SOLVING PARTIAL DIFFERENTIAL EQUATIONS IN MATLAB

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Abstract:

This paper aims to describe the application of educational tools with which it can develop interactivity and help pupils and students to better and more clearly understand mathematics and to understand that it is all around us, that it is our everyday life. The paper will show the ways of creating mathematical educational materials and their use. The calculation of partial differential equations has a wide application. Many problems in scientific research and engineering can be mathematically modeled by partial differential equations. A differential equation containing multiple independent variables is called a PDE (partial differential equation).

Keywords: educational tools, partial differential equations, mathematics teaching, MATLAB

1. INTRODUCTION

Partial differential equations (PDEs) play an important role in finance, especially in option pricing. They are a powerful apparatus for pricing complex derivatives [1].Unfortunately, analytical solutions in the general case are not possible, so numerical methods are often resorted to. Some of the problems that arise in finance are in principle easy to solve, at least in principle, using standard numerical methods. There are also certain software packages, e.g. MATLAB, which gives us the ability to face the problem of portfolio optimization and pricing.

Differential equations have numerous applications in various fields of natural, social and technical sciences [2]. When considering and solving various problems from physics, mechanics, chemistry, geometry and other scientific disciplines and their applications, one comes across equations in which, in addition to unknown functions, their arguments and known objects, there are also derivatives of these functions. Such equations are called ordinary differential equations, if the unknown functions depend on only one argument, and partial, if the unknown functions depend on several arguments. Differential equations from the point of view of practical application represent one of the most important branches of mathematics.

The goal of these methods is to understand at least the qualitative features of the solution, both from an analytical and a geometric point of view. In the last few years, computers, especially computer graphics, have given impetus to the study of systems of nonlinear differential equations. Unexpected phenomena, such as strange attractors and fractals, have been discovered and are intensively studied. Although differential equations have long been known to scientists and have been quite well studied and researched, at the beginning of the twenty-first century they are still a source of fascinating and important unsolved problems.__

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Partial differential equations can be used to describe a wide range of phenomena such as sound, diffusion, heat, electrostatics, electrodynamics, fluid dynamics, elasticity or quantum mechanics. Just as ordinary differential equations often model one-dimensional dynamical systems, partial differential equations often model multidimensional systems. Partial differential equations find their generalization in stoḥastic partial differential equations [3].

2.PARTIAL AND DIFFERENTIAL EQUATIONS

A differential equation is an equation that connects the value of the derivative of a function with the function itself, values of the independent variable, numbers (parameters). The order of derivatives included in the equation can be

different (formally it is not unlimited). Derivatives, functions, independent variables, and parameters may be included in the equation in various combinations, or all but at least one derivative may be completely absent.

Modern high-speed computers efficiently provide a numerical solution to ordinary differential equations without the need to solve them in analytical form [4]. This allowed some researchers to claim that a solution to a problem was obtained if it was possible to reduce it to the solution of an ordinary differential equation.

The order, or degree, of a differential equation is the highest order of the derivatives involved in it.

The solution (integral) of a differential equation of order *n* is a function i(*x*) that has derivatives up to order n including

on some interval (a,b) $y'(x), y''(x), ..., y^{(n)}(x)$ and satisfies this equation. The process of solving a differential equation is called integration. The problem of integrating a differential equation is considered solved if an unknown function is found that can be squared, regardless of whether the resulting integral is expressed in a finite form in the form of known functions or not.

All differential equations can be divided into ordinary differential equations (ODEs), which involve only functions (and their derivatives) of one argument, and partial differential equations (PDEs), in which the input functions depend on many variables. There are also stochastic differential equations (SDEs) involving random processes.

Depending on the combinations of derivatives, functions, independent variables, differential equations are divided into linear and non-linear, with constant or variable coefficients, homogeneous or inhomogeneous [5,6,7,8]. Due to the importance of application, quasi-linear (linear with respect to multiple derivatives) partial differential equations are separated into a special class.

The most important issue for differential equations is the existence and uniqueness of their solutions. The solution to this question is provided by the theorems of existence and uniqueness, which indicate the necessary and sufficient conditions for this. Lipšić (1864) formulated such

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conditions for ordinary differential equations. For partial differential equations, the corresponding theorem was proved by S.V. Kovalevskaya (1874).

Solutions of differential equations are divided into general and particular solutions. General solutions include undefined constants, and for partial differential equations arbitrary functions of independent variables that can be refined from additional integration conditions (initial conditions for ordinary differential equations, initial and boundary conditions for partial differential equations). After determining the form of these constant and undetermined functions, the solutions become particular.

The search for solutions to ordinary differential equations led to the establishment of a class of special functions—functions often encountered in applications that are not expressed in terms of known elementary functions. Their properties have been studied in detail, tables of values have been compiled, mutual connections have been determined, etc.

The development of the theory of differential equations made it possible in a number of cases to abandon the requirement for the continuity of the studied functions and to introduce generalized solutions of differential equations [9].

Ordinary differential equations (ODEs) are equations that depend on one independent variable; they look like

$$F\left(x, y, y', y'', ..., y^{(n)}\right) = 0 \quad \text{or} \quad F\left(x, y, \frac{dy}{dx}, \frac{d^2 y}{dx^2}, ..., \frac{d^n y}{dx^n}\right) = 0,$$

wherein *y = y(x)* an unknown function (possibly a vector function; in this case it is often referred to as a system of differential equations), depending on the independent variable *x*, the prime denotes the differentiation with respect to *x*.

The number n is called the order of the differential equation. The most important in practice are differential equations of the first and second order.

The order of the differential equation is the highest order of derivatives involved in this equation.

The simplest first-order differential equations are the class of first-order differential equations that are easiest to solve and study. Includes total differential equations, equations with

separable variables, first-order homogeneous equations, or first-order nonlinear equations. All these equations can be integrated in the final form.

The order of partial differential equations can be determined in the same way as for ordinary differential equations. Another important classification of partial differential equations is their division into elliptic, parabolic and hyperbolic type equations, especially for second order equations.

Both ordinary differential equations and partial differential equations can be divided into linear and non-linear [10,11,12]. A differential equation is linear if the unknown function and its derivatives enter the equation only to the first degree (and do not multiply each other). For such equations, the solutions form an affine subspace of the function space. The theory of linear differential equations is developed much more deeply than the theory of nonlinear equations. An overview of the nth-order linear differential equation:

$$p_n(x)y^{(n)}(x) + p_{n-1}(x)y^{(n-1)}(x) + \cdots + p_0(x)y(x) = r(x),$$

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where pi(x) are known functions of the independent variable, called the coefficients of the equation.

The function r(x) on the right is called the constant term (the only term independent of the unknown function). An important special class of linear equations are linear differential equations with constant coefficients.

Under the class of linear equations are homogeneous differential equations - equations that do not contain a free term: r(x) = 0. For homogeneous differential equations, the principle of superposition is satisfied: a linear combination of partial solutions of such an equation will also be its solution.

All other linear differential equations are called inhomogeneous differential equations.

Nonlinear differential equations in the general case do not have developed solution methods, except for some special classes. In some cases (with the use of certain approximations) they can be reduced to linear ones.

2.1. Solving differential equations with the help of MatLab

MATLAB is an incredibly flexible environment that you can use to do all kinds of mathematical tasks.Technology can be a powerful tool for transforming learning. It can help with affirmation and advancementthe relationship between educators and students, reinvent our approaches to learning and collaboration, reduce long-standing gaps in equity and accessibility, and tailor learning experiencesto meet the needs of all students.

Mathematical problems can be solved by numerical methods such as systems of linear equations, systems of differential equations and other systems of equations. Not all mathematical problems can be easily solved analytically. Numerical methods are used to find solutions to problems, especially those that are difficult to solve with analytical methods. Numerical calculations often require precision so computer programming such as Microsoft Excel helpsor MATLAB is required [13,14].

Task number 1.

Solve the differential equation *(1 + x²) y' = x (2y + 1).*

The solution

\>>dsolve('(1+t^2)*Dy=t*(2*y+1)')

ans =

(C6*(t^2 + 1))/2 - ½

\>> pretty(ans)

```
       2
C6 (t + 1)
----------- - 1/2
      2
```

\>>

Therefore, the solution of the equation is $y = c\,(x^2 + 1) - \frac{1}{2}$.

Task number 2.

Solve the differential equation $xy' - \frac{y}{x+1} = x,\ if\ it\ is\ y(1) = -1$.

The solution

\>>dsolve('t*Dy-(y/(t+1))=t','y(1)=-1')

ans =

(t*(t + log(t)))/(t + 1) - (3*t)/(t + 1)

\>> pretty(ans)

```
t (t + ln(t))    3 t
------------- - -----
   t + 1        t + 1
```

\>>

Therefore, the solution of the differential equation is $y = \frac{x}{x+1} = x + ln,\ |x| - 3$.

Task number 3.

Solve the differential equation $y' + y\cos x = 0,5\sin 2x$.

The solution

\>>dsolve('Dy+y*cos(t)=(1/2)*sin(2*t)')

ans =

sin(t) + C8/exp(sin(t)) - 1

\>> pretty(ans)

sin(t) + C8 exp(-sin(t)) - 1

\>>

Therefore, the solution of the differential equation is $y = \sin x + ce^{-\sin x} - 1$.

3. BASICS OF WORKING IN THE MATLAB ENVIRONMENT

MATLAB is a mathematical calculation system with powerful computing capabilities, advanced tools for graphical data visualization, and a set of extension packages (tools and function

libraries) for solving problems invarious fields of science. The MATLAB environment also

[13] D. Arnold & J. C. Polking (1999).

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a high-level programming language. The following is a summary of the basic methods of working with the MATLAB system interface [15].

Work with the MATLAB system begins by starting the program from the main Windows menu or using a shortcut on the Windows desktop. As a result, the main MATLAB window opens.
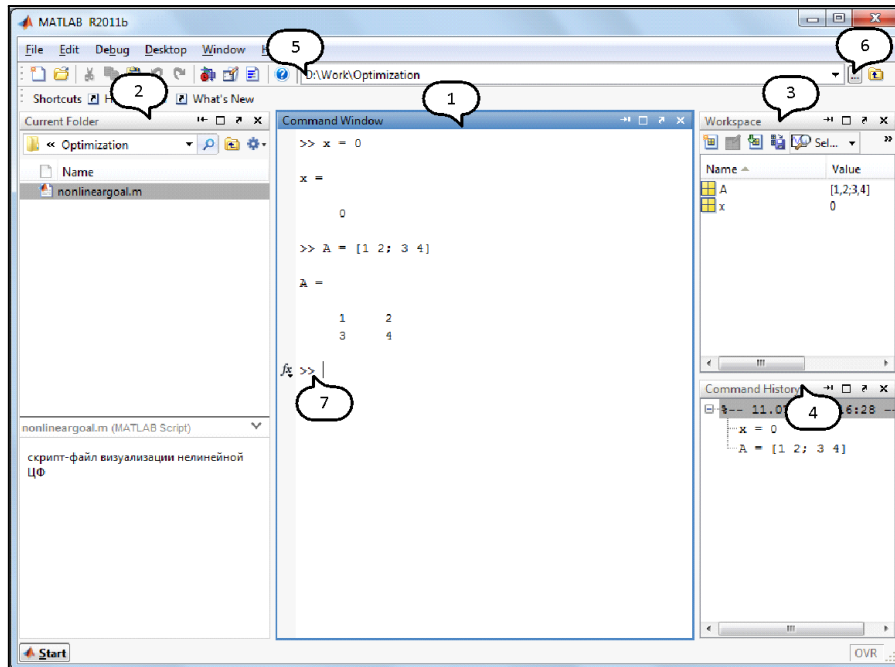
Figure 1. MATLAB Environment Interface

Access to the main tools of the MATLAB system is possible using the Start button in the lower left corner of the main window. At the top, below the window title, is the main MATLAB menu. The appearance of the window can be customized using the Desktop main menu item. To work, you will need the following elements of the main window [16].

1. Command window (command window). The command window contains a workspace designed for entering commands from the keyboard and viewing the results. It is the main means of interaction between the user and the system and provides dialogue through the MATLAB language. This means that all information entered by the user in the command line is perceived by the system as a command to be executed.

2. Current folder (current folder). Displays a list of files contained in the current Windows folder selected by the user. These can be user programs written in MATLAB language (script files or function files), data files, files of saved graphics windows.

3. Workspace (workspace). Variables created by the user during a session or as a result of program execution are located in the MATLAB system memory - in the workspace.

[15] Rodriguez, H., (2013). Introduction to MATLAB.

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4. Command history (command history). Commands entered by the user on the command line are stored in the command buffer. A previously executed command can be executed without re-selection by recalling it on the command line using the cursor keys (↑ or ↓) or by double-clicking in the command history window.

5. A string containing the path to the current folder. You can select the desired folder that contains the user's files using the Browse for folder button.

3.1. Using the MATLAB command window

MATLAB is a popular software package for solving technical, mathematical, statistical, forecasting and modeling problems. This is also the naming of the historical name of the software package. Everywhere, let's look at some kind of work with a commanding vision and a clear environment for MATLAB.
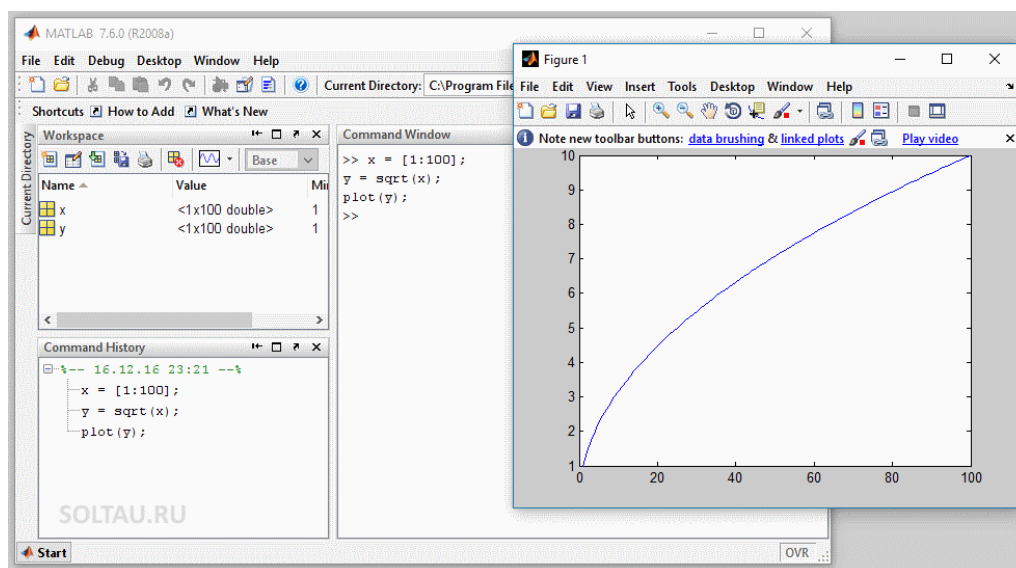

Figure 2. Command window

There are several ways to work in the MATLAB environment. The easiest way is to type the commands directly into the command window.

If it is not visible in the program interface, open it. You can find the command window through the menu Desktop → command window.

For example, enter the commands in this window one after the otherand press the Enter taster. The program calculates the value of the change X, the value of the change I and examines the harmfulness of the function for the date, and then protects the schedule.

k = [1 : 100];

u=skrt(k);

parcel (and);

Using the up and down arrows on the keyboard in the command window, we can move between entered commands, change them immediately, and press Enter to send the MATLAB environment for execution. The left and right arrows can be used to navigate through the entered command and edit it. If there is a semicolon at the end of the command, then the result will be calculated, but not displayed in the command window; otherwise, the result of the command will be displayed immediately. There is detailed built-in help for any function in the MATLAB environment. For example, to get help for the plot command, select this command, right-click on it, and choose Selection Help from the context menu that opens, or press F1.
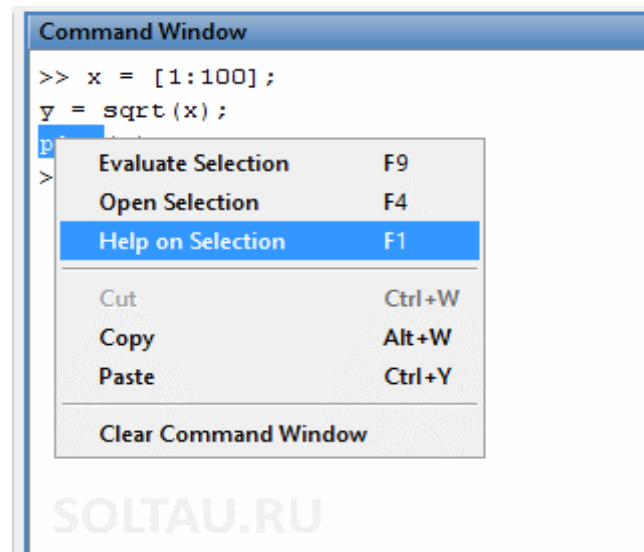
Figure 3. Getting help with MATLAB commands

Manually entering commands one by one and then searching for them in the history for a long time can be quite tedious.

4. SIGNIFICANT CONTRIBUTION OF MATLAB IN DIFFERENT COURSES OF EDUCATION

Today, MATLAB is a very popular and indispensable tool used in universities and colleges for introductory and advanced courses in mathematics, science, and especially engineering. MATLAB can also be used for economics. In industry, software is used in research, development and design. The standard MATLAB program has tools (functions) that can be used to solve common problems. In addition, MATLAB has optional toolboxes that are collections of specialized programs designed to solve specific types of problems.

MATLAB is a programming language and environment that is widely used in a variety of fields, including mathematics, engineering, and science. It provides a flexible and intuitive platform for numerical computation, data analysis, visualization and algorithm development. Today, the world's development and growth depend to a great extent on the usefulness of science, mathematics and computer knowledge. There are many streams and students who diversify the approach to science and mathematics. These students like engineering, statistics, research and analysis etc. they contribute to the great progress of techniques or you can say that they are the basis of the development and progress of the world. There are many streams that are greatly affected by the presence of massive computer applications. MATLAB is one of them. MATLAB is a fourth generation programming language used by many developing business houses to check and analyze their growth of various units. Its use is not limited to organizations only, in fact it contributes greatly in various streams of education such as engineering and scientists.

The usefulness of MATLAB can be seen in the field of statistics and mathematics as it helps a lot in the fields of scientific and mathematical approach to the real world. Nowadays, most students prefer to use MATLAB because it greatly improves their learning and computing power, and they can even complete their engineering degree without learning much from referencing sources because MATLAB is sufficient for everything [17].

To understand the usefulness and importance of MATLAB in the arena of engineering and scientific computing and analysis, it is necessary to know about its features:

- Graphical visualization of problems and their solutions.

- Help in coding and decoding facts and figures.

- Exact similarity of mathematical concepts and symbols as seen in technical reference books.

- Easy to use, write and understand MATLAB input and output.

 - Its applications are basically acceptable and applicable.

The main feature of MATLAB is its computing environment combined with a simple interface, tools, modeling capabilities, etc. MATLAB also stands as a programming language and environment that serves the technical needs of a wide range of mathematicians. MATLAB relieves you of many mundane tasks associated with numerical problem solving. This allows you to spend more time thinking and encourages you to experiment.

5. CONCLUSION

A modern approach to computer-aided development reduces the possibility of introducing errors when exchanging simulation results between different programs that previously allowed only one type of analysis.

The development of differential equations arose mostly from the needs of mechanics. The first differential equations that were studied were then current models of the problems that were being solved. During the 18th century, the theory of differential equations made it possible to solve problems from terrestrial and celestial mechanics, tides, meteorology and other fields of physics. The main development of differential equations is at the time of the discovery of differential and integral calculus, at the time of Newton and Leibniz. Attempts to solve physical problems gradually led to mathematical models; equations in which variables and their differentials occur. Differential equations represent an indispensable mathematical tool for solving a large number of problems of modern science and technology. The use of a computer and various software packages allows their solutions to be obtained with the given accuracy as well as graphically presented.

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My conclusion is that by combining under one mathematics as a tool and under two MATLAB as a software tool for solving mathematical tasks, in conjunction we get an extremely powerful tool for solving problems that any technical engineer (mechanics, architecture, design, computing...) who knows math and knows how to use MATLAB makes his job very easy.

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