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Innovative Theoretical Correlation of eπ With Mass, Number of Atoms and Enrichment Percentage (%E) Using Sayed's Enrichment Formula

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Abstract

This article introduces an innovative theoretical correlation between **eπ** and enrichment percentage (%E) assay. Based on the published Sayed`s theorem and its applications, a simple equation was derived to calculate the %E. Different formulas for enrichment percentage calculation are also presented in this work. The theoretical calculations using the eπ formula show results with deviation from 0.96 to ~2.8%. A correlation between eπ, number of atoms (N), count rate (C), half life time ($T_{1/2}$), efficiency (η) of the detector at specific energy, the branching ratio (Iγ), Avogadro`s number ($A_N$), atomic weight ($A_W$) and mass (M) was also found and elucidated. It can sharply state that this is first time to calculate the %E correlated with eπ by using what is called **Sayed`s enrichment formula**. The correlation of eπ and N A_W A_N M η Iγ $T_{1/2}$ is a unique and pioneer scientific approach in the nuclear fields.

Keywords: Sayed`s Enrichment formula, Correlation eπ and N A_W A_N M η Iγ $T_{1/2}$.

I. Introduction

It is an obligation under a comprehensive safeguards agreement (CSA) to establish and maintain an effective State system of accounting for and control (SSAC) of nuclear materials. The responsibility for establishing, implementing and maintaining an SSAC within a State party to a safeguards agreement in force with the IAEA rests entirely with the government of that State (1). Under a CSA, the following three generic safeguards objectives apply. At nuclear facilities, most safeguards activities focus on addressing the first two objectives (2):

- To detect any diversion of declared nuclear material at declared facilities or locations outside facilities (LOFs);
- To detect any undeclared production or processing of nuclear material at declared facilities or LOFs;
- To detect any undeclared nuclear material or activities in the State as a whole.

The national and international safeguards inspection is a tool to identify and determine the nuclear materials especially fissile materials content and ^{235}U enrichment percentage of the target to be investigated. As a matter of fact, natural uranium (NU) is a mixture of three radioactive isotopes: ^{238}U (99.27% abundance), ^{235}U (0.72%) and ^{234}U (0.0054%) (3,4). The ^{235}U level represents the degree of enrichment; depleted uranium (DU <0.72%), natural uranium (NU =0.72%) , low (or Slightly) enriched uranium (LEU <20%) , high enriched uranium (HEU > 20%) and weapon grade uranium (HEU > 93%).

This work focuses mainly on deduce a formula correlating the **eπ** with the enrichment percentage **(%E)** based on Sayed`s theorem and its applications to be used in the field of nuclear safeguards.

II. Theoretical Aspects and Treatment

II.1. Derivation of Sayed`s Enrichment Formula:

The enrichment percentage (%E) can be calculated using the well-known and published following equation (3,4,5,6,7).

$$\%E = \{N_5 / (N_5 + N_8 + N_4)\} \times 100 \quad (1)$$

Where, N_5, N_8 and N_4 represent the number of ^{235}U ,^{238}U and ^{234}U atoms respectively.

Equation 1 can also be expressed based on weight (mass) of ^{235}U, ^{238}U and ^{234}U as follows:

$$\%E = \{M_5 / (M_5 + M_8 + M_4)\} \times 100 \quad (2)$$

Taking into consideration the simple relation between N and activity (A); A = N λ, where λ is the decay constant, and substitute in equation 1, one gets (8,9):

$$\%E = \{A_5\, T_{1/2(5)} / (A_5 T_{1/2(5)} + A_8\, T_{1/2(8)} + A_4\, T_{1/2(4)})\}x100 \quad \textbf{(3)}$$

Where, A_5, A_8 and A_4 are the activity of ^{235}U, ^{238}U and ^{234}U respectively. While, $T_{1/2(5)}$, $T_{1/2(8)}$ and $T_{1/2(4)}$ represent half life time of ^{235}U ,^{238}U and ^{234}U respectively.

Based on the innovative correlation between **eπ** (where e equals 2.7182818 and π is 3.14159265), A and $T_{1/2}$ as given by Sayed`s theorem and its application (10,11).

$$A = N\,\lambda = N\, e\,\pi /12.32\, T_{1/2} \quad \textbf{(4)}$$

Where, λ is the decay constant (sec^{-1}) and in correlation with eπ. The half-life time is also expressed as given by Sayed`s theorem (11)

$$T_{1/2} = e\,\pi /12.32\, \lambda \quad \textbf{(5)}$$

By substituting eq.5 in equation 3, the new innovative correlation between eπ and enrichment percentage (**%E**) can be given as follows:

$$\%E = \{(e\pi\, A_5/12.32\, \lambda_{(5)}) / (A_5 T_{1/2(5)} + A_8\, T_{1/2(8)} + A_4\, T_{1/2(4)})\}x100 \quad \textbf{(6)}$$

For depleted, natural and low enriched uranium; the term $A_4\, T_{1/2(4)}$ in eq.6 can be neglected and the final form is to be equation 7.

$$\%E = \{(e\pi\, A_5/12.32\, \lambda_{(5)}) / (A_5 T_{1/2(5)} + A_8\, T_{1/2(8)})\}x100 \quad \textbf{(7)}$$

As shown from this equation, which is called **Sayed`s enrichment formula,** the **eπ** term is correlated with %E and ^{235}U, ^{238}U activities, the half-life time and the decay constant.

II.2. Correlation of (eπ) with number of atoms (N) and Mass (M) Derivation:

The relationship between activity (A) of a specific isotope in a sample and count rate (C) at specific energy, it can be expressed as (4):

$$A = C / \eta I\gamma \quad (8)$$

Where, η is the detector efficiency at specific energy and Iγ is the gamma intensity (Branching ratio). By using correlation of the activity and eπ as given by Sayed`s theorem (11):

$$A = N\, e\pi/12.32\, T_{1/2} \quad (9)$$

By equalizing equation 8 and 9 as given in the next equation number 10;

$$Ne\pi/12.32\, T_{1/2} = C / \eta\, I\gamma \quad (10)$$

Finally, The correlation between N , eπ and count rate can be stated as follow;

$$N = 12.32\, C\, T_{1/2} / e\pi\, \eta\, I\gamma \quad \textbf{(11)}$$

This equation gives the correlation between **eπ**, count rate (**C**), half life time (**$T_{1/2}$**), efficiency (**η**) of the detector at specific energy and the branching ratio (**Iγ**). By measurement the count rate of ^{235}U (e.g. 158.7, 143, 163 keV gamma energies) and ^{238}U (63, 1001keV energies), the N_5, N_8 and N_4 can be calculated and consequently the enrichment percentage can be deduced.

Considering the relation between N and mass (weight) as given below;

$$N = M.\, A_N / A_w \quad (12)$$

Where, M is the mass, A_N is the Avogadro`s number and A_w is the atomic weight. By substituting equation 12 in equation 11, the correlated eπ with the mass is obtained;

$$M = 12.32\, A_N\, A_W\, C\, T_{1/2} / e\pi\, \eta\, I\gamma \quad \textbf{(13)}$$

This simple equation which considers the correlation of M with A_N A_W C $T_{1/2}$ **eπ** η Iγ is a pioneer in the nuclear field**.**

III. Results and Discussion

Validation of the Sayed`s Enrichment formula and eπ Correlation

Validation of the deduced correlation was theoretically carried out using the data given in published papers (8,9). The results obtained by the eπ formula are compared with different calculations and given in the following table number 1.

Table 1: Results of %E correlated with eπ formula

Sample code	**%E** Previous published work	**%E** Current work eq.3	**%E** eπ Sayed formula Eq.7	% diff.
DU	0.58	0.57	0.569	1.89
NU	0.72	0.71	0.70	2.78
SEU	1.57	1,56	1.555	0.96

Based on results of this table, it was found that the maximum difference between Sayed `s enrichment formula and the other formulas are in the range from 0.96 to ~2.8%.

Generally, recently, Safeguards by Design (SbD) are considered and integrated in holistic and smart system. The preliminary design stage allows developers to innovate with respect to the safeguards by design (SbD) approach. This is a process where international safeguards design considerations are included throughout all phases of a nuclear facility life cycle, from the initial conceptual design to facility construction and to operations, including design modifications and decommissioning (12).

Figure 1 show facility life cycle stages. This includes SbD approach to be integrated with the classic design information questioner (DIQ) and verification (DIV).

Fig. (1): facility life cycle stages

Conclusion

The objective of correlation of %E with eπ has simply been achieved by deducing what is called Sayed`s enrichment formula. The calculated %E using this formula in comparison with other formulas shows error (%diff.) from 0.96 to ~2.8% only. A wonderful correlation between eπ, mass (M), atomic weight ($A_W$), Avogadro`s number (A_N), count rate (C), half life time ($T_{1/2}$), efficiency (η) of the detector, the branching ratio (Iγ), was also derived based on the previously published Sayed`s theorem.

Conflicts of interest

There are no any conflicts of interest with anyone.

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Biography

Prof. Sayed A. El-mongy was head of nuclear safeguards department and then appointed as V-Chairman of Egypt nuclear regulatory authority (ENRRA). He has three theories in the field of black hole anatomy, Revolutionary Merge of classic - relativity - quantum, in addition to eπ formula in trigonometry and astrophysics. He has a published letter to the editor concerning fission probability of ^{210}Po by antineutrino.