

DOI: <https://doi.org/10.24297/jap.v22i.9608>**A mathematical expression to predict a laser pulse shape**

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**Abstract**

A mathematical expression to predict the shape of a laser pulse is obtained. It is compatible with an experimental one published elsewhere. Comparative computations with other published trials that follow the same trend are made. As a result, our trial is recommended for forthcoming theoretical studies to be published in the field of pulsed laser heating.

**Keywords** Laser heating, Pulsed laser, Prediction of laser pulse shape, Comparative study.

**1. Introduction**

In setting up the problem of a theoretical study of heating effects induced by pulsed laser in an irradiated target it is necessary to have a mathematical expression for the shape of the laser pulse  $q(t)$ ,  $W/m^2$  emitted from the laser generator.

This prediction is important for technological applications.

Ready [1] as a result of experimental technique published a laser pulse shape.

This shape has been accepted by other authors [3,4] who published in their researches mathematical expression compatible with the shape of laser pulse published by Ready.

The aim of the present trial is to introduce a new expression to predict the same laser pulse which may be of a simpler mathematical form that is urgent for theoretical treatment of the problems dealing with pulsed laser interaction with solid targets.

This is essential to determine theoretically the temperature field within the irradiated target and to find the variation of the front surface temperature with the exposure time.

This also makes it possible to determine the critical time  $t_{cr}$  required to initiate damage (i.e. Melting) at the front layer.

This is important to avoid damage in the irradiated targets.

On the other hand, it is important to proceed to study the two phase (solid-liquid) or three phase problems (solid-liquid-vapor) at the considered irradiated materials.

This trend is of vital technological importance.

**Derivation of basic equations**

The analysis of the published laser pulse [1] revealed that  $q(t)$ ,  $W/m^2$  satisfies the following conditions:

$$i) \quad \text{at } t = 0 \quad q(0) = 0 \quad (1)$$

$$ii) \quad \text{at } t = t_0 \quad q(t_0) = q_{max} \quad (2)$$

$$\text{And} \quad \left. \frac{\partial q}{\partial t} \right|_{t=t_0} = 0 \quad (3)$$

$$iii) \quad \text{at } t = t_d \quad q(t_d) = 0 \quad (4)$$

where the pulse parameters are:

1-  $q_{max}$ ,  $W/m^2$  the maximum power density of the emitted pulse at  $t_0$ .

2-  $t_0$ , sec is the time collapsed to reach  $q_{max}$ .

3-  $\Delta t = t_d$ , sec the pulse time duration.



M.K.El-Adawi et al [4], suggests a shape in the form:

$$q(t) = \beta q_{max} \left(\frac{t}{t_d}\right) \left(1 - \frac{t}{t_d}\right) \exp \exp - B \left(\frac{t-t_0}{t_d}\right) \quad (5)$$

considering the condition (1-4) the following parameters are obtained:

$$\beta = \frac{1}{\left(\frac{t_0}{t_d}\right)\left(1-\frac{t_0}{t_d}\right)}, \quad (6)$$

$$B = \beta \left(1 - 2\left(\frac{t_0}{t_d}\right)\right) \quad (7)$$

Moreover, HASSAN et al [3] introduced a semi-empirical formula for the same laser pulse suggested in the form:

$$q(t) = q_{max} \left[ \frac{(n+1)^{n+1}}{n^n} \left(\frac{t}{t_d}\right) \left(1-\frac{t}{t_d}\right)^n \right] \quad [3] (8)$$

Computations carried out by the authors [3] for different n-values show that the function q(t) is revealed for n=3 to be more representative of the published laser pulse shape published by ready [1].

In the present trial the laser irradiance q(t), W\m<sup>2</sup> is suggested in the form:

$$q(t) = (1 - R) q_{max} \cdot A \left(\frac{t}{t_0}\right)^n \exp \exp - \left(\frac{t}{t_d-t}\right)^2 \quad (9)$$

Applying the conditions (1-4) one gets

$$A = \frac{1}{(1-R) \exp - \left(\frac{t_0}{t_d-t_0}\right)^2} \quad (10)$$

While,

$$n = \frac{2 t_0^2 t_d}{(t_d-t_0)^3} \quad (11)$$

The function q(t), W\m<sup>2</sup> is computed considering the parameters from the published pulse [1] which are as follows:

$$q_{max} = 1.38 \times 10^{11} \text{ W\m}^2$$

$$t_d = 45 \times 10^{-9} \text{ sec}$$

$$t_0 = 10 \times 10^{-9} \text{ sec}$$

While,

$$(1-R) = 0.7 [2],$$

As a result, one gets n = 0.209 and A = 1.55.

In order to modify the obtained parameters to get better fitting a least square fitting technique is applied to both data, that of Ready [1] and that computed according to equation (9) with n=0.209.

The considered technique gives a modified value of n to be n=0.28.

Thus, finally our model is rewritten in the form:

$$q(t) = q_{max} \cdot (0.7)(1.55) \left(\frac{t}{t_0}\right)^{0.28} \exp \exp - \left(\frac{t}{t_d-t}\right)^2 \quad (12)$$

Computations are made considering eq. (12) and comparison with the published data [1] is obtained and is given in table (1) and illustrated graphically in fig. (1).

Moreover, the experimental pulse published by Ready [1] is reproduced using the formulae published by El-Adawi et al. [4] and Hassan et al. [3].

For both cases the degree of fitting  $\epsilon = \frac{q_{cal} - q_{exp}}{q_{exp}} \%$  relative in each case to Ready is obtained. The whole data are given in table (1) and the scatter of such points are illustrated also in Fig. (1).

**Table (1)** Comparative computations with other authors:

t (ns)	Ready [1]	Present work	$\epsilon$ Present work	[4]	$\epsilon$	[3]	$\epsilon$
5	108.5	121.40	11.8	112.7	3.87	102.10	5.89
8	133.34	134.23	0.66	134.63	0.96	129.30	3.12
10	137.8	137.9	0.07	138	0.14	136.80	0.73
13	128.82	136.62	6.05	132.3	2.7	135.92	5.51
15	117.5	130.6	11.14	124.14	5.65	129.22	9.97
17	103.96	120.15	15.57	113.83	9.49	119.07	14.53
20	83.62	95.8	14.5	96.5	15.4	99.77	19.31
23	51.98	63.37	21.98	78.82	51.63	78.14	50.32

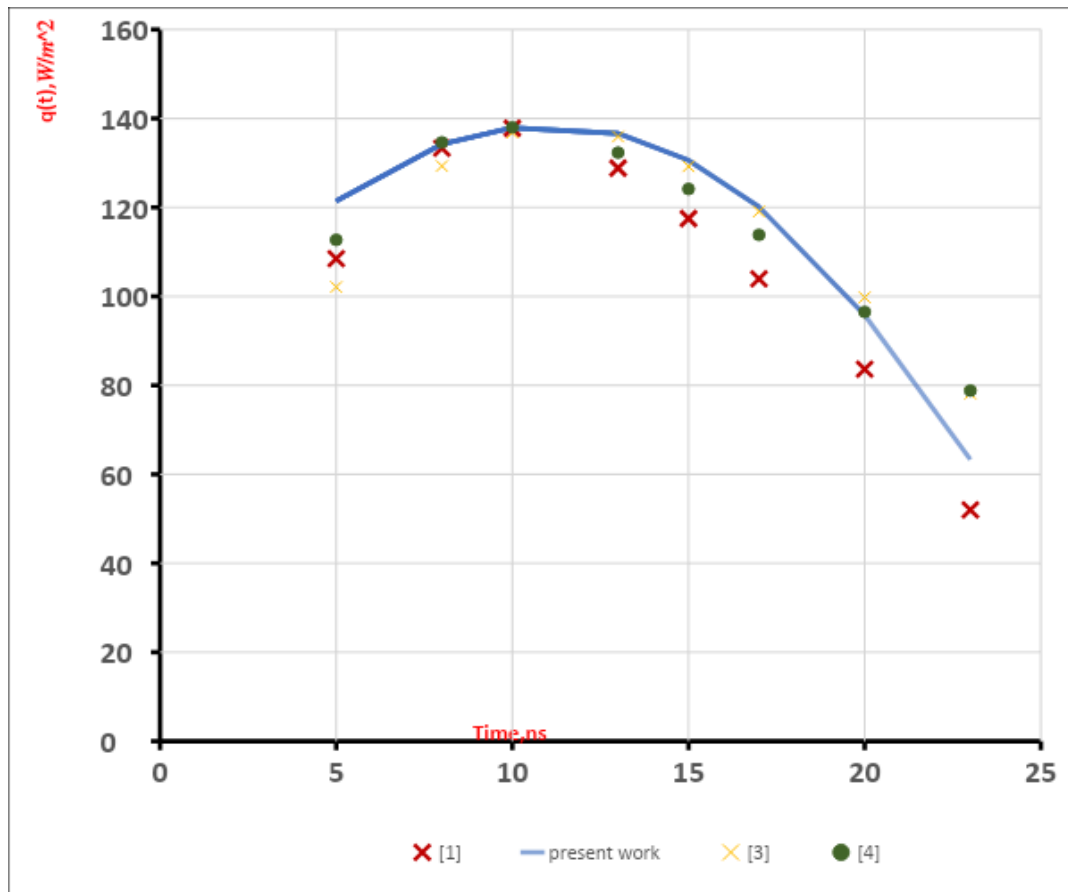


Figure (1). Laser pulse with illustrative other published data for comparison.

**Conclusions**



The comparative study revealed that the our present trial and the published trials [3,4] to predict the experimental pulse published by Ready [1] are obtained with different degrees of accuracy as shown in table (1).

Our trial is not the best, nevertheless, our suggested pulse makes the theoretical treatment of the heating problem easier from our point of view and will be accepted in our forthcoming papers. This advantage makes it possible to recommend our trial to be considered with other authors.

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