}{U} \\ C_y' - A_y' & -c_y & \frac{U_y''}{U} \\ C_z' - A_z' & -c_z & \frac{U_z''}{U} \end{vmatrix} \tag{58}$$

$$AC\_d_1 = \frac{(c)\, acD_1' + acD_1''}{(c)\, acD' + acD''} \tag{59}$$

$$acD_3 = (c)\, acD_3' + acD_3'' = \begin{vmatrix} a_x & C_x' - A_x' & u_x \\ a_y & C_y' - A_y' & u_y \\ a_z & C_z' - A_z' & u_z \end{vmatrix} = c \begin{vmatrix} a_x & C_x' - A_x' & \frac{U_x'}{U} \\ a_y & C_y' - A_y' & \frac{U_y'}{U} \\ a_z & C_z' - A_z' & \frac{U_z'}{U} \end{vmatrix} + \begin{vmatrix} a_x & C_x' - A_x' & \frac{U_x''}{U} \\ a_y & C_y' - A_y' & \frac{U_y''}{U} \\ a_z & C_z' - A_z' & \frac{U_z''}{U} \end{vmatrix} \tag{60}$$

$$AC\_d_3 = \frac{(c)\, acD_3' + acD_3''}{(c)\, acD' + acD''} \tag{61}$$

$$acD_\tau = \begin{vmatrix} ax & -c_x & C'_x - A'_x \\ ay & -c_y & C'_y - A'_y \\ az & -c_z & C'_z - A'_z \end{vmatrix} \tag{62}$$

$$\Delta\tau_{ac} = \frac{acD_\tau}{(c)\, acD' + acD''} \tag{63}$$

For the direction $\mathbf{B'C'}$ we will get the following equations:

$$d_2\, b_x - d_3\, c_x + \Delta\,\tau\, u_x = C'_x - B'_x \tag{64}$$

$$d_2\, b_y - d_3\, c_y + \Delta\,\tau\, u_y = C'_y - B'_y \tag{65}$$

$$d_2\, b_z - d_3\, c_z + \Delta\,\tau\, u_z = C'_z - B'_z \tag{66}$$

$$bcD = (c)\, bcD' + bcD'' = \begin{vmatrix} b_x & -c_x & u_x \\ b_y & -c_y & u_y \\ b_z & -c_z & u_z \end{vmatrix} = \begin{vmatrix} b_x & -c_x & \frac{c\,U'_x + U''_x}{U} \\ b_y & -c_y & \frac{c\,U'_y + U''_y}{U} \\ b_z & -c_z & \frac{c\,U'_z + U''_z}{U} \end{vmatrix} = c \begin{vmatrix} b_x & -c_x & \frac{U'_x}{U} \\ b_y & -c_y & \frac{U'_y}{U} \\ b_z & -c_z & \frac{U'_z}{U} \end{vmatrix} + \begin{vmatrix} b_x & -c_x & \frac{U''_x}{U} \\ b_y & -c_y & \frac{U''_y}{U} \\ b_z & -c_z & \frac{U''_z}{U} \end{vmatrix} \tag{67}$$

$$bcD_2 = (c)\, bcD'_2 + bcD''_2 = \begin{vmatrix} C'_x - B'_x & -c_x & u_x \\ C'_y - B'_y & -c_y & u_y \\ C'_z - B'_z & -c_z & u_z \end{vmatrix} = c \begin{vmatrix} C'_x - B'_x & -c_x & \frac{U'_x}{U} \\ C'_y - B'_y & -c_y & \frac{U'_y}{U} \\ C'_z - B'_z & -c_z & \frac{U'_z}{U} \end{vmatrix} + \begin{vmatrix} C'_x - B'_x & -c_x & \frac{U''_x}{U} \\ C'_y - B'_y & -c_y & \frac{U''_y}{U} \\ C'_z - B'_z & -c_z & \frac{U''_z}{U} \end{vmatrix} \tag{68}$$

$$BC\_d_2 = \frac{(c)\, bcD'_2 + bcD''_2}{(c)\, bcD' + bcD''} \tag{69}$$

$$bcD_3 = (c)\, bcD'_3 + bcD''_3 = \begin{vmatrix} b_x & C'_x - B'_x & u_x \\ b_y & C'_y - B'_y & u_y \\ b_z & C'_z - B'_z & u_z \end{vmatrix} = c \begin{vmatrix} b_x & C'_x - B'_x & \frac{U'_x}{U} \\ b_y & C'_y - B'_y & \frac{U'_y}{U} \\ b_z & C'_z - B'_z & \frac{U'_z}{U} \end{vmatrix} + \begin{vmatrix} b_x & C'_x - B'_x & \frac{U''_x}{U} \\ b_y & C'_y - B'_y & \frac{U''_y}{U} \\ b_z & C'_z - B'_z & \frac{U''_z}{U} \end{vmatrix} \tag{70}$$

$$BC\_d_3 = \frac{(c)\, bcD'_3 + bcD''_3}{(c)\, bcD' + bcD''} \tag{71}$$

$$bcD_\tau = \begin{vmatrix} b_x & -c_x & C'_x - B'_x \\ b_y & -c_y & C'_y - B'_y \\ b_z & -c_z & C'_z - B'_z \end{vmatrix} \tag{72}$$

$$\Delta\tau_{bc} = \frac{bcD_\tau}{(c)\, bcD' + bcD''} \tag{73}$$

4. The speed of the light regarding the referential coordinate system

First we will determine which of the two coordinate systems (K) or (K') is the referential coordinate system. It follows from equations (49) and (59) that:

$$AB\_d_1 = AC\_d_1 \tag{74}$$

$$\frac{(c)\, abD'_1 + abD''_1}{(c)\, abD' + abD''} = \frac{(c)\, acD'_1 + acD''_1}{(c)\, acD' + acD''} \tag{75}$$

$$((c)\, abD'_1 + abD''_1)((c)\, acD' + acD'') - ((c)\, acD'_1 + acD''_1)((c)\, abD' + abD'') = 0 \tag{76}$$

$$\begin{vmatrix} abD'_1 & acD'_1 \\ abD' & acD' \end{vmatrix} c^2 + \left(\begin{vmatrix} abD'_1 & acD'_1 \\ abD'' & acD'' \end{vmatrix} + \begin{vmatrix} abD''_1 & acD''_1 \\ abD' & acD' \end{vmatrix} \right) c + \begin{vmatrix} abD''_1 & acD''_1 \\ abD'' & acD'' \end{vmatrix} = 0 \tag{77}$$

$$A_0 = \begin{vmatrix} abD'_1 & acD'_1 \\ abD' & acD' \end{vmatrix} \tag{78}$$

$$B_0 = \begin{vmatrix} abD'_1 & acD'_1 \\ abD'' & acD'' \end{vmatrix} + \begin{vmatrix} abD''_1 & acD''_1 \\ abD' & acD' \end{vmatrix} \tag{79}$$

$$C_0 = \begin{vmatrix} abD''_1 & acD''_1 \\ abD'' & acD'' \end{vmatrix} \tag{80}$$

$$A_0\, c^2 + B_0\, c + C_0 = 0 \tag{81}$$

$$c_{1,2} = \frac{-B_0 \pm \sqrt{B_0^2 - 4\, A_0\, C_0}}{2\, A_0} \tag{82}$$

Applying the equations (34),(35) and (36) we can easily calculate the u_x, u_y and u_z. Applying the equations (49),(51) and (53) we can easily calculate the AB_d_1, AB_d_2 and $\Delta\tau_{ab}$. We will consider only those cases in which c_i, AB_d_1, AB_d_2 and $\Delta\tau_{ab}$ are greater than zero.

$$\Delta t_{ab} = t_B - t_A \tag{83}$$

$$\frac{AB\_d_1}{c_{AB}} + \Delta T_{ab}(\mathbf{V}) = \frac{AB\_d_2}{c_{AB}} + \Delta\tau_{ab} \tag{84}$$

$$\Delta T_{ab}(\mathbf{V}) = \frac{AB\_d_2}{c_{AB}} + \Delta\tau_{ab} - \frac{AB\_d_1}{c_{AB}} \tag{85}$$

$$\varepsilon(\mathbf{V}) = |\Delta t_{ab} - \Delta T_{ab}(\mathbf{V})| \tag{86}$$

We will define ε_{ab} in the following way:

$$\varepsilon_{ab} = \min\{|\varepsilon(\mathbf{V}),\ ||\mathbf{V}||< M\} \tag{87}$$

If ε_{ab} is much smaller compared to Δt_{ab}, then we can determine approximate values for $\mathbf{V}$ and c, which we will denote by $\mathbf{V}_i$ and c_i. But if this is not the case, then one of the possible reasons why the inequality (87) is not satisfied is that the observed star does not move uniformly in relation to the Sun. Repeating the same procedure for each star, we will get two sequences of numbers which we will denote by V and C.

$$V = \{\mathbf{V}_i\}\ ,\ (i = 1...n) \tag{88}$$

$$C = \{c_i\}\ ,\ (i = 1...n) \tag{89}$$

We will determine the arithmetic mean $\mathbf{V}$ and the standard deviation σ for the sequence of numbers V.

$$\mathbf{V} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{V}_i \tag{90}$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mathbf{V} - \mathbf{V}_i)^2} \tag{91}$$

If the standard deviation σ is less than some given positive number and $||\mathbf{V}|| >> M_0$ then we will say that $\mathbf{V}$ is the velocity with which the solar system moves in relation to the Galactic center and the (K') is referential coordinate system. If $||\mathbf{V}|| << M_0$ then we will say that the (K) is referential coordinate system. The M_0 is some positive number.

We will determine the arithmetic mean c and the standard deviation σ for the sequence of numbers C.

$$c = \frac{1}{n}\sum_{i=1}^{n}c_i \tag{92}$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(c - c_i)^2} \tag{93}$$

If the standard deviation σ is less than some given positive number, then we can conclude that the speed of light is constant.

5. Discussion

The author was not able to test this model using real data. But we tested this model by simulating reality using a computer program. If we assume that the unit with which we measure the speed is equal to $[km/sec]$, then in order to obtain satisfactory approximate values for c it is necessary that the increment by which we change the values of the components V_x, V_y and V_z is less than 1. Once we have established the value for the velocity $\mathbf{V}$, then it is obvious that it is no longer necessary to repeat each step, which simplifies the procedure.

6. Conflicts of Interest Statement

The author has no conflicts of interest to disclose.

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