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The Effect of the Earth's Rotation and Gravity on the Speed of Light

Miloš Čojanović
independent author
Email:mcojanovic@yahoo.com

Abstract

Based on the results of the $Michelson\breve{}Morley$ experiment [1] and the direct measurement of the speed of light in two directions, we can conclude that the movement of the Earth in relation to the Sun and the movement of the solar system in relation to the center of the galaxy do not affect the measurement of the speed of light in the vicinity of the Earth. But on the other hand, based on results of the $Michelson\breve{}Morley$ experiment and the direct measurement of the speed of light in two directions we cannot know with certainty whether the Earth's rotation around its axis affects the measurement of the speed of light in one direction. Thus, instead of the **motion** by which the Earth's surface moves in relation to the Sun, we will observe the **angular motion** by which the Earth rotates on its axis relative to distant stars. Instead of measuring the speed of light in two directions, we will measure the speed of light in one direction.

Keywords: *Angular motion, Michelson–Morley experiment, Michelson-Gale experiment, Speed of light*

1. Introduction

We can state that it has already been proven by the $Michelson - Gale$ experiment [2] that the rotation of the Earth affects the measurement of the speed of light. Therefore, it would be logically correct to consider the $Michelson\breve{}Morley$ experiment on the one side and the $Michelson - Gale$ experiment the other side as the two complementary experiments. Instead of that, we made the logical mistake ignoring the results of the $Michelson - Gale$ experiment, giving greater importance to the $Michelson\breve{}Morley$ experiment. Things are quite different. Based on the results of the $Michelson - Gale$ experiment [2], we can predict the result of the $Michelson\breve{}Morley$ experiment [1], but not the other way around. It has also already been proven [3] that the velocity of light, with respect to an observer resting on the Earth surface, is $c + v$ between New York and San Francisco and $c - v$ between San Francisco and New York, where $v \approx 280[m/sec]$. The speed of rotation of the Earth at the latitude of those cities is $\approx 360[m/sec]$.

2. Angular velocity

We will start with a brief introduction about angular velocity and after that derive the formula for the addition of angular velocities in the case that the center of the circle moves along an arbitrary curve. After that, we will apply the obtained results to calculate the Earth's angular velocity in relation to the stars.

The Angular motion can be defined as the motion of the point A along a Angular path, Figure[1]. We will mark the center of the circle with O, and with P the fixed point outside the circle. We will define a rectangular coordinate system so that the point O is its origin and the positive $x-axis$ is determined by the direction OP

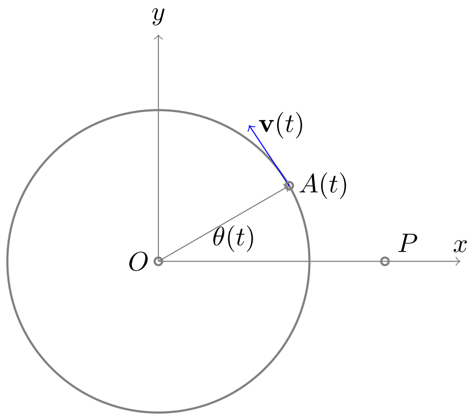

Figure 1: Point A moves with a uniform angular velocity relative to the fixed point O

The angle between the positive $x-axis$ and the direction OA will be denoted by θ. We will define the angle θ as a function of time.

$$\theta = \angle(OP, OA) \tag{1}$$

$$\theta = \theta(t) \tag{2}$$

Angular velocity ω is defined as the change in the angle of rotation of the point A with respect to time.

$$\omega(t) = \frac{\delta\theta}{\delta t} = \theta^{'}(t) \tag{3}$$

$$if \left(\omega^{'}(t) = 0\right) \Longrightarrow (\omega(t) = \omega) \tag{4}$$

If $\left(\omega^{'}(t) = 0\right)$ then we will say that point A moves with a uniform angular velocity.

$$r = OA \tag{5}$$

$$\theta = \omega t \tag{6}$$

$$\mathbf{OA} = \mathbf{r}(t) = r\left[\cos(\theta), \sin(\theta)\right] = r\left[\cos(\omega t), \sin(\omega t)\right] \tag{7}$$

$$v = r\omega \tag{8}$$

$$\mathbf{v}(t) = \frac{\delta \mathbf{r}(t)}{\delta t} = r\omega[-\sin(\omega t), \cos(\omega t)] = v[-\sin(\theta), \cos(\theta)] \tag{9}$$

3. Addition of angular velocities

We will define a rectangular coordinate system noted by (K) so that the point O is its origin and the positive $x-axis$ is determined by the direction OZ. The point Z is the "infinitely" distant point, Figure[2]. We will define a rectangular coordinate system noted by (K') so that the point S is its origin and the positive $x'-axis$ is determined by the direction OS. We will define a rectangular coordinate system noted by (K'') so that the point S is its origin and the positive $x-axis$ is determined by the direction SZ.

Let us assume that the point S moves with a uniform angular velocity w_1 about the point O and the point A moves with a uniform angular velocity w_0 about the point S .

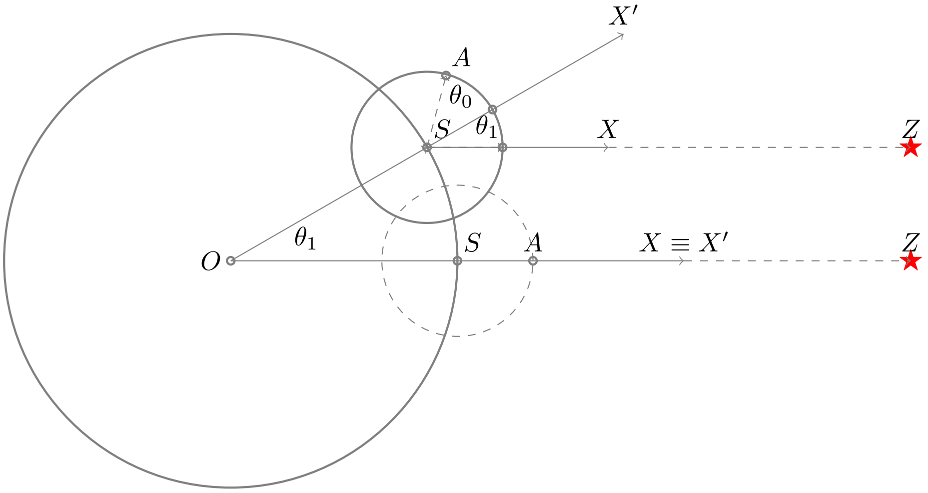

Figure 2: The point S moves with a uniform angular velocity w_1 about the point O and the point A moves with a uniform angular velocity w_0 about the point S.

We will derive a formula for the summation of the angular velocities.

$$r = SA \tag{10}$$

$$\theta_0 = \angle(SX', SA) \tag{11}$$

$$\theta_0 = \omega_0 t \tag{12}$$

$$\theta_1 = \angle(OX, OS) = \angle(SX, SX') \tag{13}$$

$$\theta_1 = \omega_1 t \tag{14}$$

$$\tag{15}$$

If the angular velocity w_1 has the same direction as the angular velocity w_0, we will have the following equations:

$$\theta = \theta_0 + \theta_1 \tag{16}$$

$$w = w_0 + w_1 \tag{17}$$

$$\tag{18}$$

If the angular velocity w_1 has the opposite direction in relation to the angular velocity w_0, then it is obvious that we have the following equations

$$\theta = \theta_0 - \theta_1 \tag{19}$$

$$w = w_0 - w_1 \tag{20}$$

The velocity $\mathbf{v}(t)$ with which the point A moves in relation to the coordinate system (K'') is given by the following equations:

$$\theta(t) = wt \tag{21}$$

$$\mathbf{r}(t) = \mathbf{SA} = r\,[\cos(\theta(t)), \sin(\theta(t))] = r\,[\cos(\omega t), \sin(\omega t)] \tag{22}$$

$$\mathbf{v}(t) = \frac{\delta \mathbf{r}(t)}{\delta t} = r\omega[-\sin(\omega t), \cos(\omega t)] \tag{23}$$

The velocity $\mathbf{v}(t)$ with which the point$A$ moves in relation to the coordinate system (K') is given by the following equations:

$$\theta(t) = w_0 t \tag{24}$$

$$\mathbf{r}(t) = \mathbf{SA} = r\,[\cos(\theta), \sin(\theta)] = r\,[\cos(w_0 t), \sin(w_0 t)] \tag{25}$$

$$\mathbf{v}(t) = \frac{\delta \mathbf{r}(t)}{\delta t} = r w_0[-\sin(w_0 t), \cos(w_0 t)] \tag{26}$$

The velocity $\mathbf{v}(t)$ with which the point$A$ moves in relation to the coordinate system (K) is given by the following equations:

$$OS = R \tag{27}$$

$$\theta(t) = wt \tag{28}$$

$$\mathbf{r}(t) = \mathbf{OA} + \mathbf{SA} \tag{29}$$

$$\mathbf{r}(t) = R\,[\cos(\theta_1), \sin(\theta_1)] + r\,[\cos(\theta), \sin(\theta)] = R\,[\cos(w_1 t), \sin(w_1 t)] + r\,[\cos(wt), \sin(wt)] \tag{30}$$

$$\mathbf{v}(t) = \frac{\delta \mathbf{r}(t)}{\delta t} = R w_1[-\sin(w_1 t), \cos(w_1 t)] + rw[-\sin(wt), \cos(wt)] \tag{31}$$

4. Addition of angular velocities-general formula

Let a $\mathbf{R}(t)$ be "sufficiently" differentiable curve. We consider the point S that moves along the curve $\mathbf{R}$ and the point A that moves uniformly on a circle whose center is given by point S, Figure[3]. Let us assume that the point S moves with a uniform angular velocity w_1 about the point O and the point A moves with a uniform angular velocity w_0 about the point S.

We will define a rectangular coordinate system noted by (K) so that the point O is its origin and the positive $x-axis$ is determined by the direction OZ. The point Z is the "infinitely" distant point. We will define a rectangular coordinate system noted by (K'') so that the point S is its origin and the positive $x-axis$ is determined by the direction SZ.

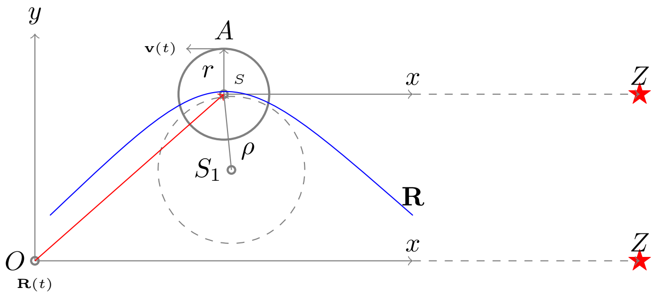

Figure 3: The point S moves along the curve $\mathbf{R}$

In relation to the coordinate system (K), We have the following equations

$$r = SA \tag{32}$$

$$\mathbf{R}(t) = \mathbf{OS} = [\mathbf{R}_x(t), \mathbf{R}_y(t)]] \tag{33}$$

$$\mathbf{v}_R(t) = \dot{\mathbf{R}}(t) = [\dot{\mathbf{R}}_x(t), \dot{\mathbf{R}}_y(t)] \tag{34}$$

$$\mid \mathbf{v}_R(t) \mid = \mid \dot{\mathbf{R}}(t) \mid \tag{35}$$

$$\ddot{\mathbf{R}}(t) = [\ddot{\mathbf{R}}_x(t), \ddot{\mathbf{R}}_y(t)] \tag{36}$$

The curvature of $\mathbf{R}$ at t denoted by κ is given by

$$\kappa = \frac{\mid \dot{\mathbf{R}}(t) \times \ddot{\mathbf{R}}(t) \mid}{\mid \dot{\mathbf{R}}(t) \mid^3} = \frac{\mid \dot{\mathbf{R}}_x(t)\ddot{\mathbf{R}}_y(t) - \dot{\mathbf{R}}_y(t)\ddot{\mathbf{R}}_x(t) \mid}{\mid \dot{\mathbf{R}}(t) \mid^3} \tag{37}$$

The radius of curvature at t, denoted by ρ is given by

$$\frac{1}{\rho} = \kappa \tag{38}$$

$$S_1 S = \rho \tag{39}$$

Let us denote by $\omega_R(t)$ the angular velocity with which the point S moves along the circle whose center is the point S_1.

$$\mid \mathbf{v}_R(t) \mid = \omega_R \rho \tag{40}$$

$$\omega_R(t) = \frac{\mid \mathbf{v}_R(t) \mid}{\rho} = \mid \dot{\mathbf{R}}(t) \mid \kappa = \frac{\mid \dot{\mathbf{R}}(t) \times \ddot{\mathbf{R}}(t) \mid}{\mid \dot{\mathbf{R}}(t) \mid^2} = \frac{\mid \dot{\mathbf{R}}_x(t)\ddot{\mathbf{R}}_y(t) - \dot{\mathbf{R}}_y(t)\ddot{\mathbf{R}}_x(t) \mid}{\dot{\mathbf{R}}_x^2(t) + \dot{\mathbf{R}}_y^2(t)} \tag{41}$$

If we apply equation (23) in relation to the coordinate system (K'') we have the following equations

$$\omega(t) = \omega_r + \omega_R(t) \tag{42}$$

$$\mid \mathbf{v}(t) \mid = \omega(t) r \tag{43}$$

5. Angular velocity at which the center of the Earth moves relative to the Sun

Denote by (Q) "The Geocentric-Equatorial Coordinate System" Figure[4]. Its origin O_q is at the center of the Earth, the fundamental plane is the equator and the positive x_q points in the vernal equinox direction. The z_q points in the direction of the north pole. By the definition the Coordinate System (Q) is non-rotating with the respect to the stars. Denote by (P) "The Heliocentric-Ecliptic Coordinate System". Its origin O_p is centered on the center of mass of the solar system, and the fundamental plane coincides with the *ecliptic* plane of the Earth's revolution about the Sun. Its $x_p - axis$ has the same direction as $x_q - axis$.

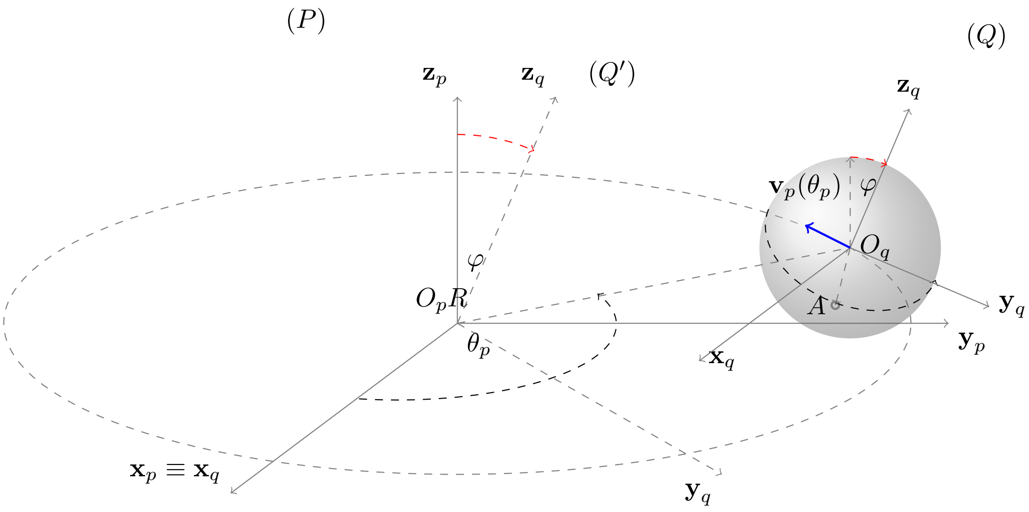

Figure 4: Ecliptic Coordinate System (P) and Equatorial Coordinate System (Q)

Let $\varphi = 23.43693 * \Pi/180$ denotes Earth's axial tilt Figure[4].

Denote by (Q') the coordinate system created by rotating the coordinate system (P) around the $\mathbf{x}_p - axis$ by an angle φ in the negative direction.

$$yearsec = 365.2422 * 24 * 3600[sec] \tag{44}$$

$$O_pO_q = AU = 1.495978707e11[m] \tag{45}$$

$$R = O_pO_q \tag{46}$$

$$\omega_p = \frac{2\Pi}{yearsec} = 1.9910637973E-07 \tag{47}$$

$$\theta_p = \angle(O_p x_p, O_pO_q) = \frac{2\Pi t}{yearsec} = \omega_p t \tag{48}$$

$$v_p = R\omega_p = 29785.89 \left[\frac{m}{sec}\right] \tag{49}$$

$$\mathbf{R}_p(\theta_p) = \mathbf{O}_p\mathbf{O}_q \tag{50}$$

$$\mathbf{R}_p(\theta_p) = R[\cos(\theta_p), \sin(\theta_p), 0] = R[\cos(\omega_p t), \sin(\omega_p t), 0] \tag{51}$$

Conversion from Ecliptic Coordinates to Equatorial Coordinates is defined as follows:

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{bmatrix} \tag{52}$$

$$\tag{53}$$

We will transform the coordinates of the vector $\mathbf{R}_p(t)$ from the coordinate system (P) to the coordinate system (Q'). The vector obtained by this transformation we denote with $\mathbf{R}_q(t)$.

$$\mathbf{R}_q(t) = \mathbf{A}_1\mathbf{R}_p(t) \tag{54}$$

$$\mathbf{R}_q(t) = \begin{bmatrix} R_x(t) \\ R_y(t) \\ R_z(t) \end{bmatrix} = R \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} \cos(\omega_p t) \\ \sin(\omega_p t) \\ 0 \end{bmatrix} = R \begin{bmatrix} \cos(\omega_p t) \\ \cos(\varphi)\sin(\omega_p t) \\ \sin(\varphi)\sin(\omega_p t) \end{bmatrix} \tag{55}$$

We can find the first and second derivative of the vector $\mathbf{R}_q(t)$.

$$\dot{\mathbf{R}}_q = \frac{\delta \mathbf{R}_p}{\delta t} = R\omega_p \begin{bmatrix} -\sin(\omega_p t) \\ \cos(\varphi)\cos(\omega_p t) \\ \sin(\varphi)\cos(\omega_p t) \end{bmatrix} = R\omega_p \begin{bmatrix} -\sin(\theta_p) \\ \cos(\varphi)\cos(\theta_p) \\ \sin(\varphi)\cos(\theta_p) \end{bmatrix} \tag{56}$$

$$\ddot{\mathbf{R}}_q = \frac{\delta \dot{\mathbf{R}}_q}{\delta t} = -R\omega_p^2 \begin{bmatrix} \cos(\omega_p t) \\ \cos(\varphi)\sin(\omega_p t) \\ \sin(\varphi)\sin(\omega_p t) \end{bmatrix} = -R\omega_p^2 \begin{bmatrix} \cos(\theta_p) \\ \cos(\varphi)\sin(\theta_p) \\ \sin(\varphi)\sin(\theta_p) \end{bmatrix} \tag{57}$$

In relation to the coordinate system (Q'), we define the three curves that we will denote by $\mathbf{R}_{xy}(t)$, $\mathbf{R}_{yz}(t)$ and $\mathbf{R}_{zx}(t)$.

$$\mathbf{R}_{xy}(t) = \begin{bmatrix} R_x(t) \\ R_y(t) \\ 0 \end{bmatrix} \tag{58}$$

$$\mathbf{R}_{yz}(t) = \begin{bmatrix} 0 \\ R_y(t) \\ R_z(t) \end{bmatrix} \tag{59}$$

$$\mathbf{R}_{zx}(t) = \begin{bmatrix} R_x(t) \\ 0 \\ R_z(t) \end{bmatrix} \tag{60}$$

The angular velocity noted by $\omega_{pxy}(t)$ of the curve $R_{xy}(t)$ is given by equation

$$\omega_{pxy} = \frac{\mid \dot{R}_x \ddot{R}_y - \dot{R}_y \ddot{R}_x \mid}{\dot{R}_x^2 + \dot{R}_y^2} = \frac{R^2\omega_p^3\left(\sin^2(\theta_p)\cos(\varphi) + \cos^2(\theta_p)\cos(\varphi)\right)}{R^2\omega_p^2\left(\sin^2(\theta_p) + \cos^2(\theta_p)\cos^2(\varphi)\right)} = \frac{\cos(\varphi)}{1 - \sin^2(\varphi)\cos^2(\theta_p)}\omega_p \tag{61}$$

$$\omega_{pxy}(t) = \frac{\cos(\varphi)}{1 - \sin^2(\varphi)\cos^2(\omega_p t)}\omega_p \tag{62}$$

$$\tag{63}$$

The angular velocity noted by $\omega_{pyz}(t)$ of the curve $R_{yz}(t)$ is given by equation

$$\omega_{pyz} = \frac{\mid \dot{R}_y \ddot{R}_z - \dot{R}_z \ddot{R}_y \mid}{\dot{R}_y^2 + \dot{R}_z^2} = \frac{-R^2\omega_p^3\left(\cos(\varphi)\cos(\theta_p)\sin(\varphi)\sin(\theta_p) - \sin(\varphi)\cos(\theta_p)\cos(\varphi)\sin(\theta_p)\right)}{R^2\omega_p^2\left(\sin^2(\theta_p) + \cos^2(\theta_p)\cos^2(\varphi)\right)} = 0 \tag{64}$$

The angular velocity noted by $\omega_{pzx}(t)$ of the curve $R_{zx}(t)$ is given by equation

$$\omega_{pzx} = \frac{\mid \dot{R}_z \ddot{R}_x - \dot{R}_x \ddot{R}_z \mid}{\dot{R}_x^2 + \dot{R}_z^2} = \frac{R^2 \omega_p^3 \left(\cos^2(\theta_p)\sin(\varphi) + \sin^2(\theta_p)\sin(\varphi)\right)}{R^2 \omega_p^2 \left(\sin^2(\theta_p) + \cos^2(\theta_p)\sin^2(\varphi)\right)} = \frac{\sin(\varphi)}{1 - \cos^2(\varphi)\cos^2(\theta_p)}\omega_p \tag{65}$$

$$\omega_{pzx}(t) = \frac{\sin(\varphi)}{1 - \cos^2(\varphi)\cos^2(\omega_p t)}\omega_p \tag{66}$$

$$\tag{67}$$

6. Angular velocity at which the Earth's surface moves relative to the stars

First, we will find the angular velocity with which the Earth's surface moves in relation to the Sun.

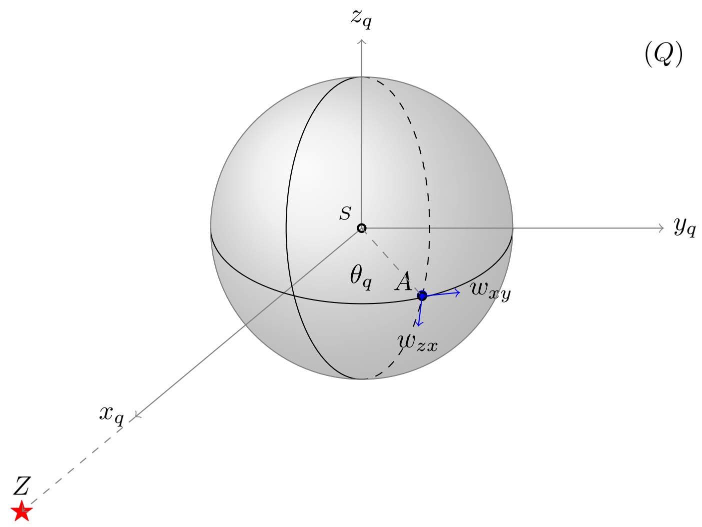

Figure 5: Angular velocity at which the Earth's surface moves relative to the stars

$$daysec = 24 * 3600[sec] \tag{68}$$

$$SA = 6.3781e6[m] \tag{69}$$

$$r = SA \tag{70}$$

$$\omega_q = \frac{2\Pi}{daysec} = 7.27221E - 05 \tag{71}$$

$$\theta_q = \angle(Sx_q, SA) = \frac{2\Pi t}{daysec} = \omega_q t \tag{72}$$

$$v_q = r\omega_q = 463.83 \left[\frac{m}{sec}\right] \tag{73}$$

$$\mathbf{r}(\theta_q) = \mathbf{r}(\omega_q t) = \begin{bmatrix} r_x(t) \\ r_y(t) \\ 0 \end{bmatrix} = \begin{bmatrix} r\cos(\theta_q) \\ r\sin(\theta_q) \\ 0 \end{bmatrix} = \begin{bmatrix} r\cos(\omega_q t) \\ r\sin(\omega_q t) \\ 0 \end{bmatrix} \tag{74}$$

We can find the first and second derivative of the vector $\mathbf{r}(t)$.

$$\dot{\mathbf{r}}(t) = \frac{\delta \mathbf{r}_q}{\delta t} = r\omega_q[-\sin(\omega_q t), \cos(\omega_q t), 0] = r\omega_q[-\sin(\theta_q), \cos(\theta_q), 0] \tag{75}$$

$$\ddot{\mathbf{r}}(t) = \frac{\delta \dot{\mathbf{r}}_q}{\delta t} = -r\omega_q^2[\cos(\omega_q t), \sin(\omega_q t), 0] = -r\omega_q^2[\cos(\theta_q), \sin(\theta_q), 0] \tag{76}$$

In relation to the coordinate system (Q), we will define three curves that we will denote by $\mathbf{r}_{xy}(t)$, $\mathbf{r}_{yz}(t)$ and $\mathbf{r}_{zx}(t)$.

$$\mathbf{r}_{xy}(t) = \begin{bmatrix} r_x(t) \\ r_y(t) \\ 0 \end{bmatrix} \tag{77}$$

$$\mathbf{r}_{yz}(t) = \begin{bmatrix} 0 \\ r_y(t) \\ r_z(t) \end{bmatrix} \tag{78}$$

$$\mathbf{r}_{zx}(t) = \begin{bmatrix} r_x(t) \\ 0 \\ r_z(t) \end{bmatrix} \tag{79}$$

if we denote the corresponding angular velocities with $\omega_{qxy}(t),\ \omega_{qyz}(t)\ and\ \omega_{qzx}(t)$ then it is easy to prove that we have the following equations

$$\omega_{qxy}(t) = \omega_q \tag{80}$$

$$\omega_{qyz}(t) = 0 \tag{81}$$

$$\omega_{qzx}(t) = 0 \tag{82}$$

Coordinate systems (Q) and (Q') are two coordinate systems that have different centers but equal corresponding axes (we assume that the axes are vectors), Figure[4]. Now, we will find the angular velocities ω_{xy}, ω_{yz} and $\omega_{zx}$with which the Earth's surface moves in relation to the stars, Fig[5].

$$\omega_{xy} = \omega_{pxy} + \omega_{qxy} = \omega_{pxy} + \omega_q = \frac{\cos(\varphi)}{1 - \sin^2(\varphi)\cos^2(\theta_p)}\omega_p + \omega_q \tag{83}$$

$$\omega_{yz} = \omega_{pyz} + \omega_{qyz} = 0 \tag{84}$$

$$\omega_{zx} = \omega_{pzx} + \omega_{qzx} = \omega_{pzx} = \frac{\sin(\varphi)}{1 - \cos^2(\varphi)\cos^2(\theta_p)}\omega_p \tag{85}$$

It is obvious that the angular velocity with which the solar system moves in relation to the center of the Galaxy is very small, so we can leave it out of the calculations.

7. The speed at which a point on the Earth's surface moves relative to the stars

We will calculate the speeds with which the a point on the Earth's surface moves in relation to the center of the Earth and the center of the solar system.

$$\varphi = 23.43693 * \Pi/180 = 4.0905107146E-01 \tag{86}$$
$$cos(\varphi) = 9.1749866161E-01 \tag{87}$$
$$sin(\varphi) = 3.9773886653E-01 \tag{88}$$
$$\omega_p = 1.9910637973E-07 \tag{89}$$
$$v_p = 29785.89 \left[\frac{m}{sec}\right] \tag{90}$$
$$r \in \{0, 6.3781\ e6\}\ [m] \tag{91}$$
$$\omega_q = 7.27221E-05 \tag{92}$$

We will assume that

$$r = 6.3781\ e6[m] \tag{93}$$
$$v_q = r\omega_q = 463.83 \left[\frac{m}{sec}\right] \tag{94}$$

First we will find the speed with which an arbitrary point on the Earth's surface moves in relation to the geocentric coordinate system (Q) Fig [5].

Applying equation (83) we will find the speed with which a point on the Earth's equator moves in the west-east direction

$$v_{xy}(\theta_p) = \omega_{xy} r = (\omega_{pxy}(\theta_p) + \omega_q)\, r = \left(\frac{\cos(\varphi)}{1-\sin^2(\varphi)\cos^2(\theta_p)}\omega_p + \omega_q\right) r \tag{95}$$
$$v_{xy}(\theta_p) \le v_{xy}(0) \approx 465.21[m/sec] \tag{96}$$
$$v_{xy}(\theta_p) \ge v_{xy}\left(\frac{\Pi}{2}\right) \approx 464.99[m/sec] \tag{97}$$

After that, applying equation (85) we will find find the maximum and minimum speed with which an arbitrary point moves in the north-south direction

$$v_{zx}(\theta_p) = \omega_{pzy}(\theta_p) r = \left(\frac{\sin(\varphi)}{1-\cos^2(\varphi)\cos^2(\theta_p)}\omega_p\right) r \tag{98}$$
$$v_{zx}(\theta_p) \le v_{zx}(0) \approx 3.19[m] \tag{99}$$
$$v_{zx}(\theta_p) \ge v_{zx}\left(\frac{\Pi}{2}\right) \approx 0.51[m] \tag{100}$$

Now we will find the speed with which the point A moves in relation to the heliocentric coordinate system (Q') Fig [4],Fig [5]

$$\mathbf{O}_p\mathbf{A} = \mathbf{O}_p\mathbf{O}_q + \mathbf{O}_q\mathbf{A} \tag{101}$$
$$\mathbf{v}(t) = \frac{\delta \mathbf{O}_p\mathbf{A}(t)}{\delta t} = \dot{\mathbf{R}}_q(t) + \dot{\mathbf{r}}_q(t) \tag{102}$$
$$\mathbf{v}(t) = v_p \begin{bmatrix} -\sin(\omega_p t) \\ \cos(\varphi)\cos(\omega_p t) \\ \sin(\varphi)\cos(\omega_p t) \end{bmatrix} + v_q \begin{bmatrix} -\sin(\omega_q t) \\ \cos(\omega_q t) \\ 0 \end{bmatrix} \tag{103}$$

8. The two-way speed of light in the vicinity of the Earth

Now we will analyze how the movement of the Earth's surface in relation to the stars affects the measurement of the speed of light in the vicinity of the Earth. Denote by Δt the time it takes for the signal to travel from point A to point B and back from point B to point A.

Now we can find two-way speed of light denoted by c_H.

$$l = AB \tag{104}$$

$$c_H = \frac{2l}{\Delta t} \tag{105}$$

Denote by t_+ the time it takes for the signal to travel from point A to point B and by t_- the time it takes for the signal to travel from point B to point A. The values of t_+ and t_- are still unknown.

$$c_+ = \frac{l}{\Delta t_+} \tag{106}$$

$$c_- = \frac{l}{\Delta t_-} \tag{107}$$

Let's define the constant speed c as the arithmetic mean of the c_+ and the c_-

$$c_+ = c + v \tag{108}$$

$$c_- = c - v \tag{109}$$

$$c = \frac{c_+ + c_-}{2} \tag{110}$$

Based on equation (110), it is obvious that if we want to accurately calculate the constant c, then it is necessary to know the speeds c_+ and c_-

$$\Delta t_+ = \frac{l}{c+v} \tag{111}$$

$$\Delta t_- = \frac{l}{c-v} \tag{112}$$

$$\Delta t = \Delta t_+ + \Delta t_- = \frac{l}{c_+} + \frac{l}{c_-} = \frac{l(c_+ + c_-)}{c_+ c_-} \tag{113}$$

$$c_H = \frac{2l}{\Delta t} \tag{114}$$

$$c_H = \frac{2l}{\Delta t} = \frac{2c_+ c_-}{c_+ + c_-} = \frac{c^2 - v^2}{c} = c - \frac{v^2}{c} \tag{115}$$

When we measure c_H, the speed of light in two directions, we actually measure the harmonic mean of the c_+ and the c_-. Based on equation (114), it is obvious that we do not need to know the c_+ nor c_- in order to be able to calculate c_H.

We will assume that c_H is known and we will compare its value with the value of unknown constant c.

$$\Delta c = c - c_H = \frac{v^2}{c} \tag{116}$$

Suppose we measure the two-way speed of light regarding to the heliocentric coordinate system (Q') and apply Equation (103).

$$c_H = 299,792,458[m/sec] \tag{117}$$

$$c \approx c_H \tag{118}$$

$$t = 0 \tag{119}$$

$$\mathbf{v}(0) = 29785.89[0, \cos(\varphi), \sin(\varphi)] + 465[0, 1, 0] \tag{120}$$

$$\mathbf{e} = [0, 1, 0] \tag{121}$$

$$v_{xy} = \mathbf{v}(0) \cdot \mathbf{e} \approx 27792.34[m/sec] \tag{122}$$

$$\Delta c_{xy} = \frac{v_{xy}^2}{c} \approx 2.57[m/sec] \tag{123}$$

$$\mathbf{e} = [0, 0, 1] \tag{124}$$

$$v_z = \mathbf{v}(0) \cdot \mathbf{e} \approx 11847[m/sec] \tag{125}$$

$$\Delta c_z = \frac{v_z^2}{c} \approx 0.47[m/sec] \tag{126}$$

It was measured that the two-way speed of light in the vicinity of the Earth is equal to c_H. Since the measurement error is less than 1 $[m/sec]$, then taking into account the values for Δc_{xy} and Δc_z we can conclude that the movement of the Earth's surface in relation to the coordinate system (Q') does not affect the measurement of the speed of light in two direction. If we took into account the movement of the solar system in relation to the center of the galaxy, it would mean that the v_{xy} and v_z actually would have significantly greater values.

Suppose we measure the two-way speed of light along the equator regarding to the geocentric coordinate system (Q) and apply Equation (96).

$$v_{xy} = 465[m/sec] \tag{127}$$

$$\Delta c = \frac{v_{xy}^2}{c} \approx 7.2E - 04[m/sec] = 0.72[mm/sec] \tag{128}$$

In this case, the difference Δc between the c and the measured value c_H is less than 1 $[mm/sec]$. Since the difference Δc is practically immeasurable, it means that it is impossible to know how the movement of the Earth's surface in relation to the coordinate system (Q) affects the measurement of the speed of light in two direction. Obviously, the movement of the solar system in relation to the center of the galaxy does not significantly affect the value of v_{xy}.

9. The one way speed of light in the vicinity of the Earth

Measuring the speed of light in one direction is not so important to get a more accurate value for the constant c, as it is to answer the question about the constancy of the one-way speed of light regarding to the coordinate system (Q). We proved in paper [4] that measuring the speed of light in one direction is possible by proving two things:

i) Synchronization between two clocks can be established without the assumption that the one-way speed of light must be isotropic in any given inertial frame(actually in the vicinity of the Earth).

ii) Slow movement of the clock does not significantly affect the measurements.

Based on the obtained measurements, there are three possibilities:

i) The results are inconsistent. This means that due to the slow movement of the clocks, it would be impossible to maintain their synchronization, so measuring the speed in one direction is impossible.

ii) The speed of light is invariant regardless of the direction of measurement. This would show that the postulate about the constant speed of light is correct, at least in the vicinity of the Earth.

iii) The speed of light depends on the direction of measurement. That would mean that we would have to reject the postulate about the constancy of light in all inertial frames of reference, because it is not valid in the vicinity of the Earth.

Regardless of the measurement result, we still cannot make a final conclusion. Because we don't know what the result would be if the experiment were done in a place where gravity is almost equal to zero.

10. Discussion

The First Postulate of Special Relativity [5], can be formulated in the following way: "The laws of physics are the same and can be stated in their simplest form in all inertial frames of reference". Based on that, we can assume that the result of the $Michelson\breve{}Morley$ experiment would be the same no matter how we choose the inertial frame of reference. All the "lab" experiments, which are directly or indirectly related to the speed of light, were made in the vicinity of the Earth, where its gravitational field is much stronger compared to the solar or galactic gravitational field. Whether the reference frame is determined by the sum of all gravitational fields acting on it? What would be the outcome of the $Michelson\breve{}Morley$ experiment if it was performed in an artificial satellite in a solar orbit far from any planet? If there is no gravitational field, what should be taken as a reference frame? $Aether$ (the "material" that fills the universe)? The Second Postulate of Special Relativity [5], can be formulated in the following way: "The speed of light in vacuum is the same in all inertial reference frames". Using $Maxwell's$ equations, it follows that the speed of electromagnetic waves c in empty space is given by the following equation.

$$c = \sqrt{\frac{1}{\epsilon_0 \mu_0}} \tag{129}$$

where

$$\epsilon_0 - vacuum\ permittivity\ (in\ the\ vicinity\ of\ the\ Earth) \tag{130}$$

$$\mu_0 - vacuum\ permeability\ (in\ the\ vicinity\ of\ the\ Earth) \tag{131}$$

Are the values of ϵ_0 and μ_0 invariant with respect to gravity? If the answer is negative, then the question arises whether the speed of light that we measure in an artificially created vacuum near the Earth is equal to the speed of light in space ($aether$) where gravity is (almost) equal to zero? We can only hope that in the near future we will start conducting experiments in space far from the Earth, so that the possible influence of its gravity on the outcome of the experiment would be nullified and in that way give a precise answer to those questions.

Conflict of interest statement

I have no conflicts of interest to disclose.

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