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Relativity: An Alternative Interpretation In the Light of The Existence of An Extra Spatial Dimension – A Systematic Review

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Abstract

This paper represents the latest revision of a portion of the research work, still in progress, carried out by the author during the last four years. The overall aim of the study fundamentally consists in showing how, while postulating the absoluteness of time, the validity of the relativistic equations may be formally preserved. Starting from the writing of the first Friedmann – Lemaître Equation (and therefore from General Relativity), a Simple-Harmonically Oscillating Universe (flat, upper-bounded, conventionally singular at $t = 0$) is obtained. Subsequently, the existence of a further spatial dimension is postulated. The Universe is identified with a 4-Ball involved in an (apparent) cyclic evolution and the concept of (material) point is replaced by the one of (material) segment. This scenario, combined with the absoluteness of time, allows an alternative derivation of the equations that characterize Special Relativity (including Lorentz Transformations), although with a different connotation. Amongst the significant consequences that arise from our approach, the possibility of (apparently) moving faster than light stands out.

Keywords: Relativity, Oscillating Universe, Extra Dimensions, Global Symmetry, Mass-Energy Equivalence, Space Quantization, Relativistic Energy, Lorentz Transformations, Faster than Light

1. Introduction

This paper represents the latest (systematic) review of (a portion of) the research work carried out by the author during the last four years (Cataldo, 2019a, 2019b, 2017a, 2017b, 2016) (in detail, this article must be regarded as a revised and updated version of a portion of the paper entitled *"Relativity: Towards a New Interpretation"*) (Cataldo, 2019a). The overall aim of the study fundamentally consists in showing how, while postulating the absoluteness of time, the validity of the relativistic equations may be formally preserved.

Starting from the writing of the first Friedmann – Lemaître Equation (Friedmann, 1922) (and therefore from General Relativity) (Einstein, 1916; Cheng, 2005), a Simple-Harmonically Oscillating Universe (flat, upper-bounded, conventionally singular at $t = 0$) is obtained (Harrison, 1967). Subsequently, the existence of a further spatial dimension (not directly perceivable) is postulated (Cataldo, 2019a, 2019b, 2017a, 2017b, 2016).

The Universe is identified with a 4-Ball involved in an (apparent) cyclic evolution and the concept of (material) point is replaced by the one of (material) segment. In detail, what is perceived as being a point may actually be a segment crossing the centre of the 4-Ball. Two antipodal points, since they evidently represent the end-points of the same segment, must be considered as being a unique entity: in other terms, the Universe may be characterized by a (Global) Central Symmetry (Cataldo, 2019a, 2019b, 2017a, 2017b, 2016).

Space is considered as being a quantized physical quantity: the minimal length is derived by resorting to the Generalised Uncertainty Principle (Shalit-Margolyn, 2018).

This background, combined with the absoluteness of time, allows an alternative derivation of all the equations that characterize Special Relativity (including Lorentz Transformations) (Cataldo, 2019a, 2019b, 2017a, 2017b, 2016), albeit with a different connotation.

Amongst the significant consequences that arise from our approach, the possibility of (apparently) moving faster than light stands out (Cataldo, 2019a, 2016a). It is worth specifying how Relativity, at least in its original formulation, is clearly a *3D+1* theory, while we propose a *4D+1* model. However, all the features of our model are deduced by resorting to the superposition of three *3D+1* sub-models (Cataldo, 2019a, 2019b, 2017b). Gravity is herein not addressed.

2. Relativistic Background

2.1 Uniform Cosmological Models

According to Harrison's classification (Harrison, 1967), there are four groups of uniform cosmological models compatible with general relativity (from now onwards GR): static, asymptotic, monotonic, and oscillatory (each of this groups, in turn, may be subdivided into sub-groups or classes).

For a uniform Universe, with the usual hypotheses of homogeneity and isotropy, the first Friedmann – Lemaître Equation (Friedmann, 1922; Cheng, 2005) is commonly written as follows:

$$\dot{R}^2 = \left(\frac{dR}{dt}\right)^2 = \frac{1}{3}(8\pi G\rho + \Lambda c^2)R^2 - kc^2. \tag{2.1}$$

In the previous Equation, R represents the scale factor (Cheng, 2005), G the gravitational constant, ρ the density, Λ the cosmological constant (Cheng, 2005), k the curvature parameter, whose value depends on the hypothesized geometry (Cheng, 2005), and c the speed of light.

Denoting with p the pressure, the Fluid Equation (Cheng, 2005), can be written as follows:

$$\dot{\rho} = \frac{d\rho}{dt} = -\frac{3}{R}\frac{dR}{dt}\left(\rho + \frac{p}{c^2}\right) = -3\frac{\dot{R}}{R}\left(\rho + \frac{p}{c^2}\right). \tag{2.2}$$

According to Zeldovich (Zeldovich, 1961), the relation between pressure and density (the Equation of State) can be expressed in the underlying form:

$$p = (\nu - 1)\rho c^2. \tag{2.3}$$

The value of ν, hypothesized as being constant, exclusively depends on the type of fluid we take into consideration (matter, radiation, relativistic gas, dark energy, etc.): the commonly accepted values lie in the range $1 \leq \nu \leq 4/3$ (Zeldovich, 1961).

From Eq. (2.2), taking into account Eq. (2.3), we obtain:

$$\frac{d\rho}{\rho} = -3\nu\frac{dR}{R}. \tag{2.4}$$

Consequently, denoting with C the constant of integration, we have:

$$\rho R^{3\nu} = C. \tag{2.5}$$

Eq. (2.1) can be rewritten as follows:

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G\rho R^{3\nu}}{3}R^{2-3\nu} + \frac{1}{3}\Lambda c^2 R^2 - kc^2. \tag{2.6}$$

By virtue of Eq. (2.5), a new constant, denoted by C_ν, can be now defined:

$$C_\nu = \frac{8\pi G\rho R^{3\nu}}{3} = \frac{8\pi GC}{3}. \tag{2.7}$$

By substituting the previous identity into Eq. (2.6) we obtain:

$$\dot{R}^2 = C_\nu R^{2-3\nu} + \frac{1}{3}\Lambda c^2 R^2 - kc^2. \tag{2.8}$$

2.2 Oscillatory Class with $\mathbf{k} = \mathbf{0}$

If we denote with ω the pulsation of the universe we aim to describe, we can set:

$$\Lambda = -3\left(\frac{\omega}{c}\right)^2. \tag{2.9}$$

If we set $k = 0$, by substituting Eq. (2.9) into Eq. (2.8), we have:

$$\dot{R}^2 = C_\nu R^{2-3\nu} - \omega^2 R^2. \tag{2.10}$$

From the previous Equation, we obtain:

$$\frac{dR}{dt} = \sqrt{C_\nu} R^{1-\frac{3}{2}\nu} \sqrt{1 - \left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right)^2}, \tag{2.11}$$

$$\frac{1}{\sqrt{C_\nu} R^{1-\frac{3}{2}\nu}} \frac{dR}{\sqrt{1 - \left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right)^2}} = dt, \tag{2.12}$$

$$\frac{d\left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right)}{\sqrt{1 - \left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right)^2}} = \frac{3}{2}\nu\omega dt. \tag{2.13}$$

If we impose that $R = 0$ when $t = 0$, from the previous Equation we have:

$$arcsin\left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right) = \frac{3}{2}\nu\omega t, \tag{2.14}$$

$$R^{3\nu} = \frac{C_\nu}{\omega^2}\sin^2\left(\frac{3}{2}\nu\omega t\right) = \frac{C_\nu}{2\omega^2}[1 - \cos(3\nu\omega t)], \tag{2.15}$$

$$R = \left(\frac{C_\nu}{2\omega^2}\right)^{\frac{1}{3\nu}}[1 - \cos(3\nu\omega t)]^{\frac{1}{3\nu}}. \tag{2.16}$$

According to Eq. (2.16), we have formally obtained a Universe belonging to the oscillatory class ("O Type" in Harrison's classification) (Harrison, 1967).

From Eqs. (2.7) and (2.15) we obtain:

$$\rho = \frac{3}{8\pi G}\frac{C_\nu}{R^{3\nu}} = \frac{3\omega^2}{4\pi G}\frac{1}{1-\cos(3\nu\omega t)}. \tag{2.17}$$

Finally, by taking into account Eq. (2.9), we can write Eq. (2.17) as follows:

$$\rho = -\frac{\Lambda c^2}{4\pi G}\frac{1}{1-\cos(3\nu\omega t)}. \tag{2.18}$$

2.3 A Simple-Harmonically Oscillating Universe

If we set $\nu = 1/3$, from Equation (2.16) we have:

$$R = \frac{C_{1/3}}{2\omega^2}[1-\cos(\omega t)]. \tag{2.19}$$

We have just achieved a simple-harmonically oscillating universe (flat, upper-bounded, conventionally singular at $t = 0$), characterized by a variable density whose value, setting $\nu = 1/3$ in Eq. (2.18), is provided by the underlying relation:

$$\rho = -\frac{\Lambda c^2}{4\pi G}\frac{1}{1-\cos(\omega t)}. \tag{2.20}$$

Denoting with A the amplitude of the motion, by virtue of Eq. (2.19) we can write:

$$A = \frac{C_{1/3}}{2\omega^2}. \tag{2.21}$$

Denoting with R_m the mean radius ($\omega t = \pi/2$), Eqs. (2.19) and (2.21) give:

$$R\left(\frac{\pi}{2}\right) = A = R_m. \tag{2.22}$$

By virtue of the previous, from Eq. (2.20) we obtain:

$$\rho_m = \rho(R_m) = \rho\left(\frac{\pi}{2}\right) = -\frac{\Lambda c^2}{4\pi G}. \tag{2.23}$$

Taking into account Eqs. (2.21) and (2.22), Eq. (2.19) acquires the underlying form:

$$R = R_m[1-\cos(\omega t)]. \tag{2.24}$$

From Eq. (2.5), being $\nu = 1/3$, we have:

$$\rho R = \rho_m R_m. \tag{2.25}$$

From Equations (2.7), (2.21), (2.22) and (2.25) we deduce:

$$\omega^2 = \frac{C_{1/3}}{2A} = \frac{C_{1/3}}{2R_m} = \frac{4\pi G}{3}\rho\frac{R}{R_m} = \frac{4\pi G\rho_m}{3}, \tag{2.26}$$

$$(\omega R_m)^2 = \frac{2\left(\frac{2}{3}\pi R_m^3 \rho_m\right) G}{R_m}. \tag{2.27}$$

Now, let us carry out the following positions (Cataldo, 2019a, 2019b, 2017a):

$$M_m = \frac{2}{3}\pi R_m^3 \rho_m, \tag{2.28}$$

$$\omega R_m = c. \tag{2.29}$$

The position in Eq. (2.28) will be better understood in *Section 4*, by resorting to the concept of "global central symmetry" (Cataldo, 2019a, 2019b, 2017b, 2016).

From Eq. (2.27), taking into account Eqs. (2.28) and (2.29), denoting with R_s the so-called Schwarzschild Radius (Schwarzschild, 1016; Cheng, 2005), we have:

$$R_m = \frac{2M_m G}{c^2} = R_s(M_m). \tag{2.30}$$

In the light of the results so far achieved, we can now write the following:

$$\omega t = \frac{ct}{R_m} = \alpha, \tag{2.31}$$

$$R = R_m(1 - \cos\alpha), \tag{2.32}$$

$$\cos\alpha = 1 - \frac{R}{R_m}, \tag{2.33}$$

$$\dot{R} = \frac{dR}{dt} = c\sin\alpha, \tag{2.34}$$

$$\ddot{R} = \frac{d\dot{R}}{dt} = c\omega\cos\alpha = \frac{c^2}{R_m}\left(1 - \frac{R}{R_m}\right). \tag{2.35}$$

The beginning of a new cycle ($t = 0$) occurs when the radius assumes a null value.

The evolution of the Universe we have obtained is characterized by four consecutive phases: accelerated expansion, decelerated expansion, decelerated contraction, accelerated contraction.

All the phases have the same duration.

By virtue of Eqs. (2.31), (2.32) and (2.34), the Hubble Parameter (Hubble, 1929), commonly denoted by H, can be written as follows:

$$H = \frac{\dot{R}}{R} = \frac{c}{R_m}\frac{2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)}{2\sin^2\left(\frac{\alpha}{2}\right)} = \frac{c}{R_m}\frac{1}{\tan\left(\frac{ct}{2R_m}\right)}. \tag{2.36}$$

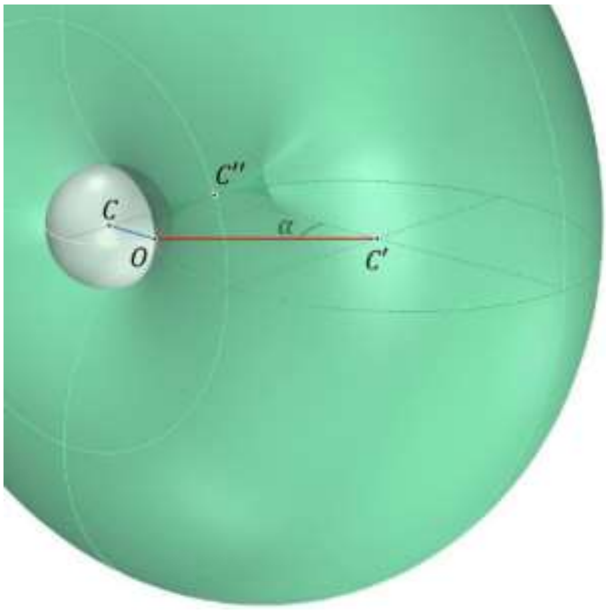

Figure 1. *Cyclic Universe*

In *Figure 1*, the Universe is portrayed as a 3 – Ball with centre C. The point C'' represents the centre of a (virtual) horn torus. The straight line segment bordered by C' and O (the radius of the section of the torus) represents the mean radius (R_m) defined in Eq. (2.22). When $t = 0$ (at the beginning of a new cycle), $C \equiv C''$. The point O, which belongs both to the boundary of the ball (a spherical surface) and to the torus, moves tangentially (following the poloidal direction) with a constant speed equal to c. The angle α is defined in Eq. (2.31). The straight line segment bordered by C and O represents the variable radius (R) defined in Eq. (2.32).

3. Mechanical Background

Let us consider a material point whose motion is defined by Eq. (2.32) (in other terms, a simple harmonic oscillator consisting of a mass and an ideal spring).

Denoting with m the mass of the point, taking into account Eq. (2.29), the elastic constant, denoted by k_e, can be written as follows:

$$k_e = m\omega^2 = m\left(\frac{c}{R_m}\right)^2. \tag{3.1}$$

Consequently, the total (mechanical) energy acquires the underlying form:

$$E_{R_m-point} = \frac{1}{2}k_e R_m^2 = \frac{1}{2}mc^2. \tag{3.2}$$

Now, by solely modifying the amplitude of the motion, denoted by R'_m, keeping the values of mass and pulsation constant, we can generalize Eq. (2.32) as follows:

$$R' = R'(R'_m, \alpha) = R'_m(1 - \cos\alpha), \qquad R'_m \in \,]0, R_m]. \tag{3.3}$$

From Eqs. (2.32) and (3.3) we have:

$$\frac{R'_m}{R_m} = \frac{R'}{R}. \tag{3.4}$$

At any given time, the value of R is univocally determined by means of Eq. (2.32), being R_m constant. On the contrary, the value of R', provided by Eq. (3.3), clearly depends on the amplitude of the motion (R'_m).

Taking into account Eqs. (3.1) and (3.4), the total energy of a material point, whose motion is described by Eq. (3.3), acquires the following form:

$$E_{R'_m-point} = \frac{1}{2}k_e R'^2_m = \frac{1}{2}\left(\frac{R'_m}{R_m}\right)^2 mc^2 = \frac{1}{2}\left(\frac{R'}{R}\right)^2 mc^2. \tag{3.5}$$

The material point can now be replaced by a homogeneous material segment (in other terms, we consider a spring, no longer ideal, with a length at rest equal to R_m).

The length of the segment (R) evolves in accordance with Eq. (2.32).

If we denote now with M the mass of the segment (the linear mass), the linear density can be defined as follows:

$$\bar{M} = \frac{M}{R}. \tag{3.6}$$

By virtue of the homogeneity, denoting with M' the mass of a portion of segment characterized, at any given time, by a length equal to R', we can write:

$$M' = \bar{M}R' = \frac{R'}{R}M, \tag{3.7}$$

$$\bar{M} = \frac{M}{R} = \frac{M'}{R'}. \tag{3.8}$$

Eq. (3.8) clearly underlines how the linear density does not vary along the segment.

Taking into account Eqs. (3.5) and (3.7), the energy related to an infinitesimal material segment can be evidently written as follows:

$$dE_{R'} = \frac{1}{2}\left(\frac{R'}{R}\right)^2 c^2 dM' = \frac{1}{2}\left(\frac{R'}{R}\right)^2 c^2 \bar{M} dR' = \frac{Mc^2}{2R^3}R'^2 dR'. \tag{3.9}$$

By virtue of Eqs. (3.7) and (3.9), the energy of a material segment, whose length, at any given time, is equal to R', can be expressed in the underlying form:

$$E_{R'} = \int_0^{R'} dE' = \frac{1}{6}\left(\frac{R'}{R}\right)^3 Mc^2 = \frac{1}{6}\left(\frac{R'}{R}\right)^2 M'c^2. \tag{3.10}$$

4. Introducing the 4th Spatial Dimension

4.1 Mass – Energy "Equivalence"

The Universe we hypothesize is identifiable with a 4-Ball.

The radius, denoted by R, evolves in accordance to Eq. (2.32). The corresponding boundary, which may represent the space we are allowed to perceive (at rest) (Cataldo 2019a, 2019b, 2017b, 2016), is a three-dimensional surface (a hyper sphere) described by the following identity:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2. \tag{4.1}$$

The 4-Ball is banally described by the following inequality:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 \leq R^2. \tag{4.2}$$

Let us consider the point P^+ defined as follows:

$$P^+ = (0,0,0,R). \tag{4.3}$$

Denoting with P^- the antipode of P^+ (the point diametrically opposite), we have:

$$P^- = (0,0,0,-R). \tag{4.4}$$

We must now consider the straight line segment bordered by the points P^+ and P^-.

Figure 2 provides a representation of the segment, obtained from Eq. (4.2) by setting equal to zero, one at a time, all the four coordinates.

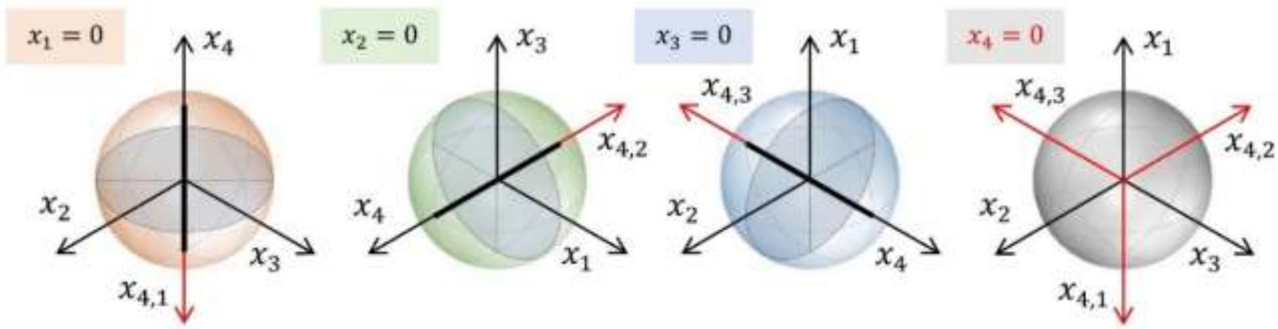

Figure 2. *Representation of a Material Segment*

As shown in *Figure 2*, by setting $x_4 = 0$ we obtain nothing but a single point. Therefore, we have to examine the three-dimensional scenarios that arise from the underlying identity:

$$x_i = 0, \qquad i = 1,2,3. \tag{4.5}$$

Let us set, e.g., $x_1 = 0$. Consequently, from Eqs. (4.2), (4.3) and (4.4) we have:

$$x_2^2 + x_3^2 + x_4^2 \leq R^2, \tag{4.6}$$

$$P_1^+ = (0,0,R), \tag{4.7}$$

$$P_1^- = (0,0,-R). \tag{4.8}$$

Let us now consider the straight line segment bordered by the centre of the 4-ball and the point defined by Eq. (4.7). If the segment in question, the length of which evolves in accordance with Equation (2.32), is provided with a mass M, its energy can be deduced from Eq. (3.10) by setting $R' = R$.

Consequently, underlining how the same procedure can be obviously adopted for the point defined by Eq. (4.8), we can write, with obvious meaning of the notation, as follows:

$$E_{R,1}^+ = E_{R,1}^- = \frac{1}{6} M c^2, \tag{4.9}$$

$$E_1 = E_{R,1} = E_{R,1}^{+} + E_{R,1}^{-} = \frac{1}{3} Mc^2. \tag{4.10}$$

Generalizing, we have:

$$E_i = E_{R,i} = E_{R,i}^{+} + E_{R,i}^{-} = \frac{1}{3} Mc^2, \qquad i = 1,2,3. \tag{4.11}$$

Finally, by superposition, we can easily write the total amount of energy related to the material segment bordered by the points P^+ and P^- defined, respectively, by Eqs. (4.3) and (4.4):

$$E = \sum_{i=1}^{3} E_i = Mc^2. \tag{4.12}$$

As far as our perception is concerned, each point and its antipode are to be considered as being the same entity, since they both belong to the same straight line segment. In other terms, according to our model, the Universe may be characterized by a global central symmetry (Cataldo, 2019a, 2019b, 2017a, 2017b, 2016).

4.2 Imposing the Conservation of Energy

Let us consider one amongst the scenarios defined by Eq. (4.5). E.g., we can set, once again, $x_1 = 0$. Initially, the homogenous material segment, bordered by the points P_1^+ and P_1^- defined in Eqs. (4.7) and (4.8), is characterized by a length equal to $2R$ and a mass equal to $2M$.

Let us suppose that the segment starts rotating around the centre of the ball.

If we impose the conservation of energy, the motion must necessarily modify length and/or mass of the segment: otherwise the kinetic energy would be simply added to the energy defined in Eq. (4.11), and the total energy could no longer be regarded as being constant.

Obviously, the length of the segment in motion cannot increase: otherwise, the inequality in Eq. (4.6) would be violated (in other terms, the points P^+ and P^- would end up with being paradoxically placed beyond the boundary).

Let us impose the two following conditions (Cataldo, 2019a):

Condition 1. The tangential speed of the endpoints (of the segment), from now onwards denoted by v, is less than the speed of light ($v < c$);

Condition 2. The motion does not cause any linear density variations (this condition will be later legitimized): therefore, the value of the linear mass must keep on abiding by the simple rule established in Eq. (3.7).

Ultimately, according to our model, the motion may produce, concurrently, a loss of linear mass and a (symmetric) reduction of the length of the segment.

If $2R'$ represents the total length of the segment in motion (with $0 < R' \leq R$), denoting with I the moment of inertia, we can write the kinetic energy as follows:

$$E_{k,1} = \frac{1}{2} I \left(\frac{v}{R'}\right)^2. \tag{4.13}$$

If $2M'$ represents the (reduced) mass of the segment in motion, we have:

$$I = \frac{1}{12}(2M')(2R')^2 = \frac{2}{3}M'R'^2. \tag{4.14}$$

From the two previous Equations we immediately obtain:

$$E_{k,1} = \frac{1}{3}M'v^2. \tag{4.15}$$

From Eq. (3.10), taking into account the symmetry, we can state that the segment, since it is involved in the cyclic evolution described by Eq. (3.3), is also provided with the following energetic amount:

$$E_{R',1} = E^{+}_{R',1} + E^{-}_{R',1} = \frac{1}{3}\left(\frac{R'}{R}\right)^2 M'c^2 = E_{p,1}. \tag{4.16}$$

From Eqs. (4.15) and (4.16), taking into account the condition in Eq. (3.7), we have:

$$E_{k,1} + E_{p,1} = \frac{1}{3}M'\left[v^2 + \left(\frac{R'}{R}\right)^2 c^2\right] = \frac{1}{3}M\left[v^2 + \left(\frac{R'}{R}\right)^2 c^2\right]\frac{R'}{R}. \tag{4.17}$$

Since $v < c$, when R' approaches 0 (when the segment tends to completely lose its mass), $E_{k,1} + E_{p,1}$ tends to vanish. Therefore, in order to safeguard the conservation of energy, we need to introduce a further energetic term, denoted by $E_{w,1}$ (Cataldo 2019a, 2019b). Obviously, when R' approaches 0 (when the segment tends to completely lose its mass), $E_{w,1}$ must tend to the value provided by Eq. (4.11); on the contrary, when $R' = R$ (when $M' = M$), $E_{w,1}$ must vanish.

By imposing a linear dependence between $E_{w,1}$ and M', we have (Cataldo, 2019a, 2019b, 2017b):

$$E_{w,1} = \frac{1}{3}(M - M')c^2. \tag{4.18}$$

Taking into account Eqs. (4.11), (4.15), (4.16) and (4.18), the conservation of energy, for the considered scenario ($x_1 = 0$), can be finally written as follows:

$$E_1 = \frac{1}{3}Mc^2 = \frac{1}{3}M'v^2 + \frac{1}{3}\left(\frac{R'}{R}\right)^2 M'c^2 + \frac{1}{3}(M - M')c^2. \tag{4.19}$$

By multiplying by three all the members of the previous Equation, taking into account Eq. (4.12), we finally obtain the underlying general relation:

$$E = Mc^2 = M'v^2 + \left(\frac{R'}{R}\right)^2 M'c^2 + (M - M')c^2 = E_k + E_p + E_w. \tag{4.20}$$

E_k represents the (real) kinetic energy, E_p the potential (background) energy (related to the cyclic evolution of the Universe), E_w (a "non-material" aliquot, related to the so-called "quantum potential") (Bohm, 1952a, 1952b) represents the energy needed to obtain the motion (to obtain the mass reduction) (Cataldo, 2019a, 2019b, 2017b).

From Eq. (4.20) we immediately deduce the underlying noteworthy relation:

$$M'c^2 = M'v^2 + \left(\frac{R'}{R}\right)^2 M'c^2 = E_k + E_p = E'. \tag{4.21}$$

The linear dependence between $E_{w,1}$ and M' has allowed the writing of E' as the product between the (reduced) mass and c^2. In other terms, net of the non-material aliquot, the energy can be still expressed in the form achieved in Eq. (4.12).

4.3 Specific Energies

According to the definition of Lorentz Factor (Lorentz, 1904), we have:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}, \tag{4.22}$$

$$\left(\frac{v}{c}\right)^2 = \beta^2 = 1 - \frac{1}{\gamma^2}. \tag{4.23}$$

From Equation (4.21), exploiting Eqs. (4.22) and (4.23), we easily obtain:

$$R' = R\sqrt{1-\left(\frac{v}{c}\right)^2} = R\sqrt{1-\beta^2} = \frac{R}{\gamma}. \tag{4.24}$$

We have just found the relation between the tangential speed (of the endpoints) and the radial extension (net of the symmetry) of the segment in motion.

From Eq. (3.6), by virtue of Eq. (4.24), we obtain:

$$\frac{M}{R'} = \frac{R}{R'}\bar{M} = \frac{\bar{M}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma\bar{M}. \tag{4.25}$$

Consequently, taking into account Eqs. (3.7), (3.8), (4.20), (4.23), (4.24) and (4.25), the specific energies (the energies per unit of length) can now be written, with obvious meaning of the notation, as follows:

$$\bar{E} = \frac{Mc^2}{R'} = \frac{\bar{M}c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma\bar{M}c^2, \tag{4.26}$$

$$\bar{E}_k = \frac{M'v^2}{R'} = \bar{M}\beta^2c^2 = \left(1-\frac{1}{\gamma^2}\right)\bar{M}c^2, \tag{4.27}$$

$$\bar{E}_p = \left(\frac{R'}{R}\right)^2\frac{M'c^2}{R'} = \frac{\bar{M}c^2}{\gamma^2}, \tag{4.28}$$

$$\bar{E}_w = \left(\frac{M}{M'}-1\right)\frac{M'c^2}{R'} = \left(\frac{R}{R'}-1\right)\frac{M'c^2}{R'} = (\gamma-1)\bar{M}c^2. \tag{4.29}$$

Therefore, by dividing both members of Eq. (4.20) by R', we obtain:

$$\bar{E} = \bar{E}_k + \bar{E}_p + \bar{E}_w, \tag{4.30}$$

$$\gamma\bar{M}c^2 = \left(1-\frac{1}{\gamma^2}\right)\bar{M}c^2 + \frac{\bar{M}c^2}{\gamma^2} + (\gamma-1)\bar{M}c^2. \tag{4.31}$$

If we denote with E_0 the energy at rest ($R' = R$), whose value has been obtained in Eq. (4.12), by virtue of Eqs. (3.6), (4.26) and (4.29) we have:

$$\bar{E}_0 = \frac{Mc^2}{R} = \bar{M}c^2, \tag{4.32}$$

$$\bar{E} = \gamma \bar{M}c^2 = \bar{E}_0 + (\gamma - 1)\bar{M}c^2 = \bar{E}_0 + \bar{E}_w. \tag{4.33}$$

Now, by dividing both members of Eq. (4.21) by R', taking into account Eqs. (3.8) and (4.24), we easily obtain:

$$\bar{M}c^2 = \bar{M}v^2 + \frac{\bar{M}c^2}{\gamma^2}. \tag{4.34}$$

Finally, by multiplying both members of the foregoing Equation by γ, by virtue of Eq. (4.26) we have:

$$\gamma \bar{M}c^2 = \gamma \bar{M}v^2 + \frac{\bar{M}c^2}{\gamma}, \tag{4.35}$$

$$\bar{E} = \frac{\bar{M}c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{\bar{M}v^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} + \sqrt{1 - \left(\frac{v}{c}\right)^2}\,\bar{M}c^2. \tag{4.36}$$

4.4 Punctual Mass and Space Quanta: the "Relativistic" Energy

In order to obtain the formal definition of the so-called relativistic energy, we have to impose a space quantization.

If R is regarded as a primary measurable quantity, denoting with ΔR_{min} the (radial) quantum of space (Cataldo, 2019a) and with $\mathcal{N}$ an integer, we can write:

$$R = \mathcal{N}\Delta R_{min}. \tag{4.37}$$

According to the generalized uncertainty principle (Shalit-Margolyn, 2018), for ΔR_{min} we have:

$$\Delta R_{min} = 2\sqrt{\alpha'}\ell_P. \tag{4.38}$$

In the previous, ℓ_P represents the so-called Planck Length and α' a constant. There are several methods to estimate the value of α' (Cataldo, 2019a; Veneziano, 1986; Adler et al., 1999; Maggiore, 1994; Capozziello et al., 2000): in any case, $\alpha' \cong 1$. Consequently, from Eq. (4.37) and (4.38), making explicit the expression of ℓ_P and setting $\alpha' = 1$, we obtain:

$$R = \mathcal{N}\Delta R_{min} = 2\mathcal{N}\sqrt{\frac{\hbar G}{c^3}}. \tag{4.39}$$

The punctual (three-dimensional) mass, denoted by m, can be defined as follows:

$$m = \bar{M}\Delta R_{min}. \tag{4.40}$$

Condition 2 can be now fully understood. From Eqs. (3.6) and (4.40), in fact, we can immediately deduce how the value of the punctual mass is not influenced by the motion. In other terms, by virtue of the constancy of the linear density, m is considered as being constant (and the misleading concept of relativistic mass can be definitively rejected). Now, taking into account Eq. (4.30), for a material point, with obvious meaning of the notation, we can write:

$$E_m = \bar{E}\Delta R_{min} = \left(\bar{E}_k + \bar{E}_p + \bar{E}_w\right)\Delta R_{min} = E_{k,m} + E_{p,m} + E_{w,m}. \tag{4.41}$$

By multiplying both members of Equation (4.31) by ΔR_{min}, we have:

$$E_m = \gamma mc^2 = \left(1 - \frac{1}{\gamma^2}\right)mc^2 + \frac{mc^2}{\gamma^2} + (\gamma - 1)mc^2. \tag{4.42}$$

By multiplying all the members of Eq. (4.36) by ΔR_{min}, taking into account Eq. (4.40), the well-known relation for the relativistic energy is finally obtained (Einstein, 1916; Cheng, 2005):

$$E_m = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{mv^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} + \sqrt{1 - \left(\frac{v}{c}\right)^2}\, mc^2. \tag{4.43}$$

Denoting with p_m the momentum, with $\mathcal{L}$ the (relativistic) Lagrangian, and with $\mathcal{H}$ the Hamiltonian, we have:

$$p_m = \frac{mv}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \tag{4.44}$$

$$\mathcal{L} = -\sqrt{1 - \left(\frac{v}{c}\right)^2}\, mc^2, \tag{4.45}$$

Consequently, Eq. (4.43) can be concisely rewritten as follows:

$$E_m = \mathcal{H} = p_m v - \mathcal{L}. \tag{4.46}$$

4.5 Relativistic Phenomenology: Towards a New Interpretation

According to the results up to now obtained, what we perceive as being a (material) point may actually be a straight line (material) segment crossing the centre of the 4-Ball described by the inequality in Eq. (4.2). The endpoints represent all we are allowed to perceive of any segment. Coherently with the hypothesized central symmetry, moreover, the endpoints are to be considered as being a unique entity.

The Uniform Linear Motion of a punctual mass may actually be a rotation (with a constant angular speed) of the corresponding material segment around the centre of the 4-Ball. The rotation produce, concurrently, a loss of linear mass (although the value of the punctual mass is clearly preserved) and a (symmetric) reduction of the length: the new radial extension of the segment (half its length), denoted by R', depends on the value of the tangential speed acquired by its endpoints (the constant speed, denoted by v, which characterizes the apparent linear uniform motion. The relation between R' and v is expressed by Eq. (4.24) (Cataldo, 2019a, 2019b, 2017a, 2017b, 2016).

The Universe perceived by an observer involved in a linear uniform motion, characterized by a constant speed equal to v (being $0 \leq v < c$), may be closed and curved (Di Valentino at al., 2019), and therefore described by the following equality:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = R'^2. \tag{4.47}$$

5. The Lorentz Transformations

5.1 Bringing into Question the Real Meaning of the Lorentz Transformations

The Lorentz Transformations (Lorentz, 1904) can be considered, without any doubt, as the backbone of Special Relativity (from now onwards SR). Nonetheless, both the conventional derivation of the transformations and the meaning usually assigned to them have been often brought into question (Cataldo 2019a, 2019b, 2016).

Firstly, it is worth underlining how, as Lorentz himself was forced to admit at a later time (Lorentz 1909), the transformations had been already conceived, several years before the publication of the famous paper (Lorentz, 1904), by Voigt (Voigt, 1887). Secondly, the work of Lorentz was anything but concretely linked to relativistic issues, at least in the Einsteinian sense of the term (Einstein, 1916).

Very simply, Lorentz's aim fundamentally lay in finding some transformations able to formally make the Maxwell Equations (Maxwell, 1873) invariant. On this subject, moreover, it can be even proved how the Lorentz transformations are not the only ones able to preserve the formal validity of the Maxwell equations (Cataldo, 2019a, 2019b, 2016; Di Mauro et al., 1997).

The so-called direct transformations are usually written as follows:

$$x = \frac{x' + vt'}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \tag{5.1}$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}. \tag{5.2}$$

The so-called inverse transformations are usually written in the following form:

$$x' = \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \tag{5.3}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}. \tag{5.4}$$

It is commonly said that, when the speed assumed by the mobile frame of reference is far less than that of light, the Lorentz Transformations tend to the Galilean ones. In other terms, Galilean Relativity should be regarded as a particular case of the Einsteinian one. This is an erroneous conviction (Cataldo, 2016; Ghosal et al. 1991). In fact, referring to the ratio that appears in the numerator of Eqs. (5.2) and (5.4), it is easy to understand how no limitation turns out to be imposed, respectively, on the variables x and x'. Therefore, since the above mentioned variables should be allowed to assume arbitrarily large values, the ratio we have taken into consideration could even not tend to zero, so making de facto impossible a real identification of the Lorentz Transformations with the Galilean ones (Cataldo, 2019a, 2019b, 2016; Di Mauro et al., 1991).

5.2 Lorentz Transformations: Alternative Derivation and Different Meaning

In order to deduce the direct transformations, we will consider the scenario depicted in *Figure 3*. The deduction will be carried out net of the symmetry.

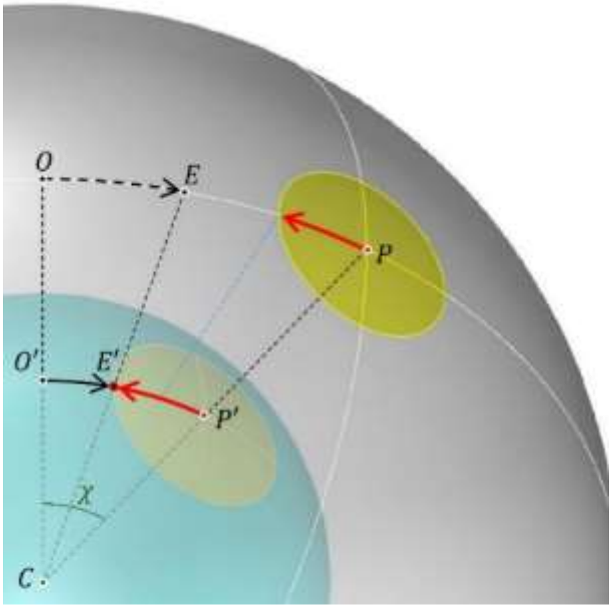

Figure 3. *First Scenario: Direct Transformations*

We denote with O the origin of the frame of reference at rest, and with O' the origin of the frame in motion. At the beginning O and O' coincide. We have to hypothesize that when O' starts moving, with a constant speed v, a light signal is simultaneously sent from a source, which both the observer at rest and the one in motion perceive as being punctual. The initial angular distance between the origins and the source is denoted by χ. The signal is actually sent, with a constant speed c, from each of the points that belong to the straight line segment bordered by the centre, denoted by C, and P (which represents the source as perceived by an observer at rest). The radial extension of any point at rest is equal to the radius of the 4-Ball (R).

As soon as O' starts moving, its radial extension, denoted by R', assumes the value provided by Eq. (4.24).

If we denote with l_{OP} the arc bordered by O and P, which represents the distance at rest from the source, and with $l_{O'P'}$ the arc bordered by O' and P', which represents the distance between O' and the source as soon as the motion occurs, taking into account Eq. (4.24), we can write the following:

$$\overline{CO} = \overline{CP} = R, \tag{5.5}$$

$$\overline{CO'} = \overline{CP'} = R', \tag{5.6}$$

$$l_{OP} = R\chi, \tag{5.7}$$

$$l_{O'P'} = R'\chi, \tag{5.8}$$

$$\frac{l_{OP}}{l_{O'P'}} = \frac{R}{R'} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}, \tag{5.9}$$

$$l_{OP} = \frac{l_{O'P'}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}. \tag{5.10}$$

The coordinate of the source as measured by the observer at rest, up until now denoted by l_{OP}, can be replaced by x. After a certain time, denoted by t', the observer in motion intercepts the signal in E'. The distance covered by the observer in motion is equal to $l_{O'E'}$ (the arc bordered by O' and E').

The time elapsed is equal to the time taken by light to cover the distance $l_{E'P'}$ (the arc bordered by E' and P'): this distance coincides with the coordinate of the source, denoted by x', as measured by the observer in motion (as soon as the signal is received). We have:

$$l_{OP} = x, \tag{5.11}$$

$$l_{E'P'} = x', \tag{5.12}$$

$$t' = \frac{l_{E'P'}}{c} = \frac{x'}{c}, \tag{5.13}$$

$$l_{O'E'} = vt', \tag{5.14}$$

$$l_{O'P'} = l_{E'P'} + l_{O'E'} = x' + vt'. \tag{5.15}$$

From Eqs. (5.10) and (5.15), exploiting the position in Eq. (5.11), we can deduce Eq. (5.1), which represents the first direct Lorentz Transformation.

By dividing both members of Equation (5.1) by c, we obtain:

$$\frac{x}{c} = \frac{\frac{x'}{c} + \frac{vt'}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}. \tag{5.16}$$

The first member of the previous equation, which can be denoted by t, represents the time elapsed between the light signal emission and the moment in which the observer at rest succeeds in seeing it. From Eqs. (5.13) and (5.16) we can obtain Eq. (5.2), which represents the second direct Lorentz Transformation.

In order to deduce the inverse transformations, we will consider the scenario depicted in *Figure 4*.

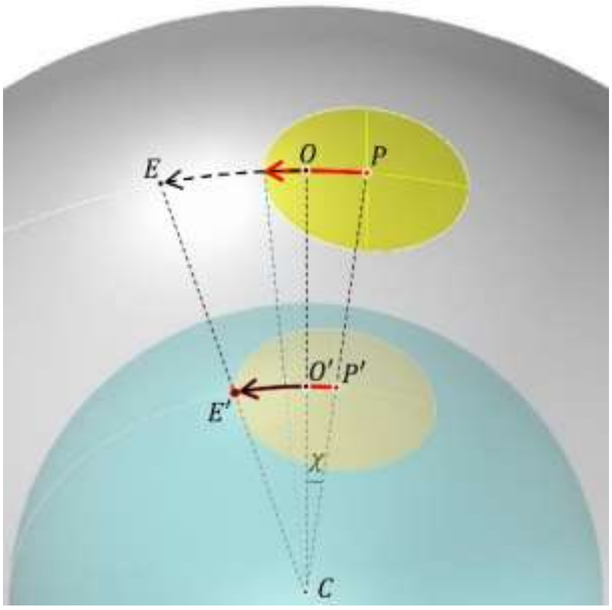

Figure 4. *Second Scenario: Inverse Transformations*

This time, we have to suppose that the motion occurs anti-clockwise (once again, with a constant speed equal to v). Obviously, the Eqs. from (5.5) to (5.10) are still valid.

We can exploit the line of reasoning previously followed in deriving the direct transformations, being careful to switch the superscripts: from the point of view of the observer in motion, in fact, the one at rest, placed in O, seems to approach the light source (moving with a constant speed equal to v). Therefore, we can write:

$$l_{OP} = x', \tag{5.17}$$

$$l_{E'P'} = x, \tag{5.18}$$

$$t = \frac{l_{E'P'}}{c} = \frac{x}{c}, \tag{5.19}$$

$$l_{E'O'} = vt, \tag{5.20}$$

$$l_{O'P'} = l_{E'P'} - l_{E'O'} = x - vt. \tag{5.21}$$

From Eqs. (5.10) and (5.21), exploiting the position in Eq. (5.17), we can deduce Eq. (5.3), which represents the first inverse Lorentz Transformation.

By dividing both members of Equation (5.3) by c, we obtain:

$$\frac{x'}{c} = \frac{\frac{x}{c} - \frac{vt}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}. \tag{5.22}$$

The first member of the previous equation, which can be denoted by t', represents the time elapsed between the light signal emission and the moment in which the observer placed in O, actually at rest but considered as being in relative motion towards the source, succeeds in seeing it. From Eqs (5.19) and (5.22) we can obtain Eq. (5.4), which represents the second direct Lorentz Transformation.

It is fundamental to underline that, if we take into account the symmetry, both the direct transformations and the inverse ones can be simultaneously applied to whatever point in motion with a constant speed equal to v.

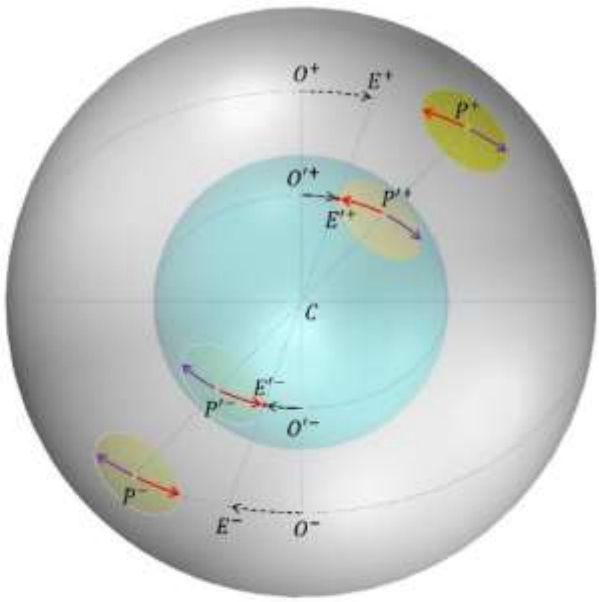

Figure 5. *First Scenario with Symmetry*

Referring to *Figure 5* (which represents just a modified version of *Figure 3*), in fact, we can easily notice how, due to the symmetry, the light signals start not only from P^+ and P'^+, but also from P^- and P'^-, moving both clockwise and counterclockwise. Very simply, the observer in motion travels towards the signal that propagates anti-clockwise (starting from P^+ and P'^+), so making possible the adoption of the direct transformations; simultaneously, the same observer moves away from the signal that propagates clockwise (starting from P^- and P'^-), so making possible the adoption of the inverse transformations.

5.3 Apparent Speed and Reduced Distances

The distance between the origin of the mobile frame of reference and the light source undergoes a reduction as soon as the motion takes place: the higher the value of the speed, the higher the entity of the reduction. For example, referring to the first of the two cases previously examined, we can state that the observer in motion is able to cover the distance $l_{O'P'}$ by taking a time, denoted by t_{mob}, provided by the following relation:

$$t_{mob} = \frac{l_{O'P'}}{v}. \tag{5.23}$$

However, once the observer in motion reaches the light source, the observer at rest believes that the covered distance may be equal to l_{OP}. As a consequence, from the point of view of the observer at rest, the mobile frame is moving with an "apparent" speed, denoted by v_{app}, provided by the following relation:

$$v_{app} = \frac{l_{OP}}{t_{mob}} = \frac{l_{OP}}{l_{O'P'}} v. \tag{5.24}$$

From the foregoing, taking into account Equation (5.9), we immediately obtain:

$$v_{app} = \frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}. \tag{5.25}$$

Consequently, the observer at rest will measure, in any case, a speed greater than v. Obviously, the "real" speed (v) can never equate that of light. On the contrary, the apparent speed (v_{app}) tends to infinity when the speed tends to that of light.

From Eq. (5.25) we can deduce the relation that expresses v as a function of v_{app}:

$$v = \frac{v_{app}}{\sqrt{1+\left(\frac{v_{app}}{c}\right)^2}}. \tag{5.26}$$

Let us choose a generic "destination". Generalizing Eq. (5.10), if we denote with l the distance (at rest) from the point that we have to reach, and with l_{mob} the corresponding reduced distance (the distance that a traveler, who starts moving with a constant speed v, should actually cover in order to reach the destination), we have:

$$l_{mob} = l\sqrt{1-\left(\frac{v}{c}\right)^2}. \tag{5.27}$$

6. Short Remarks and Conclusions

- The variation of the cosmological distances is actually considered as being an apparent phenomenon: in other terms, we postulate that the amount of space between whatever couple of points remains the same with the passing of time (on this subject, it could be worth bearing in mind how Hubble himself started bringing into question the relation between redshift and recessional velocity of the astronomical objects) (Hubble, 1947).

More precisely, we hypothesize that the so-called cosmological redshift may banally related to the conservation of energy.

As is well known, the energy of a quantum of light can be expressed as the product between the value of its frequency and the Plank Constant.

On the one hand, as an alternative to the conventional interpretation of the cosmological redshift, we could imagine that, in travelling through the interstellar vacuum, light may somehow "get tired", so losing part of its energy (Zwicky, 1929; Geller, 1972; LaViolette, 1986).

On the other hand, we may accept that the Plank Constant could vary over time (Seshavatharam, 2013; Seshavatharam et al., 2013; Mangano et al., 2015): in this case a photon, in order to preserve its energy, would be forced into modifying its frequency. In the latter case, all the cosmological equations can be rewritten as a function a Plank parameter.

From Eq. (2.36), taking into account Equation (4.39), we can easily deduce:

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2 = \frac{1}{4}\left(\frac{\dot{h}}{h}\right)^2. \tag{6.1}$$

By virtue of the previous, Eq. (2.1) (with $k = 0$), can be rewritten as follows:

$$\left(\frac{dh}{dt}\right)^2 = \frac{4}{3}(8\pi G\rho + \Lambda c^2)h^2. \tag{6.2}$$

The problem related to the singularity (Harrison, 1967; Turok, 2015), herein not addressed, may be solved by resorting to the (space and time) quantization: a “quantum bounce” (Gielen et al., 2016; Ijjas et al. 2016), in fact, may prevent the radius from assuming a null value. To extremely simplify, the time at which the Planck Constant should mathematically assume a null value would fall between two consecutive time-steps.

- Referring to each of the three-dimensional scenarios that arise from Eqs. (4.2) and (4.5), the position $\nu = 1/3$ banally entails the constancy of the surface energy.

Denoting with σ the surface tension, the Young-Laplace Equation (Young, 1805) can be written as follows:

$$p = \frac{2\sigma}{R}. \tag{6.3}$$

From the foregoing, by virtue of Eqs. (2.3) and (2.5) (being $\nu = 1/3$), we have:

$$\frac{d\sigma}{dt} = \frac{1}{2}\frac{d}{dt}(pR) = -\frac{1}{3}c^2\frac{d}{dt}(\rho R) = -\frac{1}{3}c^2\frac{dC}{dt} = 0. \tag{6.4}$$

- The Universe is identified with a 4-Ball (involved in an apparent cyclic evolution). The concept of material point is replaced by the one of material segment: what is perceived as being a point may actually be a segment crossing the centre of the 4-Ball. Two antipodal points, since they evidently represent the end-points of the same segment, must be considered as being a unique entity: in other terms, the Universe may be characterized by a (global) central symmetry.

Although the topic is herein not addressed, it is worth underlining how the centre of the 4-Ball with which we identify the Universe cannot be exactly located. The quantization proposed in Eq. (4.37), in fact, must be regarded as approximate. Actually, the “centre” may consist in a minimal 4-Ball, characterized by a diameter equal to the minimal length defined in Eq. (4.38): inside the corresponding border (a minimal hyper-sphere), the concept of "separation" may become de facto meaningless.

- The extra spatial dimension and the space quantization allow the writing of all the relativistic equations concerning energy, although with different connotations. The conservation of energy is derived by considering an additional non-material component, introduced in Eq. (4.18), related to the concept of Quantum Potential (Bohm, 1952a, 1952b). In particular, albeit the punctual mass is considered as being constant, the linear mass may undergo a reduction with the increasing of speed. (Cataldo, 2019a, 2019b, 2017b, 2016). The non-material component may simply compensate for the linear mass reduction. In particular, there is no mass in the range $]R', R]$. Ultimately, any rotating segment (perceived as a translating point) may also exhibit a wave-like behavior. On this subject, taking into account Eq. (5.25), denoting with h the Planck Constant and with λ_{Rel} the de Broglie (relativistic) Length (de Broglie, 1970), we have:

$$\lambda_{Rel} = \frac{h}{mv_{app}} = \frac{h}{mv}\sqrt{1-\left(\frac{v}{c}\right)^2}. \tag{6.5}$$

- As underlined in the introduction, time is considered as being absolute. Nevertheless, instruments and devices of whatever kind, commonly employed to measure time, may be (even significantly) influenced by motion (and gravity). To be clearer, time does not slow down with the increasing of speed (or in approaching a gravitational source) (Cataldo, 2019a). Muons succeed in covering a distance not compatible with their mean life-time (Rossi at al., 1941): this phenomenon is commonly legitimised by resorting to SR. Actually, muons may simply exploit a "shortcut", as it were. According to our model, in fact, the proper distance between whatever couple of points depends on their speed and, what is more, is no longer symmetric. In other terms, the very moment a particle starts moving, all the proper distances must be redefined as a function of the speed: on the contrary, all the angular distances preserve their initial value. Obviously, both the angular distances and the proper ones vary during the motion. This scenario is formally coherent with the Lorentz Transformations, the meaning of which, however, is completely altered. Amongst the significant consequences that arise from our approach, the possibility of (apparently) moving faster than light stands out.

Acknowledgements

This paper is dedicated to my father, Antonio Cataldo, who unexpectedly passed away on the 11th of June 2016.

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