DOI; https://doi.org/10.24297/jap.v22i.9597

Gauge Abelian Models for Symetry Objects

Renato M. Doria, Samuel d. S. Freitas

Abstract:

This work explores gauge symmetry for the so-called symmetry object. The investigation associates gauge symmetry to a collection of fields. Different fields are boxed inside of a symmetry object as a matrix. A systemic gauge symmetry is proposed. It introduces a new procedure where gauge symmetry should not be restricted to Yang-Mills procedure. Two fields arrangements are taken. The N-matter fields and M- mediators fields. These two symmetry objects are connected through an abelian gauge symmetry. The corresponding generator becomes a matricial charge Q rotating as $U = e^{iQ\alpha}$ where a means the abelian gauge parameter. It yields a collective gauge transformation.

Thus, instead of considering the Yang-Mills canonical procedure where the number of gauge fields is equal to the number of gauge generators, this work develops an abelian gauge theory for symmetry objects. The cases where Q is 2x2 and 3x3 matrices are studied. By consequence, instead of developing gauge models where three and eight mediators should be associated to SU(2) and SU(3) groups, are develops their behaviors in terms M an abelian

Introduction

Yang Mills theory expresses a golden rule for symmetry in physics[]. It connects gauge group and conservation laws given that every symmetry groups contains parameters, generators, generators algebra, one gets that following the YM procedure the corresponding physics will depend on the group choice.

For YM methodology, the phenomenology will depend on the group choice. The cross sections will depend on the corresponding structure constants, the number of gauge fields must be equal to the number of group generators, and so on.

Our argument here is different. We disconnect this strict connection between gauge symmetry and group theory. Our starting point is that the number of mediators is indif- ferent to the number of gauge parameters. Our principle is that symmetry is independent to the number of fields involved.

At this work are studies a non-defined number of fields through a common gauge parameter. For this, are defines as symmetry objetcs as fields encapsulated in mathematical elements of a matrix. Thus instead of looking for just are original simetries as string theories we are considering are gauge parameter associated to symmetry objetcs.

Focusing on the abelian case, one takes as origin a matricial charge Q, which means a gauge transformation $U = e^{iQ\alpha}$ on N-matter fields. The cases 2x2 and 3x3 will be studied at sections 2 and 3. Instead of considering the classical situation where three and eight mediators should be associated to SU(2) and SU(3), they will be performed under a common abelian gauge paremeter. At conclusion.

We studied the following cases:

A study was made between sets of mediating fields and their generators.

Study with 4 fields and Pauli's matrices:

The goal in this step is to place any generic 2x2 matrix in terms of the Pauli and Idetity matrices.

Pauli Matrix Properties:

$$tr(\sigma_i) = 0,\ tr(\sigma_i)\sigma_j = 2\delta_{ij}$$

A generic "M" array can be rewritten as:

$$\mathrm{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \alpha \mathbb{1} + \beta_i \sigma i$$

Calculating the trace of "M" to find the value of "α"

$$tr(M) = \alpha tr(\mathbb{1}) + \beta_i \underbrace{tr(\sigma_i)}_{=0} \tag{1}$$

$$a + d = \alpha 2 \tag{2}$$

$$\alpha = \frac{a+d}{2} \tag{3}$$

To find the value of "β_i", we must manipulate "M" with "σ_i" and calculate the trace of this operation.

$$\sigma_i M = \alpha \sigma_i \mathbb{1} + \beta_i \sigma_j \sigma_i \tag{4}$$

$$tr(\sigma_j M) = \alpha tr \underbrace{(\sigma_j \mathbb{1})}_{=0} + \beta_i tr(\sigma_j \sigma_i) \tag{5}$$

$$tr(\sigma_j M) = \beta_i 2\delta_{ij} \tag{6}$$

contracting indices with the Kronecker delta:

$$tr(\sigma_i M) = \beta_i 2 \tag{7}$$

$$\beta_i = \frac{tr(\sigma_i M)}{2} \tag{8}$$

Expanding $M = \alpha\mathbb{1} + \beta_i\sigma_i$:

$$\text{M} * \sigma_1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

$$=> \beta_1 = \frac{b+c}{2} \tag{9}$$

$$\text{M} * \sigma_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} ib & -ia \\ id & -ic \end{bmatrix}$$

$$=> \beta_2 = \frac{i(b-c)}{2} \tag{10}$$

$$\text{M} * \sigma_3 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} a & -b \\ c & -d \end{bmatrix}$$

$$=> \beta_3 = \frac{a-d}{2} \tag{11}$$

$$\therefore M = \alpha\mathbb{1} + \beta_i\sigma_i = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$(\frac{a+d}{2}) * \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (\frac{b+c}{2}) * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + (\frac{i(b-c)}{2}) * \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + (\frac{a-d}{2}) * \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Proving that $M = \alpha\mathbb{1} + \beta_i\sigma_i$:

$$- > M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \ \alpha = \frac{tr(M)}{2}, \ \beta_i = \frac{tr(M\sigma_i)}{2}$$

$$=> M = \underbrace{\frac{tr(M\mathbb{1})}{2}}_{=0} + \frac{tr(M\sigma_1)}{2}\sigma_1 + \frac{tr(M\sigma_2)}{2}\sigma_2 + \frac{tr(M\sigma_3)}{2}\sigma_3$$

Calculating the values of "β_i":

$$M * \sigma_1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\beta_1 = \frac{tr(M\sigma_1)}{2} = 0 \tag{12}$$

$$M * \sigma_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

$$\beta_2 = \frac{tr(M\sigma_2)}{2} = 0 \tag{13}$$

$$M * \sigma_3 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\beta_3 = \frac{tr(M\sigma_3)}{2} = 1 \tag{14}$$

$$\therefore M = 1 * \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The coefficients "α" and "β_i" actually satusfy the matrix equation.

Writing "M" in terms of Pauli's Sigmas arrays and "$\mathbb{1}$" in a new example:

$$- > M = \frac{tr(M\mathbb{1})}{2} + \frac{tr(M\sigma_i)}{2}\sigma_i$$

$$- > M = \begin{bmatrix} i & 3+4i \\ \alpha i + \beta & c + di \end{bmatrix}, \; tr(M) = c + i + di$$

$$\alpha = \frac{c + (d+1)i}{2} \tag{15}$$

$$M * \sigma_1 = \begin{bmatrix} i & 3+4i \\ \alpha i + \beta & c + di \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3+4i & i \\ c+di & \alpha i + \beta \end{bmatrix}$$

$$\beta_1 = \frac{tr(M\sigma_1)}{2}$$

$$\beta_1 = \frac{3 + \beta + i(4 + \sigma)}{2} \tag{16}$$

$$M * \sigma_2 = \begin{bmatrix} i & 3+4i \\ \alpha i + \beta & c + di \end{bmatrix} * \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 3i - 4 & 1 \\ ci - d & \alpha - \beta i \end{bmatrix}$$

$$\beta_2 = \frac{tr(M\sigma_2)}{2}$$

$$=> \beta_2 = \frac{\alpha - 4 + i(3-\beta)}{2} \tag{17}$$

$$M * \sigma_3 = \begin{bmatrix} i & 3+4i \\ \alpha i + \beta & c+di \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} i & -3-4i \\ \alpha i + \beta & -c-di \end{bmatrix}$$

$$\beta_3 = \frac{tr(M\sigma_3)}{2}$$

$$=> \beta_3 = \frac{c - i(1+d)}{2} \tag{18}$$

$$\therefore M = [\frac{c+(d+1)i}{2}]\mathbb{1} + [\frac{3+\beta+i(4+\alpha)}{2}]\sigma_1 + [\frac{\alpha-4+i(3-\beta}{2}]\sigma_2 + [\frac{-c-i(1+d)}{2}]\sigma_3$$

To put a matrix in terms of Pauli's Sigmas matrices is to give it a physical interpretation.

Placing a Matrix in tems of Pauli's Sigmas matrices and "Ladder Operators" Will be essential in calculations involving rotations over the studied fields, since a matrix in these terms acts as a generator of the rotation group.

An aray composed of field loads:

$$Q = \begin{bmatrix} q1 & q2 \\ q3 & q4 \end{bmatrix},\ \alpha = \frac{q1+q2}{2} = a_0,\ \beta_i = a_i$$

Finding the coefficiets:

$$Q * \sigma_1 = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} q_2 & q_1 \\ q_4 & q_3 \end{bmatrix}$$

$$a_1 = \frac{tr(Q\sigma_1)}{2}$$

$$=> a_1 = \frac{q_2 + q_3}{2} \tag{19}$$

$$Q * \sigma_2 = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} * \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} q_2 i & q_1 i \\ q_4 i & q_3 i \end{bmatrix}$$

$$a_2 = \frac{tr(Q\sigma_2)}{2}$$

$$=> a_2 = \frac{(q_2 + q_3)i}{2} \tag{20}$$

$$Q * \sigma_3 = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} q_1 & -q_2 \\ q_3 & -q_4 \end{bmatrix}$$

$$a_3 = \frac{tr(Q\sigma_3)}{2}$$

$$=> a_3 = \frac{q_1 - q_4}{2} \tag{21}$$

$$\therefore Q = a_0\mathbb{1} + a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3 \tag{22}$$

Using the properties of the ladder operators:

$$- > \sigma_+ = \sigma_1 + i\sigma_2 \; and \; \sigma_- = \sigma_1 - i\sigma_2$$

$$Q = a_0\mathbb{1} + a_3\sigma_3 + a_+\sigma_+ + a_-\sigma_- \tag{23}$$

An "N" field of matter is given by:

$$N = \begin{bmatrix} a \\ b \end{bmatrix}$$

And a set of mediating fields is given by:

$$\mathbf{A}_{\mu I} = \begin{bmatrix} \gamma A_\mu + \delta A^3_\mu & A^1_\mu + iA^2_\mu \\ A^1_\mu + iA^2_\mu & \gamma A_\mu + \delta A^3_\mu \end{bmatrix}$$

Both fields will be rotated by group $U = e^{iQ\alpha}$, where Q it is the group generator, as previously studied. And the goal is to find the gauge transformation for these fields,

A covariant derivative is made to analyze how N behaves over curvatures of the $\mathbf{A}_{\mu I}$ field, according to their interactions.

The N gauge transformation is given by:

$$N^{'} = UN \tag{24}$$

$$(\mathbf{D}_\mu N)' = [\partial_\mu + i\mathbf{A}^{'}_{\mu I}]N^{'} \tag{25}$$

Using similarity transformation:

$$\mathbf{A}'_{\mu I} = U\mathbf{A}_{\mu I}U^{-1} + ai\partial_\mu UU^{-1} \tag{26}$$

Replacing $A'_{\mu I}$ in $(\mathbf{D}_\mu N)'$

$$- > (\mathbf{D}_\mu N)' = [\partial_\mu + iU\mathbf{A}_{\mu I}U^{-1} - a\partial_\mu UU^{-1}]UN$$

$$- > (\mathbf{D}_\mu N)' = U\partial_\mu N + \partial_\mu UN + iU\mathbf{A}_{\mu I}U^{-1}UN - a\partial_\mu UU^{-1}UN$$

$$=> (\mathbf{D}_\mu N)' = U\partial_\mu N + \partial_\mu UN + iU\mathbf{A}_{\mu I}N - aU\partial_\mu N + a\partial_\mu UN$$

To satisfy the operation, the value from "a" must be 1:
a=1:

$$\therefore (\mathbf{D}_\mu N)' = U[\partial_\mu + i\mathbf{A}_{\mu I}]N = U(\mathbf{D}_\mu N)$$

So the field $\mathbf{A}_{\mu I}$ is now given by:

$$=> \mathbf{A}'_{\mu I} = U\mathbf{A}_{\mu I}U^{-1} + i\partial_\mu UU^{-1} \tag{27}$$

Now you have to manipulate $\partial_\mu U$, to start puting $\mathbf{A}_{\mu I}$ in terms of the α symmetry parameter and the Q generator.

Remembering that:

$$U = e^{iQ\alpha} \; and \; Q = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix}$$

Since $e^{iQ\alpha}$ is a matrix exponential, it can be rewritten as:

$$e^{iQ\alpha} = 1 + iQ\alpha + ... + \frac{(iQ\alpha)^n}{n!}$$

Therefore:

$$- > \partial_\mu U = iQ\partial_\mu\alpha + ... + \frac{(iQ)^n}{n!}\partial_\mu\alpha^n$$

$$\partial_\mu U = iQ\partial_\mu \alpha[1 + ... + \frac{(iQ)^{n-1}}{(n-1)!}\alpha^{n-1}]$$

$$=> \partial_\mu U = iQ\partial_\mu \alpha U$$

The derivative occours only in α is a function of space and time, $\alpha = \alpha(x, t)$.

Note that by deriving U from μ, you get U again. So, $\mathbf{A}_{\mu I}$ is:

$$- > \mathbf{A}'_{\mu I} = U\mathbf{A}_{\mu I}U^{-1} + i^2 Q\partial_\mu \alpha U U^{-1}$$

$$=> \mathbf{A}'_{\mu I} = U\mathbf{A}_{\mu I}U^{-1} - Q\partial_{\mu\alpha}$$

We now have $\mathbf{A}_{\mu I}$, partly, in terms of α and Q

Using U as $e^{iQ\alpha}$ and adding a term $\mathbf{O}(\alpha^2)$ infinitesimal, to put all $\mathbf{A}_{\mu I}$ in terms of α and Q

$$- > \mathbf{A}'_{\mu I} = e^{iQ\alpha}\mathbf{A}_{\mu I}e^{-iQ\alpha} - Q\partial_\mu \alpha$$

$$- > \mathbf{A}'_{\mu I} = (1 + iQ\alpha)\mathbf{A}_{\mu I}(1 - iQ\alpha) - Q\partial_\mu \alpha + \mathbf{O}(\alpha^2)$$

$$- > \mathbf{A}'_{\mu I} = (\mathbf{A}_{\mu I} + iQ\alpha\mathbf{A}_{\mu I})(1 - IQ\alpha) - Q\partial_\mu \alpha$$

$$- > \mathbf{A}'_{\mu I} = \mathbf{A}_{\mu I} - \mathbf{A}_{\mu I}iQ\alpha + iQ\alpha\mathbf{A}_{\mu I} - \underbrace{i^2 Q^2 \mathbf{A}_{\mu I}\alpha^2}_{=0} - Q\partial_\mu \alpha$$

$$- > \mathbf{A}'_{\mu I} = \mathbf{A}_{\mu I} - i\alpha[\mathbf{A}_{\mu I}, Q] - Q\partial_\mu \alpha$$

$$=> \mathbf{A}'_{\mu I} = \mathbf{A}_{\mu I} + i\alpha[Q, \mathbf{A}_{\mu I}] - Q\partial_\mu \alpha \tag{28}$$

We then get an expression for $\mathbf{A}_{\mu I}$ field gauge transformation. Note a switch in the expression. It must be calculated.

Using the switches:

$$[\sigma_3, \sigma_+] = \sigma_+, [\sigma_3, \sigma_-] = -\sigma_-, [\sigma_+, \sigma_-] = \sigma_3$$

We calculate:

$$-> [Q, \mathbf{A}_{\mu I}] = [a_0 \mathbb{1} + a_3\sigma_3 + a_+\sigma_+ a_-\sigma_-, \gamma\mathbf{A}_\mu + \delta\mathbf{A}^3_\mu\sigma_3 + \mathbf{A}^+_\mu\sigma_+ + \mathbf{A}^-_\mu\sigma_-]$$

$$-> [Q, \mathbf{A}_{\mu I}] = a_0\gamma\mathbf{A}_\mu \underbrace{[\mathbb{1},\mathbb{1}]}_{=0} + a_0\delta\mathbf{A}^3_\mu \underbrace{[\mathbb{1},\sigma_3]}_{=0} + a_0\mathbf{A}^+_\mu \underbrace{[\mathbb{1},\sigma_+]}_{=0} + a_0\mathbf{A}^-_\mu \underbrace{[\mathbb{1},\sigma_-]}_{=0}$$

$$+a_3\gamma\mathbf{A}_\mu \underbrace{[\sigma_3,\mathbb{1}]}_{=0} + a_3\delta\mathbf{A}^3_\mu \underbrace{[\sigma_3,\sigma_3]}_{=0} + a_3\mathbf{A}^+_\mu \underbrace{[\sigma_3,\sigma_+]}_{=0} + a_3\mathbf{A}^-_\mu \underbrace{[\sigma_3,\sigma_-]}_{=0}$$

$$+a_+\gamma\mathbf{A}_\mu \underbrace{[\sigma_+,\mathbb{1}]}_{=0} + a_+\delta\mathbf{A}^3_\mu \underbrace{[\sigma_+,\sigma_3]}_{=0} + a_+\mathbf{A}^+_\mu \underbrace{[\sigma_+,\sigma_+]}_{=0} + a_+\mathbf{A}^-_\mu \underbrace{[\sigma_+,\sigma_-]}_{=0}$$

$$+a_-\gamma\mathbf{A}_\mu \underbrace{[\sigma_-,\mathbb{1}]}_{=0} + a_-\delta\mathbf{A}^3_\mu \underbrace{[\sigma_-,\sigma_3]}_{=0} + a_-\mathbf{A}^+_\mu \underbrace{[\sigma_-,\sigma_+]}_{=0} + a_-\mathbf{A}^-_\mu \underbrace{[\sigma_-,\sigma_-]}_{=0}$$

$$=> [Q, \mathbf{A}_{\mu I}] = (a_+A^-_\mu - a_-A^+_\mu)\sigma_3 + (a_3A^+_\mu - a_+\delta A^3_\mu)\sigma_+ + (a_\delta - A^3_\mu - a_3A^-_\mu)\sigma_-$$

From this result we can rewrite $\mathbf{A}'_{\mu I}$ in matrix form:

$$-> \mathbf{A}'_{\mu I} = \mathbf{A}_{\mu I} + i\alpha[Q, \mathbf{A}_{\mu I}] - Q\partial_\mu\alpha$$

$$\begin{bmatrix} \gamma\mathbf{A}'_\mu + \delta\mathbf{A^3}'_\mu & \mathbf{A}^{+'}_\mu \\ \mathbf{A}^{-'}_\mu & \gamma\mathbf{A}'_\mu - \delta\mathbf{A^3}'_\mu \end{bmatrix} = \begin{bmatrix} \gamma\mathbf{A}_\mu + \delta\mathbf{A^3}_\mu & \mathbf{A}^+_\mu \\ \mathbf{A}^-_\mu & \gamma\mathbf{A}_\mu - \delta\mathbf{A^3}_\mu \end{bmatrix}$$

$$+i\alpha \begin{bmatrix} a_+A^-_\mu - a_-A^+_\mu & a_3A^+_\mu - a_+\delta A^3_\mu \\ a_-\delta A^3_\mu - a_3A^-_\mu & -(a_+A^-_\mu - a_-A^+_\mu) \end{bmatrix} - \begin{bmatrix} a_0 + a_3 & a_+ \\ a_- & a_0 - a_3 \end{bmatrix} \partial_\mu\alpha$$

a system of 4 equations is obtained when we analyze each component:

$$\gamma A'_\mu + \delta A^{3'}_\mu = \gamma A_\mu + \delta A^3_\mu + i\alpha(a_+A^-_\mu - a_-A^+_\mu) - (a_0 + a_3)\partial_\mu\alpha \tag{29}$$

$$\gamma A'_\mu - \sigma A^{3'}_\mu = \gamma A_\mu - i\alpha(a_+A^-_\mu - a_-A^+_\mu) - (a_0 - a_3)\partial_\mu\alpha \tag{30}$$

$$A^{+'}_\mu = A^+_\mu + i\alpha(a_3A^+_\mu - a_+\delta A^3_\mu) - a_+\partial_\mu\alpha \tag{31}$$

$$A^{-'}_\mu = A^-_\mu - i\alpha(a_-A^3_\mu - a_-\delta A^-_\mu) - a_-\partial_\mu\alpha \tag{32}$$

You need to isolate the fields and try to make them as independent as possible.

The general form of gauge transformations in 4 fields is given by:

$$-> A'_\mu = A_\mu + e^{i\alpha(A_\mu)}A_\mu + A_\mu\partial_\mu\alpha$$

0.1 Manipulating the equations:

Adding (29) and (30) and multiplying by a_3:

$$- > a_3 \gamma A_\mu^{'} = a_3 \gamma A_\mu - a_3 a_0 \partial_\mu \alpha \tag{33}$$

$$A_\mu^{'} = A_\mu + k_1 \partial_\mu \alpha$$

Subtracting (29) and (30) and multiplying by a_0:

$$a_0 \delta A_\mu^{3'} = a_0 \delta A_\mu^3 + i\alpha a_0 (a_+ A_\mu^- - a_- A_\mu^+) - a_0 a_3 \partial \mu \alpha \tag{34}$$

Expression (33) is a good expression because it depends only on the field itself and the gauge parameter.

Subtracting (33) and (34):

$$- > a_3 \gamma A_\mu^{'} - a_0 \delta A_\mu^{3'} - a_3 \gamma A_\mu - a_0 \delta A_\mu^3 - i\alpha a_0 (a_+ A_\mu^- - a_- A_\mu^+)$$

$$Z_\mu^{'} = Z_\mu - i\alpha a_0 (a_+ A_\mu^- - a_- A_\mu^+) \tag{35}$$

Multiplying (31) by a_- and (32) by a_+:

$$a_- A_\mu^{+'} = a_- A_\mu^+ + i\alpha a_- (a_3 A_\mu^+ - a_+ \delta A_\mu^3) - a_- a_+ \partial_\mu \alpha \tag{36}$$

$$a_+ A_\mu^{-'} = a_+ A_\mu^- - i\alpha a_+ (a_- A_\mu^3 - a_- \delta A_\mu^-) - a_+ a_- \partial_\mu \alpha \tag{37}$$

Subtracting (36) and (37):

$$a_- A_\mu^{+'} - a_+ A_\mu^{-'} = a_- A_\mu^+ - a_+ A_\mu^- + i\alpha a_3 (a_- A_\mu^+ + a_+ A_\mu^-) \tag{38}$$

One can, through this expression, obtain the fields $A_\mu^{+'}$ and $A_\mu^{-'}$:

$$- > A_\mu^{+'} = A_\mu^+ + i\alpha a_3 A_\mu^+$$

$$=> A_\mu^{+'} = A_\mu^+ e^{i\alpha a_3} \tag{39}$$

$$-> A_\mu^{-'} = A_\mu^- - i\alpha a_3 A_\mu^-$$

$$=> A_\mu^{-'} = A_\mu^- e^{-i\alpha a_3} \tag{40}$$

We then get a new set of fields:

$$=> A_{\mu I}\{A_\mu^{'}, Z_\mu^{'}, A_\mu^{+'}, Z_\mu^{-'}, \} \tag{41}$$

Study whith 3 fields and Pauli's matrice:

$$-> A_{\mu I}^{'} = A_{\mu I} + i\alpha[Q, A_{\mu I}] - Q\partial_\mu \alpha$$

$$\begin{bmatrix} A_\mu^{3'} & A_\mu^{+'} \\ A_\mu^{-'} & -A_\mu^{3'} \end{bmatrix} = \begin{bmatrix} A_\mu^3 & A_\mu^+ \\ A_\mu^- & -A_\mu^3 \end{bmatrix} + i\alpha \begin{bmatrix} a_+A_\mu^- - a_-A_\mu^+ & a_3A_\mu^+ - a_+A_\mu^3 \\ a_-A_\mu^3 - a_3A_\mu^- & -(a_+A_\mu^- - a_-A_\mu^+) \end{bmatrix}$$
$$- \begin{bmatrix} a_0 + a_3 & a_+ \\ a_- & a_0 - a_3 \end{bmatrix} \partial_\mu \alpha$$

The system of 4 equations obtained when we analyze each component:

$$=> A_\mu^{3'} + A_\mu^3 + i\alpha(a_+A_\mu^- - a_-A_\mu^+) - (a_0 + a_3)\partial_\mu \alpha \tag{42}$$

$$=> -A_\mu^{3'} = -A_\mu^3 - i\alpha(a_+A_\mu^- - a_-A_\mu^+) - (a_0 + a_3)\partial_\mu \alpha \tag{43}$$

$$=> A_\mu^{+'} = A_\mu^+ + i\alpha(a_3A_\mu^+ - a_+A_\mu^3) - a_+\partial_\mu \alpha \tag{44}$$

$$=> A_\mu^{-'} = A_\mu^- - i\alpha(a_3A_\mu^- - a_-A_\mu^3) - a_-\partial_\mu \alpha \tag{45}$$

Subtracting (42) and (43):

$$2A_\mu^{3'} = 2A_\mu^3 + i\alpha 2(a_+A_\mu^- - a_-A_\mu^+) - 2a_3\partial_\mu \alpha$$

$$A_\mu^{3'} = A_\mu^3 + i\alpha(a_+A_\mu^- - a_-A_\mu^+) + k_1\partial_\mu \alpha \tag{46}$$

Multiplying (44) by a_- and (45) by a_+

$$=> a_- A_\mu^{+'} = a_- A_\mu^+ + i\alpha a_-(a_3 A_\mu^+ - a_+ A_\mu^3) - a_- a_+ \partial_\mu \alpha \tag{47}$$

$$=> a_+ A_\mu^{-'} = a_+ A_\mu^- - i\alpha a_+(a_3 A_\mu^- - a_- A_\mu^3) - a_+ a_- \partial_\mu \alpha \tag{48}$$

Subtracting (47) and (48):

$$=> a_- A_\mu^{+'} - a_+ A_\mu^{-'} = a_- A_\mu^+ - a_+ A_\mu^- + i\alpha a_3(a_- A_\mu^+ + a_+ A_\mu^-) \tag{49}$$

getting the fields $A_\mu^{+'}$ and $A_\mu^{-'}$

$$- > A_\mu^{+'} = A_\mu^+ + i\alpha a_3 A_\mu^+$$

$$=> A_\mu^{+'} = A_\mu^+ e^{i\alpha a_3} \tag{50}$$

$$- > A_\mu^{-'} = A_\mu^- - i\alpha a_3 A_\mu^-$$

$$=> A_\mu^{-'} = A_\mu^- e^{-i\alpha a_3} \tag{51}$$

The new set of fields:

$$a_{\mu I} : \{A_\mu^3, A_\mu^+, A_\mu^-\} \tag{52}$$

Study for 4 fields and the Q $= \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$ **generator, through Fierz representation and decomposition:**

$$- > Q = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \alpha = \frac{tr(Q)}{2} = -\frac{1}{2}, Q = \alpha \mathbb{1} + \beta_i \sigma_i$$

Calculating the values of "β_i"

$$Q * \sigma_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$$

$$=> \beta_1 = \frac{tr(Q\sigma_1)}{2} = 0 \tag{53}$$

$$Q * \sigma_2 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -i & 0 \end{bmatrix}$$

$$=> \beta_2 = \frac{tr(Q\sigma_2)}{2} = 0 \tag{54}$$

$$Q * \sigma_3 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=> \beta_3 = \frac{tr(Q\sigma_3)}{2} = \frac{1}{2} \tag{55}$$

$$\therefore Q = -\frac{1}{2}\mathbb{1} + \frac{1}{2}\sigma_3$$

Set of 4 fields:

$$A_{\mu I} = \begin{bmatrix} A_\mu + A_\mu^3 & A_\mu^1 + iA_\mu^2 \\ A_\mu^1 + iA_\mu^2 & A_\mu + A_\mu^3 \end{bmatrix}$$

Calculating the swtich $[Q, A_{\mu I}]$:

$$[Q, A_{\mu I}] = [-\frac{1}{2}\mathbb{1} + \frac{1}{2}\sigma_3, A_\mu\mathbb{1} + A_\mu^3\sigma_3 + A_\mu^+\sigma_+ + +A_\mu^-\sigma_-]$$

$$- > [Q, A_{\mu I}] = -\frac{1}{2}A_\mu \underbrace{[\mathbb{1}, \mathbb{1}]}_{=0} - \frac{1}{2}A_\mu^3 \underbrace{[\mathbb{1}, \sigma_3]}_{=0} - \frac{1}{2}A_\mu^+ \underbrace{[\mathbb{1}, \sigma_+]}_{=0} - \frac{1}{2}A_\mu^- \underbrace{[\mathbb{1}, \sigma_-]}_{=0}$$
$$+\frac{1}{2}A_\mu \underbrace{[\sigma_3, \mathbb{1}]}_{=0} + \frac{1}{2}A_\mu^3 \underbrace{[\sigma_3, \sigma_3]}_{=0} + \frac{1}{2}A_\mu^+ \underbrace{[\sigma_3, \sigma_+]}_{=\sigma_+} + \frac{1}{2}A_\mu^- \underbrace{[\sigma_3, \sigma_-]}_{=-\sigma_-}$$

Using gauge transformation:

$$- > A'_{\mu I} = A_{\mu I} + i\alpha[Q, A_{\mu I}] - Q\partial\mu\alpha$$

$$=> \begin{bmatrix} A'_\mu + A_\mu^{3'} & A_\mu^{+'} \\ A_\mu^{-'} & -A'_\mu - A_\mu^{3'} \end{bmatrix} = \begin{bmatrix} A_\mu + A_\mu^3 & A_\mu^+ \\ A_\mu^- & A_\mu - A_\mu^3 \end{bmatrix}$$
$$+i\alpha \begin{bmatrix} 0 & \frac{1}{2}A_\mu^+ \\ -\frac{1}{2}A_\mu^- & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \partial_\mu\alpha$$

The system of equations:

$$=> A_\mu^{'} + A_\mu^{3'} = A_\mu + A_\mu^3 \tag{56}$$

$$=> A_\mu^{'} - A_\mu^{3'} = A_\mu - A_\mu^3 + \partial_\mu \alpha \tag{57}$$

$$- > A_\mu^{+'} = A_\mu^+ + i\alpha \frac{1}{2} A_\mu^+$$

$$A_\mu^{+'} = A_\mu^+ e^{i\alpha \frac{1}{2}} \tag{58}$$

$$- > A_\mu^{-'} = A_\mu^- - i\alpha \frac{1}{2} A_\mu^-$$

$$A_\mu^{-'} = A_\mu^- e^{-i\alpha \frac{1}{2}} \tag{59}$$

Adding (56) and (57):

$$- > 2A_\mu^{'} = 2A_\mu + \partial_\mu \alpha$$

$$=> A_\mu^{'} = A_\mu + k_1 \partial_\mu \alpha \tag{60}$$

Subtracting (56) and (57):

$$- > 2A_\mu^{3'} = 2A_\mu^3 + \partial_\mu \alpha$$

$$=> A_\mu^{'} = A_\mu + k_1 \partial_\mu \alpha \tag{61}$$

The new set of fields:

$$=> A_{\mu I} : \{A_\mu, A_\mu^3, A_\mu^+, A_\mu^-\} \tag{62}$$

THe result obtained with 3 fields is similar except that it does not contain $A_\mu^{'} = A_\mu + k_1 \partial_\mu \alpha$

Study with the triplet of fields and the "Σ" matrices.:

$$- > A_{\mu I} = \begin{bmatrix} 0 & -iA_\mu^3 & iA_\mu^2 \\ iA_\mu^3 & 0 & -iA_\mu^1 \\ -iA_\mu^2 & iA_\mu^1 & 0 \end{bmatrix}$$

$$- > Q = \begin{bmatrix} 0 & -ia_3 & ia_2 \\ ia_3 & 0 & -ia_1 \\ -ia_2 & ia_1 & 0 \end{bmatrix} = a_i\Sigma_i$$

Calculating the switch:

$$- > [Q, A_{\mu I}] = [a_1\Sigma_1 + a_2\Sigma_2 + a_3\Sigma_3, A_\mu^1\Sigma_1 + A_\mu^2\Sigma_2 + A_\mu^3\Sigma_3]$$

$$- > [Q, A_{\mu I}] = a_1A_\mu^1\underbrace{[\Sigma_1,\Sigma_1]}_{=0} + a_1A_\mu^2\underbrace{[\Sigma_1,\Sigma_2]}_{=\Sigma_3} + a_1A_\mu^3\underbrace{[\Sigma_1,\Sigma_3]}_{=-\Sigma_2}$$
$$a_2A_\mu^1\underbrace{[\Sigma_2,\Sigma_1]}_{=-\Sigma_3} + a_2A_\mu^2\underbrace{[\Sigma_2,\Sigma_2]}_{=0} + a_2A_\mu^3\underbrace{[\Sigma_2,\Sigma_3]}_{=\Sigma_1}$$
$$a_3A_\mu^1\underbrace{[\Sigma_3,\Sigma_1]}_{=\Sigma_2} + a_3A_\mu^2\underbrace{[\Sigma_3,\Sigma_2]}_{=-\Sigma_1} + a_3A_\mu^3\underbrace{[\Sigma_3,\Sigma_3]}_{=0}$$

$$=> [Q, A_{\mu I}] = (a_2A_\mu^3 - a_3A_\mu^2)\Sigma_1 + (a_3A_\mu^1 - a_1A_\mu^3)\Sigma_2 + (a_1A_\mu^2 - a_2A_\mu^1)\Sigma_3$$

The gauge transformation:

$$- > A_{\mu I}^{'} = A_{\mu I} + i\alpha[Q, A_{\mu I}] - Q\partial_\mu\alpha$$

$$=> \begin{bmatrix} 0 & -iA_\mu^{3'} & iA_\mu^{2'} \\ iA_\mu^{3'} & 0 & -iA_\mu^{1'} \\ -iA_\mu^{2'} & iA_\mu^{1'} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -iA_\mu^3 & iA_\mu^2 \\ iA_\mu^3 & 0 & -iA_\mu^1 \\ -iA_\mu^2 & iA_\mu^1 & 0 \end{bmatrix} +$$
$$i^2\alpha \begin{bmatrix} 0 & -(a_1A_\mu^2 - a_2A_\mu^1) & a_3A_\mu^1 - a_1A_\mu^3 \\ a_1A_\mu^2 - a_2A_\mu^1 & 0 & -(a_2A_\mu^3 - a_3A_\mu^2) \\ -(a_3A_\mu^1 - a_1A_\mu^3) & a_2A_\mu^3 - a_3A_\mu^2 & 0 \end{bmatrix} - i\begin{bmatrix} 0 & -ia_3 & ia^2 \\ ia^3 & 0 & -ia^1 \\ -ia^2 & ia^1 & 0 \end{bmatrix}\partial_\mu\alpha$$

The system of equations:

$$- > -iA_\mu^{3'} = -iA_\mu^3 - i^2\alpha(a_1A_\mu^2 - a_2A_\mu^1) + ia_3\partial_\mu\alpha$$

Dividing for "$(-i)$":

$$=> A_{\mu}^{3'} = A_{\mu}^{3} + i\alpha(a_1 A_{\mu}^{2} - a_2 A_{\mu}^{1}) - a_3 \partial_{\mu}\alpha \tag{63}$$

$$- > -iA_{\mu}^{2'} = -iA_{\mu}^{2} - i^2\alpha(a_3 A_{\mu}^{1} - a_1 A_{\mu}^{3}) + ia_2 \partial_{\mu}\alpha$$

$$=> A_{\mu}^{2'} = A_{\mu}^{2} + i\alpha(a_3 A_{\mu}^{1} - a_1 A_{\mu}^{3}) - a_2 \partial_{\mu}\alpha \tag{64}$$

$$- > -iA_{\mu}^{1'} = -iA_{\mu}^{1} - i^2\alpha(a_2 A_{\mu}^{3} - a_3 A_{\mu}^{2}) + ia_1 \partial_{\mu}\alpha$$

$$=> A_{\mu}^{1'} = A_{\mu}^{1} + i\alpha(a_2 A_{\mu}^{3} - a_3 A_{\mu}^{2}) - a_1 \partial_{\mu}\alpha \tag{65}$$

The results obtained are not good for collective field theory, with a symmetry.

Study with 0 fields, 9 generators and the Gell-Mann matrices:

$$- > A_{\mu I} = \frac{1}{2}\begin{bmatrix} A_{\mu}^{3} + \frac{1}{\sqrt{3}}A_{\mu}^{8} & A_{\mu+}^{1} & A_{\mu+}^{2} \\ A_{\mu-}^{1} & -A_{\mu}^{3} + \frac{1}{\sqrt{3}}A_{\mu}^{8} & A_{\mu+}^{3} \\ -A_{\mu-}^{2} & A_{\mu-}^{3} & -\frac{2}{\sqrt{3}}A_{\mu}^{8} \end{bmatrix}$$

$$- > Q = \begin{bmatrix} a_3 + a_8 + a_9 & a_{+}^{1} & a_{+}^{2} \\ a_{-}^{1} & a_8 + a_9 - a_3 & a_{+}^{3} \\ a_{-}^{2} & a_{-}^{3} & a_9 - a_8 \end{bmatrix}$$

Calculating the switch:

$$- > [Q, A_{\mu I}] = [a_3\lambda_3 + a_8\lambda_8 + a_9\lambda_9 + a_{+}^{1}\lambda_{+}^{1} + a_{+}^{2}\lambda_{+}^{2} + a_{+}^{3}\lambda_{+}^{3} + a_{-}^{1}\lambda_{-}^{1} + a_{-}^{2}\lambda_{-}^{2} + a_{-}^{3}\lambda_{-}^{3},$$

$$A_{\mu}^{3}\lambda_3 + A_{\mu}^{8}\lambda_8 + A_{\mu+}^{1}\lambda_{+}^{1} + A_{\mu+}^{2}\lambda_{+}^{2} + A_{\mu+}^{3}\lambda_{+}^{3} + A_{\mu-}^{1}\lambda_{-}^{1} + A_{\mu-}^{2}\lambda_{-}^{2} + A_{\mu-}^{3}\lambda_{-}^{3}]$$

$$- > [Q, A_{\mu I}] = a_3(A_{\mu+}^{1}2\lambda_{+}^{1} + A_{\mu+}^{2}\lambda_{+}^{2} + A_{\mu+}^{3}(-\lambda_{+}^{3}) + A_{\mu-}^{1}(-2\lambda_{-}^{1}) + A_{\mu-}^{2}(-\lambda_{-}^{2}) + A_{\mu-}^{3}\lambda_{-}^{3})$$

$$+a_8(A_{\mu+}^{2}\sqrt{3}\lambda_{+}^{2} + A_{\mu+}^{3}\sqrt{3}\lambda_{+}^{3} + A_{\mu-}^{2} - \sqrt{3}\lambda_{-}^{2} + A_{\mu-}^{3} - \sqrt{3}\lambda_{-}^{3}) + a_9(0)$$

$$+a_{+}^{1}(A_{\mu}^{3}(-2)\lambda_{+}^{1} + A_{\mu+}^{3}\lambda_{+}^{2} + A_{\mu-}^{1}\lambda_3 + A_{\mu-}^{2}(-\lambda_{-}^{3}))$$

$$+a_{+}^{2}(A_{\mu}^{3}(-\lambda_{+}^{2})+A_{\mu}^{8}(-\sqrt{3})\lambda_{+}^{2}+A_{\mu-}^{1}(-\lambda_{+}^{3})+A_{\mu-}^{2}M_{1}+A_{\mu-}^{3}\lambda_{+}^{1})$$

$$+a_{+}^{3}(A_{\mu}^{3}\lambda_{+}^{3}+A_{\mu}^{8}(-\sqrt{3})\lambda_{+}^{3}+A_{\mu+}^{1}(-\lambda_{+}^{2})+A_{\mu-}^{2}\lambda_{-}^{1}+A_{\mu-}^{3}M_{2})$$

$$+a_{-}^{1}(A_{\mu}^{3}2\lambda_{-}^{1}+A_{\mu+}^{1}(-\lambda_{3})+A_{\mu+}^{2}\lambda_{+}^{3}+A_{\mu-}^{3}(-\lambda_{-}^{2}))$$

$$+a_{-}^{2}(A_{\mu}^{3}\lambda_{-}^{2}+A_{\mu}^{8}\sqrt{3}\lambda_{-}^{2}+A_{\mu+}^{1}\lambda_{-}^{3}+A_{\mu+}^{2}(-M1)+A_{\mu+}^{3}(-\lambda_{-}^{1}))$$

$$+a_{-}^{3}(A_{\mu}^{3}(-\lambda_{-}^{3})+A_{\mu}^{8}\sqrt{3}\lambda_{-}^{3}+A_{\mu+}^{2}(-\lambda_{+}^{1})+A_{\mu+}^{3}(-M_{2})+A_{\mu-}^{1}\lambda_{-}^{2})$$

$$=>[Q,A_{\mu I}]=\lambda_{3}(a_{+}^{1}A_{\mu-}^{1}-a_{-}^{1}A_{\mu+}^{1}+\lambda_{8}(0)$$

$$+\lambda_{+}^{1}(a_{3}A_{\mu+}^{1}2+a_{+}^{1}A_{\mu}^{3}(-2)+a_{+}^{2}A_{\mu-}^{3}-a_{-}^{3}A_{\mu+}^{2})$$

$$+\lambda_{+}^{2}(a_{3}A_{\mu+}^{2}+a_{8}A_{\mu+}^{2}\sqrt{3}+a_{+}^{1}A_{\mu+}^{3}-a_{+}^{2}A_{\mu}^{3}+a_{+}^{2}A_{\mu}^{8}(-\sqrt{3})-a_{+}^{3}A_{\mu+}^{1})$$

$$+\lambda_{+}^{3}(-a_{3}A_{\mu+}^{3}+a_{8}A_{\mu+}^{3}\sqrt{3}-a_{+}^{2}A_{\mu-}^{1}+a_{+}^{3}A_{\mu}^{3}+a_{+}^{3}A_{\mu}^{8}(-\sqrt{3})+a_{+}^{1}A_{\mu+}^{2})$$

$$+\lambda_{-}^{1}(a_{3}A_{\mu-}^{1}(-2)+a_{+}^{3}A_{\mu-}^{2}+a_{-}^{1}A_{\mu}^{3}2-a_{-}^{2}A_{\mu+}^{3})$$

$$+\lambda_{-}^{2}(-a_{3}A_{\mu-}^{2}+a_{8}A_{\mu-}^{2}(-\sqrt{3})-a_{-}^{1}A_{\mu-}^{3}+a_{-}^{2}A_{\mu}^{3}+a_{-}^{2}A_{\mu}^{8}\sqrt{3}+a_{-}^{3}A_{\mu-}^{1})$$

$$+\lambda_{-}^{3}(a_{3}A_{\mu-}^{3}+a_{8}A_{\mu-}^{3}(-\sqrt{3})-a_{+}^{1}A_{\mu-}^{2}+a_{-}^{2}A_{\mu+}^{1}+a_{-}^{3}A_{\mu}^{3}+a_{-}^{3}A_{\mu}^{8}\sqrt{3})$$

$$M_{1}(a_{+}^{2}A_{\mu-}^{2}-a_{-}^{2}A_{\mu+}^{2})$$

$$M_{2}(a_{+}^{3}A_{\mu-}^{3}-a_{-}^{3}A_{\mu+}^{3})$$

Arrays, "M_1" and "M_2", obtained from the switch:

$$- > M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The matrix equation:

$$=> \frac{1}{2}\begin{bmatrix} A_\mu^{3'} + \frac{1}{\sqrt{3}}A_\mu^{8'} & A_{\mu+}^{1'} & A_{\mu+}^{2'} \\ A_{\mu-}^{1'} & -A_\mu^{3'} + \frac{1}{\sqrt{3}}A_\mu^{8'} & A_{\mu+}^{3'} \\ A_{\mu-}^{2'} & A_{\mu-}^{3'} & -\frac{2}{\sqrt{3}}A_\mu^{8'} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} A_\mu^{3} + \frac{1}{\sqrt{3}}A_\mu^{8} & A_{\mu+}^{1} & A_{\mu+}^{2} \\ A_{\mu-}^{1} & -A_\mu^{3} + \frac{1}{\sqrt{3}}A_\mu^{8} & A_{\mu+}^{3} \\ A_{\mu-}^{2} & A_{\mu-}^{3} & -\frac{2}{\sqrt{3}}A_\mu^{8} \end{bmatrix}$$

$$i\alpha\frac{1}{2}\begin{bmatrix} \gamma & \tau & \phi \\ \rho & \theta & \xi \\ \eta & \varsigma & \omega \end{bmatrix} + \begin{bmatrix} a_3 + a_8 + a_9 & a_+^1 & a_+^2 \\ a_-^1 & a_8 + a_9 - a_3 & a_+^3 \\ a_-^2 & a_-^3 & a_8 - a_9 \end{bmatrix} \partial_\nu \alpha$$

Where:

$$- > \gamma = (a_+^1 A_{\mu-}^1 - a_-^1 A_{\mu+}^1 + a_+^2 A_{\mu-}^2 - a_-^2 A_{\mu+}^2)$$

$$- > \tau = (a_3 A_{\mu+}^1 2 + a_+^1 A_{\mu+}^3(-2) + a_+^2 A_{\mu-}^3 - a_-^3 A_{\mu+}^2)$$

$$- > \phi = (a_3 A_{\mu+}^2 + a_8 A_{\mu+}^2 \sqrt{3} + a_+^1 A_{\mu+}^3 - a_+^2 A_\mu^3 + a_+^2 A_\mu^8(-\sqrt{3}) - a_+^3 A_{\mu+}^1)$$

$$- > \xi = (-a_3 A_{\mu+}^3 + a_8 A_{\mu+}^3 \sqrt{3} - a_+^2 A_{\mu-}^1 + a_+^3 A_\mu^3 + a_+^3 A_\mu^8(-\sqrt{3}) + a_-^1 A_{\mu+}^2)$$

$$\theta = (a_+^3 A_{\mu-}^3 - a_-^3 A_{\mu+}^3) - (a_+^1 A_{\mu-}^1 - a_-^1 A_{\mu+}^1)$$

$$\rho = (a_3 A_{\mu-}^1(-2) + a_+^3 A_{\mu-}^2 - a_-^1 A_\mu^3 2 - a_-^2 A_{\mu+}^3)$$

$$\eta = (-a_3 A_{\mu-}^2 + a_8 A_{\mu-}^2(-\sqrt{3}) - a_-^1 A_{\mu-}^3 + a_-^2 A_\mu^3 2 + a_-^2 A_\mu^8 \sqrt{3} + a_-^3 A_{\mu-}^1)$$

$$\varsigma = (a_3 A_{\mu-}^3 + a_8 A_{\mu-}^3(-\sqrt{3}) - a_+^1 A_{\mu-}^2 + a_-^2 A_{\mu+}^1 - a_-^3 A_\mu^3 + a_-^3 A_\mu^8 \sqrt{3})$$

$$\omega = -(a_+^2 A_{\mu-}^2 - a_-^2 A_{\mu+}^2) - (a_+^3 A_{\mu-}^3 - a_-^3 A_{\mu+}^3)$$

The resulting equation:

$$=> A_{\mu}^{3'} = A_{\mu}^{3} + i\alpha(2(a_{+}^{1}A_{\mu-}^{1} - a_{-}^{1}A_{\mu+}^{1}) + a_{+}^{2}A_{\mu-}^{2} - a_{+}^{2}A_{\mu-}^{2} - a_{+}^{3}A_{\mu-}^{3} - a_{-}^{3}A_{\mu+}^{3})a^{3}4\partial_{\mu}\alpha \tag{66}$$

$$=> A_{\mu}^{8'} = A_{\mu}^{8} + i\alpha\frac{\sqrt{3}}{2}(a_{+}^{2}A_{\mu-}^{2} - a_{-}^{2}A_{\mu+}^{2} + a_{+}^{3}A_{\mu-}^{3} - a_{-}^{3}A_{\mu+}^{3}) - a_{8}4\sqrt{3}\partial_{\mu}\alpha \tag{67}$$

$$=> A_{\mu-}^{1'} = A_{\mu-}^{1} + i\alpha(a_{3}A_{\mu-}^{1}(-2) + a_{+}^{3}A_{\mu-}^{2} + a_{-}^{1}A_{\mu}^{3}2 - a_{-}^{2}A_{\mu+}^{3}) + a_{-}^{1}2\partial_{\mu}\alpha \tag{68}$$

$$=> A_{\mu-}^{2'} = A_{\mu-}^{2} + i\alpha(-a_{3}A_{\mu-}^{2} + a_{8}A_{\mu-}^{2}(-\sqrt{3}) - a_{-}^{1}A_{\mu-}^{3} + a_{-}^{2}A_{\mu}^{3} + a_{-}^{2}A_{\mu}^{8}\sqrt{3} + a_{-}^{3}A_{\mu-}^{1}) + a_{-}^{2}2\partial_{\mu}\alpha \tag{69}$$

$$=> A_{\mu-}^{3'} = A_{\mu-}^{3} + i\alpha(a_{3}A_{\mu-}^{3} + a_{8}A_{\mu-}^{3}(-\sqrt{3}) - a_{+}^{1}A_{\mu-}^{2} + a_{-}^{2}A_{\mu+}^{1} - a_{-}^{3}A_{\mu}^{3} + a_{-}^{3}A_{\mu}^{8}\sqrt{3}) + a_{-}^{3}2\partial_{\mu}\alpha \tag{70}$$

$$=> A_{\mu+}^{1'} = A_{\mu+}^{1} + i\alpha(a_{3}A_{\mu+}^{1}2 + a_{+}^{1}A_{\mu}^{3}(-2) + a_{+}^{2}A_{\mu-}^{3} - a_{-}^{3}A_{\mu+}^{2}) + a_{+}^{1}2\partial_{\mu}\alpha \tag{71}$$

$$=> A_{\mu+}^{2'} = A_{\mu+}^{2} + i\alpha(a_{3}A_{\mu+}^{2} + a_{8}A_{\mu+}^{2}\sqrt{3} - a_{+}^{1}A_{\mu+}^{3} - a_{+}^{2}A_{\mu}^{3} + a_{+}^{2}A_{\mu}^{8}(-\sqrt{3}) - a_{+}^{3}A_{\mu+}^{1}) + a_{+}^{2}2\partial_{\mu}\alpha \tag{72}$$

$$=> A_{\mu+}^{3'} = A_{\mu+}^{3} + i\alpha(-a_{3}A_{\mu+}^{3} + a_{8}A_{\mu+}^{3}\sqrt{3} - a_{+}^{2}A_{\mu-}^{1} + a_{+}^{3}A_{\mu}^{3} + a_{+}^{3}A_{\mu}^{8}(-\sqrt{3}) + a_{-}^{1}A_{\mu+}^{2}) + a_{-}^{3}2\partial_{\mu}\alpha \tag{73}$$

$$T = \begin{bmatrix} a & b & 0 & \cdots & 0 & a & b & 0 & \cdots & 0 \\ c & a & b & \ddots & \vdots & c & a & b & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 & 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & c & a & b & v\vdots & \ddots & c & a & b \\ 0 & \cdots & 0 & c & a & 0 & \cdots & 0 & c & a \end{bmatrix}_{NxN} \tag{74}$$

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \alpha\mathbb{1} + \beta_{i}\sigma i\epsilon_{0} \tag{75}$$

$$\begin{bmatrix} \mathbf{A'} \\ vw' \\ w' \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

References

[1] La Plaga de Tux, LaTeX: Ecuaciones con matrices, http://plagatux.es/2008/11/latex-ecuaciones-con-matrices 12 de noviembre de 2008.