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## Couple Stresses and Thermophoresis Influences on Free Convective Heat and Mass Transfer of Viscoelastic Fluid with Hall and Ion-Slip Effects.

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### Abstract

An analysis is presented to investigate the Hall and Ion-slip effects on non-Newtonian couple stress fluid flow between vertical channel in the presence of thermophoresis phenomena. The considered fluid obeys to the viscoelastic second-grade model. The problem is modulated mathematically to describe continuity, momentum, temperature, and concentration equations. The influences of physical parameters of dimensionless equations of velocity components ( $u$ ) and ( $w$ ), temperature ( $\theta$ ) and concentration ( $\phi$ ) have been shown in some of figures, as well as skin friction ( $\tau^*$ ), Nusselt number ( $Nu$ ) and Sherwood number ( $Sh$ ) have been computed. It is found that the velocity  $u(y)$  decreases with increasing in Hall parameter and Ion slip parameters.

**Keywords:** Hall and ion-slip; Thermophoresis; Couple stresses; Viscoelastic.

### Introduction

During recent years the study of convection heat and mass transfer in non-Newtonian fluids has received much attention, and this is because the traditional Newtonian fluids cannot precisely describe the characteristics of the real fluids. The spin couple stresses the theory of fluids showed size-dependent due to microrotation of freely suspended particles set up an antisymmetric stress, known as couple-stress, and thus forming couple-stress fluid. These fluids such as lubricants, blood, suspension fluids, etc. and their applications industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of the metallic plate in a bath, and colloidal solutions, etc. Stokes [1] discussed the hydromagnetic steady flow of a fluid with a couple stress effects. In this paper, we have investigated the Hall and Ion-slip effects on steady free convective heat transfer flow between two vertical parallel plates in a couple of stress fluid. Piazza [2] has displayed thermophoresis which mean moving particles with thermal gradients, consequently, particle thermophoresis is a non-equilibrium cross-flow effect between mass and heat transport. In addition, it is already exploited as a novel tool in macromolecular fractionation, micro-fluidic manipulation, and selective tuning of colloidal structures. Analysis of thermophoretic MHD slip flow over a permeable surface has been presented by Das et al [3]. Chamkha et al [4] study the Hall current, thermal radiation, and chemical reaction effects on heat and mass transfer. Hayat and Nawaz [5] have analyzed second-grade fluid flow in three dimensional under the Hall and ion slip effects. Chaudhary and Jain [6] have elucidated the effect of Hall current on MHD mixed convection flow of a viscoelastic fluid past an infinite vertical porous plate with mass transfer and radiation. Similarly, Kumar and Chand [7] have investigated the effect of slip conditions and Hall current on unsteady MHD flow of a viscoelastic fluid past an infinite porous plate embedded in a porous medium. Sahoo [8] has studied heat and mass transfer effect on MHD flow of a viscoelastic fluid through a porous medium bounded by an oscillating porous plate in slip flow regime. Furthermore, Jha et al. [9] have examined the effect of Soret and Hall current on MHD mixed convection flow of visco-elastic fluid past a vertical surface. Recently (in 2016), K.Das et al [10] have studied influences of thermophoresis and thermal radiation on heat and mass transfer of second grade MHD fluid flow past a semi-infinite stretching sheet with convective surface heat flux. Also, K. Das [11] has investigated the combined effects of thermophoresis and thermal radiation on MHD mixed convective heat and mass transfer flow of a second-grade fluid past a semi-infinite stretching sheet in the presence of viscous dissipation and Joule heating. M.VeeraKrishna and B.V.Swarnalathamma [12] have studied Hall effects on unsteady MHD free convection flow of an incompressible electrically conducting Second-grade fluid through a porous medium over an infinite

rotating vertical plate fluctuating with Heat Source/Sink and chemical reaction. Nayak and Panda [13] have presented an interesting result on mixed convective MHD flow of second-grade fluid past an infinite vertical plate with mass transfer, Joule heating, and viscous dissipation. Recently, El-Dabe et al.[14] have analyzed the effects of thermophoresis and hall currents on non-linear heat and mass transfer of second-grade fluid flow over a permeable infinite vertical plate with the presence of viscous dissipation and chemical reaction. The focus of this study extends the work of Srinivasacharya and Kaladhar [15], in addition, to study a non-Newtonian model which obeyed to viscoelastic second-grade fluid and Thermophoresis phenomena.

**Problem Formulation:**

Consider an incompressible couple stress fluid flow between two vertical parallel plates distance  $2d$  apart. Choose the coordinate system such that the  $x$ -axis is taken along vertically upward direction through the central line of the channel,  $y$  is perpendicular to the plates, and the two plates are infinitely extended in the direction of  $x$  and  $z$ . The plates of the channel are at  $y = \pm d$ . The effect of Hall and Ion-slip current gives rise to force in the  $z$ -direction, which induces a cross flow in that direction and hence the flow becomes three dimensional. Assume that the flow is steady and the magnetic Reynolds number is very small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. Further, assume that all the fluid properties are constant except the density in the buoyancy term of the balance of momentum equation.

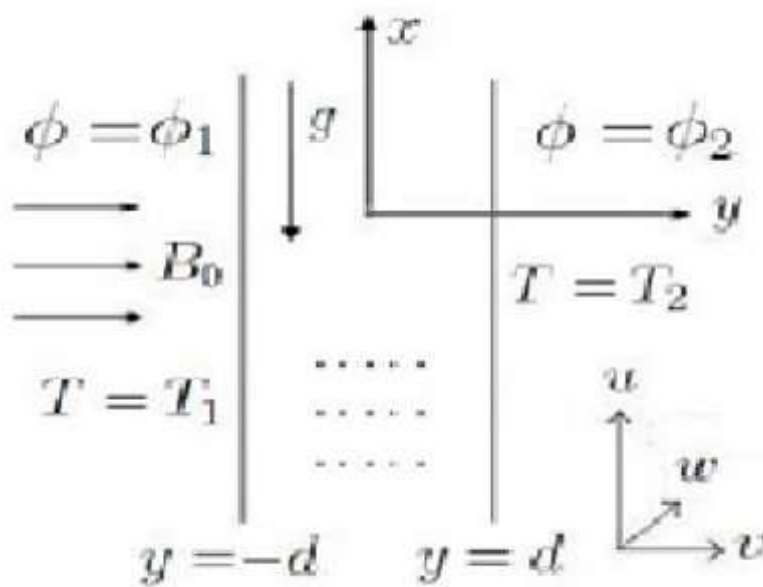


Figure 1: Physical configuration with the coordinate system

Stokes, Mindlin, and Tiersten are obtained the constitutive equations as:

$$T_{ij}^S = -P\delta_{ij} + \alpha(D_{rr})\delta_{ij} + 2\mu D_{ij} \tag{1}$$

$$M_{ij}^D = 4\eta K_{ij} + 4\eta' K_{ji} \tag{2}$$

Such that  $D_{ij}$  is the rate of deformation tensor,  $\delta_{ij}$  is the Kronecker delta,  $\alpha$  and  $\mu$  are the material

constants of viscosity,  $K_{ij}$  is the curvature-twist tensor,  $\eta$ , and  $\eta'$  are the material constants of momentum and  $M_{ij}$  is the couple stress tensor and  $P$  is the pressure. The equation of motion as obtained by Stokes in Gibbs notation as:

$$\rho a = -\nabla P + (\alpha + \mu)\nabla\nabla \cdot V + \eta\nabla^2\nabla\nabla \cdot V + \mu\nabla^2V - \eta\nabla^4V + \frac{1}{2}\nabla \wedge (\rho\ell) + \rho f \quad (3)$$

where,  $a = \frac{\partial V}{\partial t} + V \cdot \nabla V$ , For incompressible fluids  $\text{div}V = 0$  so, if the body force  $f$  and the body moment are absent, the equations of motion reduce to

$$\rho a = -\nabla P + \mu\nabla^2V - \eta\nabla^4V \quad (4)$$

where the fourth derivative in the above equation gives the effect of couple stresses.

We used a second-grade viscoelastic model For these according to Truesdell and Noll (1965):

$$T = -PI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2, \quad (5)$$

the coefficients  $\mu$ ,  $\alpha_1$  and  $\alpha_2$  describe the viscosity, visco-elasticity, and cross-viscosity, respectively and must satisfy:

$$\mu \geq 0, \quad \alpha_1 \geq 0, \quad \alpha_1 + \alpha_2 = 0. \quad (6)$$

material constants and  $A_1$  and  $A_2$  are the kinematic Rivlin-Ericksen tensors,

Wu and Greif have determined the thermophoretic velocity  $V_T$  as

$$V_T = \frac{-k_t v}{T_r} \nabla T = \frac{-k_t v}{T_r} \frac{dT}{dy}, \quad (7)$$

A thermophoretic parameter  $\tau$  can be defined as follows;

$$\tau = \frac{k_t(T_2 - T_1)}{T_r}, \quad (8)$$

where,  $T_r$  is some reference temperature,  $k_t v$  represents the thermophoretic diffusivity while,  $k_t$  is the coefficient of thermophoresis which ranges in value from 0.2 to 1.2.

the typical values of  $\tau$  are 0.01, 0.05 and 0.1 corresponding to approximate values of  $(T_2 - T_1)$  equal to 3.15 and 30k for a reference temperature of  $T_r = 300k$ .

the generalized Ohm's law taking the effects of Hall and ion-slip currents, and Maxwell's equations into account are:

$$\rho[(V \cdot \nabla)V] = \mu\nabla^2V - \mu_1\nabla^4V + J \wedge B + \nabla \cdot \tau \quad (9)$$

$$J = \frac{\sigma\alpha_e}{\alpha_e^2 + \beta_e^2}(E + V \wedge B) - \frac{\sigma\alpha_e}{\alpha_e^2 + \beta_e^2} \wedge \frac{B}{B_0} \quad (10)$$

$$\nabla \cdot V = 0, \nabla \wedge H = J, \nabla \wedge E = 0, \nabla \cdot B = 0 \quad (11)$$

where  $V$  is the velocity vector,  $\rho$  is the density of the fluid,  $t$  is the time,  $p$  is the pressure,  $\mu$  is the dynamic viscosity,  $\mu_1$  is the couple stress parameter,  $\nabla^2$  is the Laplacian operator,  $J$  is the electric current,  $B$  is the magnetic induction vector,  $E$  is the intensity of the electric field,  $\sigma$  is the electric conductivity,  $H$  is the magnetic field strength, and  $\alpha_e = 1 + \beta_i\beta_e$ ,  $\beta_i$  is the ion-slip parameter, and  $\beta_e$  is the Hall parameter. By using the relation  $\nabla \cdot B = 0$  for the magnetic field  $B = (B_x, B_y, B_z)$  we obtain  $B_z = B_0$  (imposed magnetic field)

everywhere in the field. This assumption is valid only when the magnetic Reynolds number is very small. For the current density  $J = (J_x, J_y, J_z)$ , we obtain from the relation  $\nabla \cdot J = 0$  that  $J_z = \text{constant}$ . Hence we consider that the channel is non-conducting and therefore  $J_z = 0$  at the channel and hence is zero everywhere.

Within the above framework, the governing partial differential equations of the flow under the usual Boussinesq approximation are:

continuity equation:

$$v^{*'} = 0, \quad (12)$$

momentum conservation in x-direction:

$$\rho v_o u^{*'} = \mu u^{*''} + \alpha_1 v_o u^{*'''} - \eta u^{*(iv)} + \rho g \beta_T (T^* - T_1^*) + \rho g \beta_c (\phi^* - \phi_1^*) - \frac{\sigma B_0^2}{\alpha_e^2 + \beta_h^2} (\alpha_e u + \beta_h w), \quad (13)$$

momentum conservation in z-direction:

$$\rho v_o w^{*'} = \mu w^{*''} + \alpha_1 v_o w^{*'''} - \eta w^{*(iv)} + \frac{\sigma B_0^2}{\alpha_e^2 + \beta_h^2} (\beta_h u - \alpha_e w), \quad (14)$$

heat equation:

$$v_o T^{*'} = \frac{\kappa}{\rho c_p} T^{*''} + \frac{\alpha_1}{\rho c_p} v_o u^{*'} u^{*''} + 2 \frac{\mu}{\rho c_p} (u^{*'})^2 + \frac{\eta}{\rho c_p} (u^{*''})^2, \quad (15)$$

concentration equation

$$v_o \phi^{*'} = D \phi^{*''} - (V_T (\phi^* - \phi_1^*))' - k_c (\phi^* - \phi_1^*). \quad (16)$$

The appropriate boundary conditions for the problem are:

$$\left. \begin{array}{l} \text{at } y^* = -d \quad u^* = 0, u_{yy}^{*'} = 0, T^* = T_1^*, \phi^* = \phi_1^* \\ \text{as } y^* = d \quad u^* = 0, u_{yy}^{*'} = 0, T^* = T_2^*, \phi^* = \phi_2^* \end{array} \right\} \quad (17)$$

such that all differentiations are respected to  $y^*$  and the nomenclature in the above equations are:  $\nu$  is the kinematic viscosity,  $\rho$  is the fluid density,  $g$  is the gravitational acceleration,  $\beta_T$  and  $\beta_c$  are thermal expansion volumetric coefficient and volumetric coefficient of concentration expansion, respectively.  $T^*$  is denote the dimensional temperature of the fluid,  $C^*$  is the dimensional concentration of the solute [?],  $c_p$  is the specific heat at constant pressure,  $\kappa$  is the thermal conductivity of the medium, and  $D$  is chemical molecular diffusivity. From eq.(12), it can be seen that  $v^*$  is either a constant or a function of time. Thus, assuming that the suction velocity to be constant. where,  $v_o$  is a scale of mean suction velocity, Let us obtain the dimensionless equations by using the following non-dimensional variables:

$$\left. \begin{array}{l} u = \frac{u^*}{v_o}, w = \frac{w^*}{v_o}, y = \frac{v_o y^*}{\nu} \\ \theta = \frac{T^* - T_1^*}{T_2^* - T_1^*}, \phi = \frac{\phi^* - \phi_1^*}{\phi_2^* - \phi_1^*} \end{array} \right\} \quad (18)$$

Using the above dimensionless quantities (18), the system of equations (14), (15) and (16) with boundary conditions (17) can be written in dimensionless form as:

$$u^{(iv)} - \alpha a^2 u^{*'''} - a^2 u^{*''} + a^2 u^{*'} - a^2 G_r \theta - a^2 G_m \phi + \frac{H a^2}{\alpha_e^2 + \beta_h^2} (\alpha_e u + \beta_h w) = 0 \quad (19)$$

$$w^{(iv)} - \alpha a^2 w''' - a^2 w'' + a^2 w' - \frac{Ha^2}{\alpha_e^2 + \beta_h^2} (\beta_h u - \alpha_e w) = 0 \quad (20)$$

$$-a^2 \theta'' + Pr a^2 \theta' = Pr Ec u'^2 + \alpha Ec a^2 Pr u' u'' + 2 Ec a^2 Pr u'^2 \quad (21)$$

$$\phi'' - Sc \phi' = \tau (\phi \theta'' + \phi' \theta') + \gamma \phi \quad (22)$$

with the boundary conditions :

$$\left. \begin{array}{l} \text{at } y = -1 \quad u = 0, u'' = 0, w = 0, w'' = 0, \theta = 0, \phi = 0 \\ \text{at } y = 1 \quad u = 0, u'' = 0, w = 0, w'' = 0, \theta = 1, \phi = 1 \end{array} \right\} \quad (23)$$

where primes denote differentiation with respect to  $y$ ; the dimensionless numbers are  $\alpha = \frac{\alpha_1 v_0^2}{\rho v^2}$  is the viscoelastic parameter,  $\frac{1}{a^2} = \frac{\eta v_0^2}{\rho v^3}$  is the couple stresses parameter,  $Gr = \frac{g \beta_T (T_2^* - T_1^*) \nu}{v_0^3}$  is the Grashof number,  $Gm = \frac{g \beta_c (C_2^* - C_1^*) \nu}{v_0^3}$  is the modified Grashof number,  $Ha = B_0 d \sqrt{\frac{\sigma}{\mu}}$  is the Hartmann number, Prandtl number  $Pr = \frac{\mu c_p}{k}$  that represents the ratio of momentum to thermal diffusivity,  $Ec = \frac{v_0^2}{c_p (T_2^* - T_1^*)}$  is the Eckert number, the Schmidt number  $Sc = \frac{\nu}{D}$  that represents the ratio of momentum to mass diffusivity,  $\gamma = \frac{k_c \nu}{v_0^2}$  is the non-dimensional chemical reaction parameter,  $\tau = \frac{k_t}{T_r} (T_2^* - T_1^*)$  It is the thermophoretic parameter.

### **Skin friction**

At the surface  $y = 0$ , the skin friction coefficient can be obtained by:

$$\tau^* = -\left(\frac{du}{dy}\right) \Big|_{y=0} \quad (24)$$

### **Nusselt Number**

The rate of heat transfer between plates and the fluid can be expressed in terms of non-dimensional Nusselt number as  $q_w = -\kappa \frac{d\theta}{dy} = \kappa Nu$  where the Nusselt number is given by:

$$Nu = -\left(\frac{d\theta}{dy}\right) \Big|_{y=0} \quad (25)$$

### **Sherwood Number**

The rate of mass transfer between plates and the fluid can be expressed in terms of non-dimensional Sherwood number as  $j_m = -\rho D \frac{d\phi}{dy} = \rho D Sh$  where the Sherwood number is given by:

$$Sh = -\left(\frac{d\phi}{dy}\right) \Big|_{y=0} \quad (26)$$

## **Results and discussions**

A parametric ND solve package was designed in Mathematica 10 software program to simulate the numerical computations of differential equations system which describe our problem. The purpose of these numerical solutions to illustrate the effect of different governing physical parameters such as viscoelastic second-grade parameter  $\alpha$ , Hall parameter  $\beta_e$ , Ion-slip parameter  $\beta_i$ , inverse of couple stress parameter  $a$ , Grashof number  $Gr$ , modified Grashof number  $Gm$ , Prandtl number  $Pr$ , Eckert number  $Ec$ , Schmidt number  $Sc$ , thermophoretic parameter  $\tau$  and chemical reaction parameter  $\gamma$  on the velocity  $u(y)$ , the temperature  $\theta(y)$  and the

concentration  $\phi(y)$  fields which they have been done at the following values:  $\alpha = 0.5$ ,  $a = 2$ ,  $Gr = 5$ ,  $\tau = 0.1$ ,  $\gamma = 0.2$ ,  $Gm = 5$ ,  $Pr = 0.71$ ,  $Ec = 0.01$ ,  $Sc = 0.22$ ,  $\beta h = 1$ ,  $Ha = 2$ ,  $\alpha e = 0.5$ . As well as Skin friction  $\tau^*$ , Nusselt number  $Nu$  and Sherwood number  $Sh$  have been computed and represented in tabulated forms at the same values of parameters. In order to get a clear insight into the physics of the problem, a representative set of numerical results are shown graphically in figures (2 – 21). In fig. 2, by increasing the viscoelastic second-grade parameter  $\alpha$ , the velocity profile  $u(y)$  decreases. It is observed from fig. 3 that an increase in the couple stress inverse parameter  $a$  leads to decreasing in the values of velocity  $u(y)$ . The velocity  $u(y)$  curves decrease when the thermophoretic parameter  $\tau$  increases, as seen in fig. 4. The values of Grashof number  $Gr$  are taken to be positive and negative as they respectively represent symmetric heating of the plates when  $Gr < 0$  and symmetric cooling of the plates when  $Gr > 0$ . The velocities profiles  $u(y)$  and  $w(y)$  increase by increasing the Grashof number  $Gr$  as happened in modified Grashof number  $Gm$  see fig. 5 and fig. 6, respectively. This is due to the buoyancy forces, which is acting on the fluid particles due to the gravitational force that enhances the fluid velocity. The increase in the Prandtl number  $Pr$  results in the decrease in  $w(y)$  as displayed in fig. 7. This is physically true because, the Prandtl number is a non-dimensional number which is the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. The effect of increasing Eckert number  $Ec$  is to enhance  $u(y)$  as shown in fig. 8. Higher Eckert number values lead to greater viscous dissipative heat. On the contrary, the reversed effect has been observed in fig. 9 where it indicates that velocity curves decreases with increasing of Schmidt number  $Sc$ . Physically it is true as the concentration increase the density of the fluid increases which results a decrease in fluid particles. The combination effect for hall and ion slip parameter has been shown in fig.10 fig. 11 depicts that  $w(y)$  decreases as the chemical reaction parameter  $\gamma$  increases because of the presence of viscous dissipation. As a general description, velocity curves of this fluid are increased from injected plate to reach the top of the curves (the highest value for the speed) which locate next to the center of channel and then they decrease until it reaches zero at the sucked plate. This mean that the velocity profiles of the fluid is maximum at the center of the channel and zero at the plates. Fig. 12 depicts that  $u(y)$  decreases as ion-slip parameter  $\beta_i$  increases, this also happen in fig.13but with hall parameter  $\beta_e$ .

### Velocities profiles

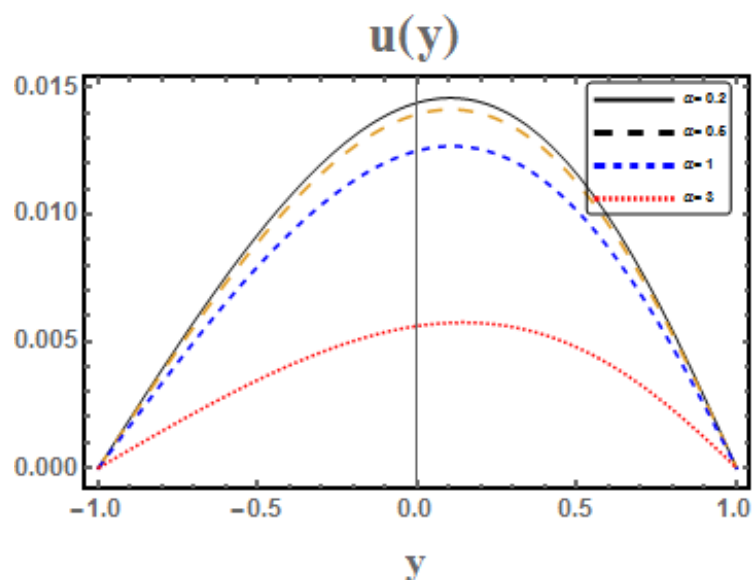


Figure 2: Effect of the viscoelastic parameter  $\alpha$  on velocity profiles  $u(y)$ .

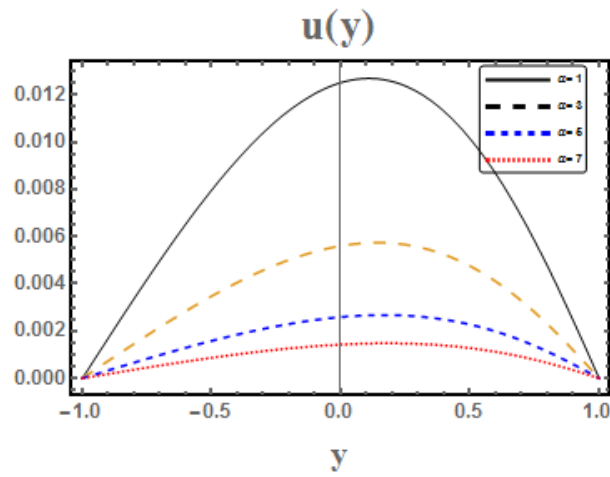


Figure 3: Effect of couple stress inverse parameter  $\alpha$  on velocity profiles  $u(y)$ .

Figure 4: Effect of thermophoretic parameter  $\tau$  on velocity profiles  $u(y)$ .

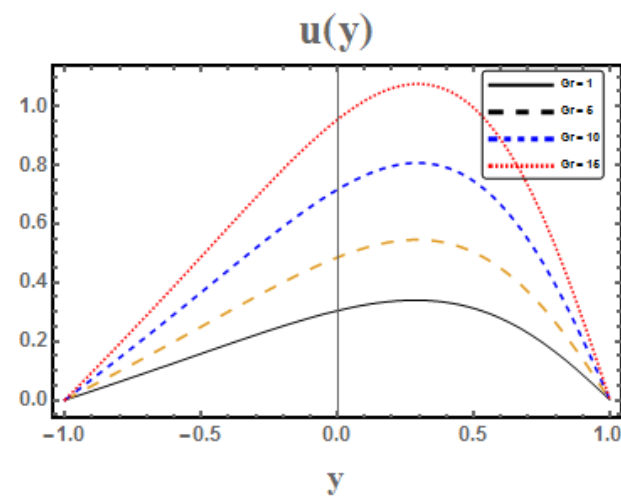
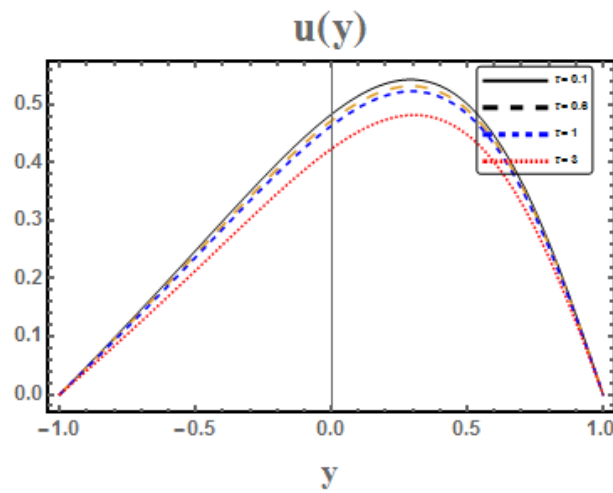


Figure 5: Effect of Grashof number  $Gr$  on velocity profiles  $u(y)$ .

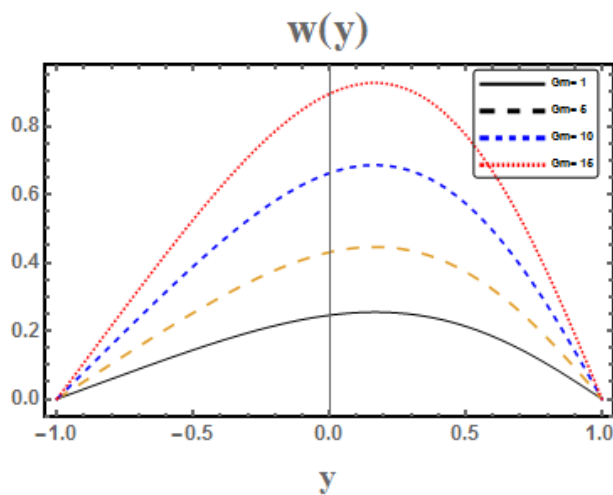


Figure 6: Effect of modified Grashof number  $Gm$  on velocity profiles  $w(y)$ .

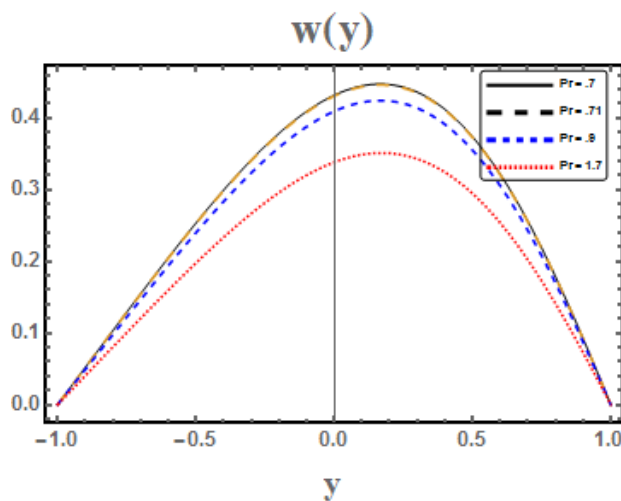


Figure 7: Influence of Prandtl parameter  $Pr$  on velocity profiles  $w(y)$ .

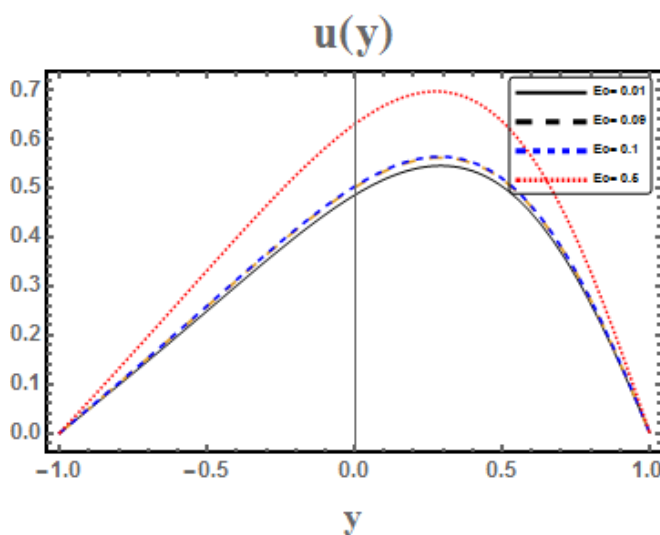


Figure 8: Effect of Eckert parameter  $Ec$  on velocity profiles  $u(y)$ .



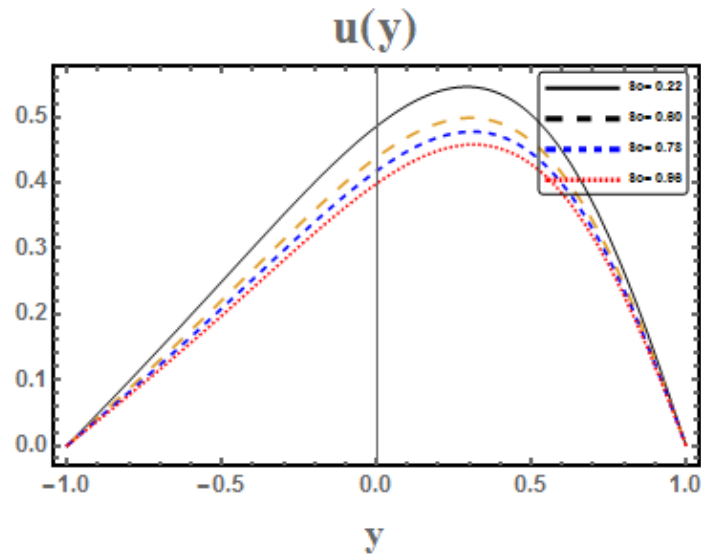


Figure 9: Effect of Schmidt parameter  $Sc$  on velocity profiles  $u(y)$ .

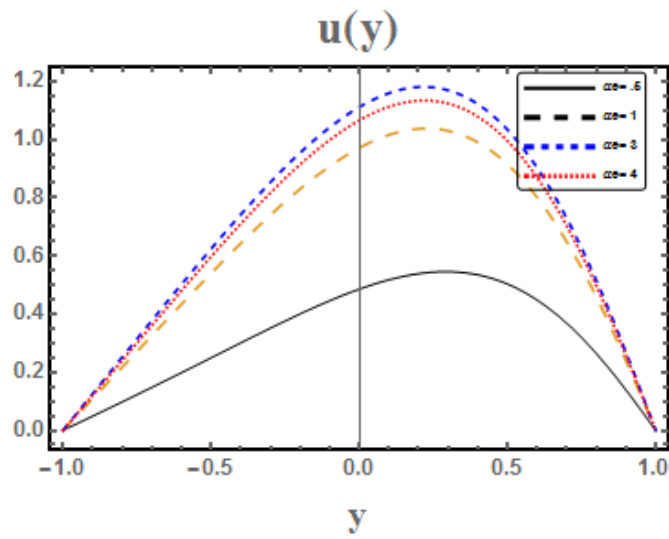


Figure 10: Effect of hall and ion slip parameter  $ae$  on velocity profiles  $u(y)$ .

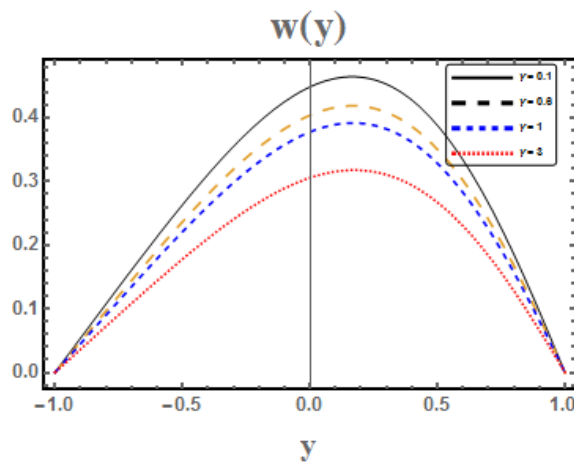


Figure 11: Effect of chemical parameter  $\gamma$  on velocity profiles  $u(y)$ .

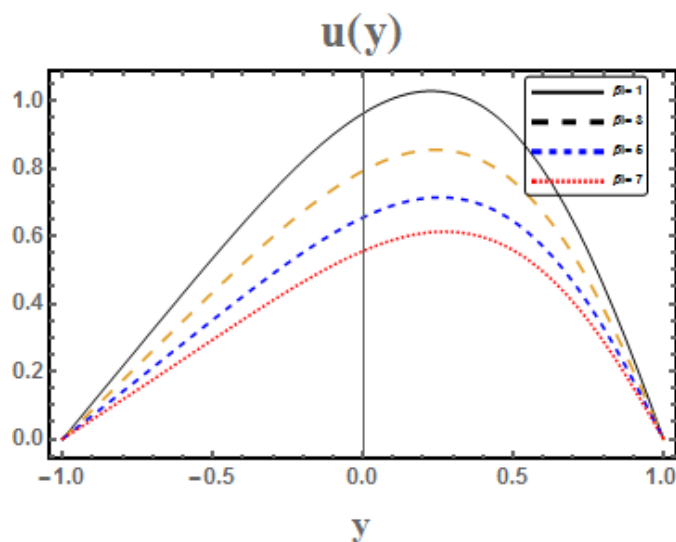


Figure 12: Effect of ion slip parameter  $\beta_i$  on velocity profiles  $u(y)$ .

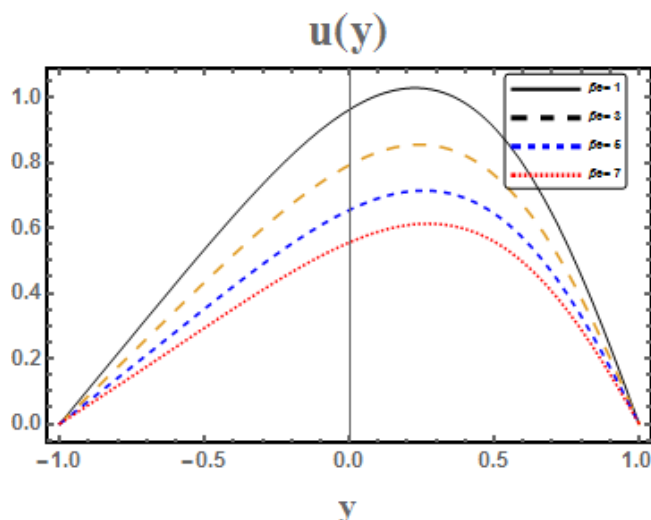


Figure 13: Effect of Hall parameter  $\beta_e$  on velocity profiles  $u(y)$ .

**Temperature profiles**

Fig. 14 exhibits the temperature profiles  $\theta(y)$  for various values of Prandtl number  $Pr$ . From this figure, it is noticed that by increasing  $Pr$ , the temperature decreases. Small values of the Prandtl number,  $Pr \ll 1$ , means that the thermal diffusivity dominates. Whereas with large values of  $Pr \gg 1$ , the momentum diffusivity dominates the behavior. By choose the values of Prandtl number where they are physically correspond to gases and fluids which are commonly used in commerce and industry. Around 0.16-0.7 for mixtures of noble gases or noble gases with hydrogen and at room temperature  $Pr$  is 0.71 . fig. 15 displays influence of Eckert parameter  $Ec$  on the temperature profiles  $\theta(y)$ . It is observed that temperature  $\theta(y)$  increases as  $Ec$  increases. Effect of Grashof number  $Gr$  and modified Grashof number  $Gm$  on temperature profiles  $\theta(y)$  were studied into figure 16 and 17 respectively . From the above figures it is noticed that the temperature distribution  $\theta(y)$  of the fluid flow increases as increase both of  $Gr$  and  $Gm$ . From these figures it is noticed that thermal boundary layer stretch when the values of  $Ec$ ,  $Gr$  and  $Gm$  parameters increase. Generally, The fluid has a high temperature at the positive side of the channel compared to the another part which it locates near to the injection process.

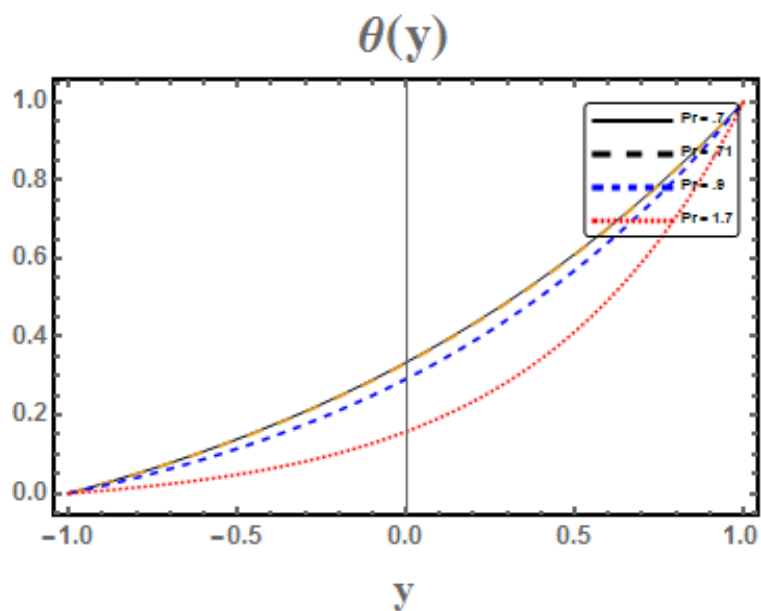


Figure 14: Influence of Prandtl parameter  $Pr$  on temperature profiles  $\theta(y)$ .

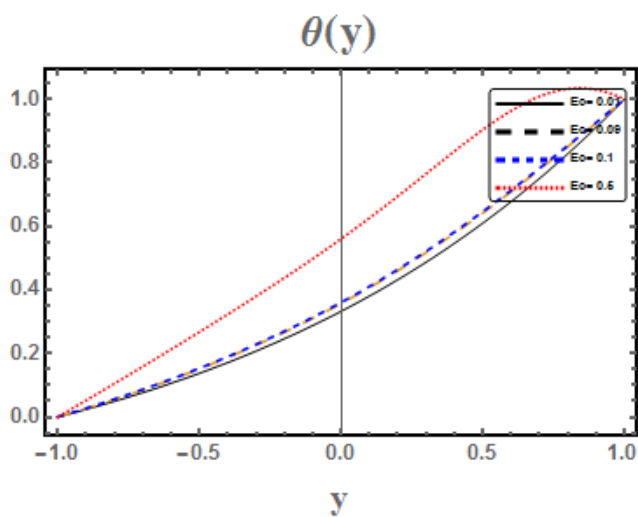


Figure 15: Influence of Eckert parameter  $Ec$  on temperature profiles  $\theta(y)$ .

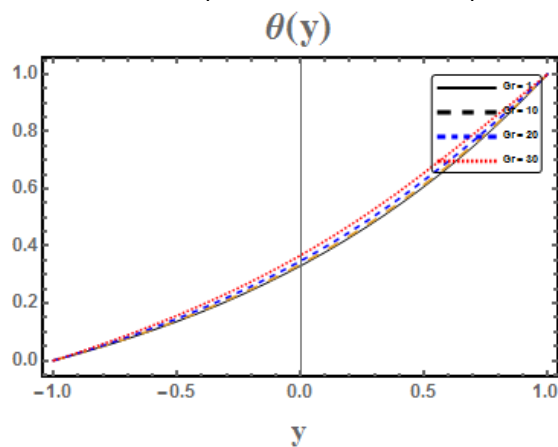


Figure 16: Influence of Grashof number  $Gr$  on temperature profiles  $\theta(y)$ .

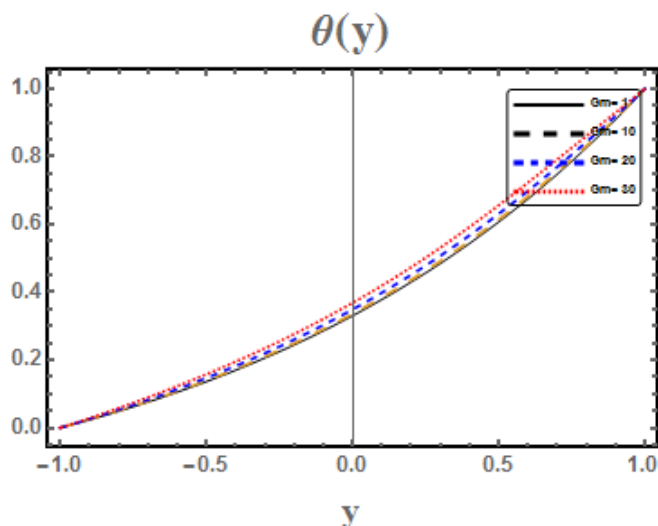


Figure 17: Influence of modified Grashof number  $Gm$  on temperature profiles  $\theta(y)$ .

**Concentration profiles**

Influence of Schmidt number  $Sc$  on concentration  $\phi(y)$  is shown in the figure 18, from this figure it is noticed that concentration decreases with an increase in  $Sc$ . Because, Schmidt number is a dimensionless number determined by the ratio of momentum diffusivity and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. Therefore, concentration boundary layer decreases with an increase in Schmidt number. The increase in chemical reaction parameter  $\gamma$  at values  $\gamma = 0.1, 0.3, 0.5, 0.8$  leads to a decrease in the concentration distributions  $\phi(y)$  as shown in fig. 19. Effect of thermophoresis  $\tau$  on concentration profiles  $\phi(y)$  is displayed in fig.20 at values of  $\tau = 0.1, 0.6, 0.9$  and  $1.2$ . It is noticed that the concentration profiles  $\phi(y)$  decreases as  $\tau$  increases. Specially, the effect of increasing the thermophoretic parameter  $\tau$  is limited to increasing the wall slope of the concentration profiles, but decreasing the concentration. This is true only for small values of Schmidt number for which the Sherwood number  $Sh$  is large as chemical reaction effect increases and compared to the convection effect. So, the thermophoretic parameter  $\tau$  is expected to alter the concentration boundary layer significantly. The concentration profiles diverge at values of couple stress inverse parameter that equal to  $a = 8$ . As a result of the effect of the previous parameters on the concentration layers, we found that the fluid be more concentrated in the adjacent side of the absorption process of th plate which located at  $y = 1$  as shown in 21.

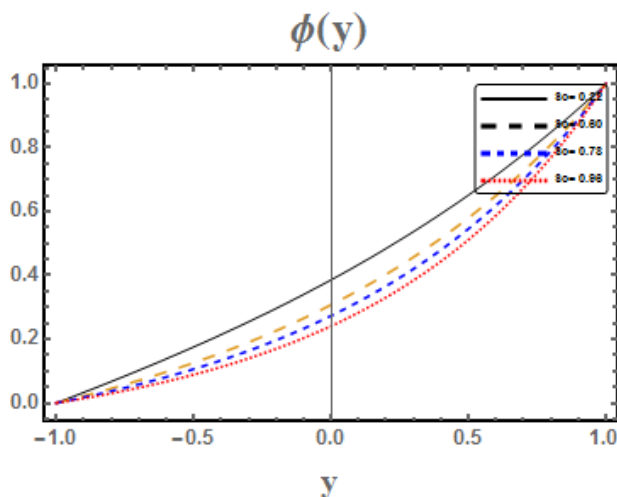


Figure 18: Effect of Schmidt parameter  $Sc$  on concentration profiles  $\phi(y)$ .

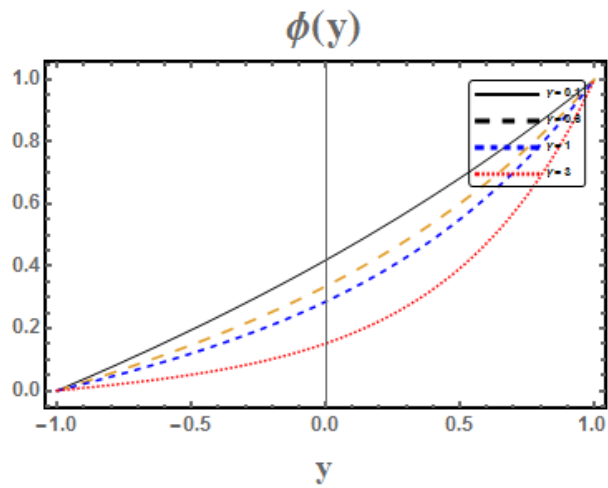


Figure 19: Influence of chemical reaction parameter  $\gamma$  on concentration profiles  $\phi(y)$ .

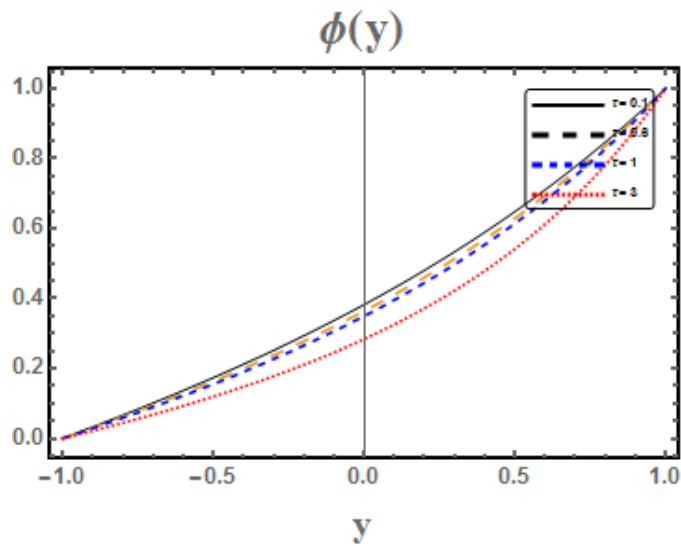


Figure 20: Influence of thermophoretic parameter  $\tau$  on concentration profiles  $\phi(y)$

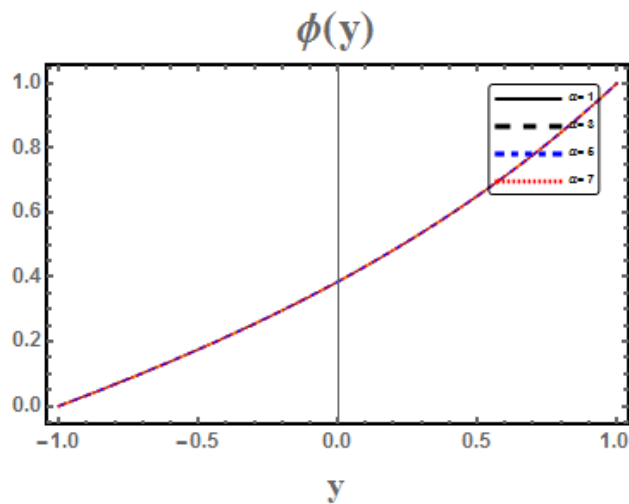


Figure 21: Effect of couple stress inverse parameter  $\alpha$  on concentration profiles  $\phi(y)$ .

Table 1: Skin friction coefficient  $\tau^*$  data for:  $\alpha = 0.5, a = 2, Gr = 5, \tau = 0.1, \gamma = 0.2, Gm = 5, Pr = 0.71, Ec = 0.01, Sc = 0.22, \beta h = 1, Ha = 2, \alpha e = 0.5$

$\alpha$	$\tau$	$Ha$	$a$	$\alpha e$	$\beta h$	$Gr$	$\tau^*$
5	0.1	2	2	0.5	1	5	-0.3661
	0.1	2	2	0.5	1	5	-0.1931
5	0.1	2	2	0.5	1	5	-0.3671
5	0.9	2	2	0.5	1	5	-0.3592
5	0.1	0.2	2	0.5	1	5	-0.2475
5	0.1	4	2	0.5	1	5	-0.1049
5	0.1	2	0.8	0.5	1	5	-0.0539
5	0.1	2	3	0.5	1	5	-0.7331
5	0.1	2	2	0.1	1	5	-0.0110
5	0.1	2	2	1.5	1	5	-0.5960
5	0.1	2	2	0.5	0.5	5	-0.6660
5	0.1	2	2	0.5	2	5	-0.4262
5	0.1	2	2	0.5	1	1	-0.3727
5	0.1	2	2	0.5	1	10	-0.9205

Table 1 display various numerical values of Skin friction coefficient  $\tau^*$  at varying the following parameters viscoelastic parameter  $\alpha$ , thermophoresis parameter  $\tau$ , Hartmann number  $Ha$ , couple stress inverse parameter  $a$ ,  $\alpha e = 1 + \beta_i \beta_e$ ,  $\beta_e$  is the Hall parameter and  $\beta_i$  is the ion-slip, Grashof number  $Gr$ . From that table, we noticed that an increase in  $\alpha, \tau, Ha$  and  $\beta_h$  increase the value of the skin friction coefficient  $\tau^*$ , while an increase in  $a, \alpha e, Gr$  decrease the value of the skin friction coefficient  $\tau^*$ .

Table 2: Nusselt number  $Nu$  data for:  $\alpha = 0.5, a = 2, Gr = 5, \tau = 0.1, \gamma = 0.2, Gm = 5, Pr = 0.71, Ec = 0.01, Sc = 0.22, \beta h = 1, Ha = 2, \alpha e = 0.5$

$Pr$	$Ec$	$Gr$	$\alpha e$	$\beta h$	$Nu$
.70	0.01	5	0.5	1	-0.4708
.7	0.01	5	0.5	1	-0.3366
.71	0.07	5	0.5	1	-0.5453
.71	0.9	5	0.5	1	-0.6693
.71	0.01	1	0.5	1	-0.4640
.71	0.01	15	0.5	1	-0.5028
.71	0.01	5	0.1	1	-0.4604
.71	0.01	5	3	1	-0.4696
.71	0.01	5	0.5	0.5	-0.4724
.71	0.01	5	0.5	3	-0.4613

Nusselt number  $Nu$  is given in table2. It can be clearly seen that any increase in Prandtl number  $Pr$  or Hall parameter  $\beta_h$  result in an increase in Nusselt number  $Nu$ . But the increasing of Eckert number  $Ec$ , Grashof number  $Gr$  and  $\alpha e$  lead to Nusselt number  $Nu$  decreases.

Table 3: Sherwood number  $Sh$  data for  $\alpha = 0.5$ ,  $a = 2$ ,  $Gr = 5$ ,  $\tau = 0.1$ ,  $\gamma = 0.2$ ,  $Gm = 5$ ,  $Pr = 0.71$ ,  $Ec = 0.01$ ,  $Sc = 0.22$ ,  $\beta_h = 1$ ,  $Ha = 2$ ,  $\alpha e = 0.5$

$\gamma$	$Ec$	$Sc$	$\tau$	$\beta_h$	$Sh$
.1	0.01	0.22	0.1	1	-0.4842
.6	0.01	0.22	0.1	1	-0.4404
.2	0.01	0.22	0.1	1	-0.4730
.2	0.09	0.22	0.1	1	-0.4777
.2	0.01	0.30	0.1	1	-0.4670
.2	0.01	1.3	0.1	1	-0.3300
.2	0.01	0.22	0.3	1	-0.4705
.2	0.01	0.22	0.9	1	-0.4772
.2	0.01	0.22	0.1	0.5	-0.4731
.2	0.01	0.22	0.1	3	-0.4726

Table 3 shows numerical values of mass transfer coefficient in terms of sherwood number  $Sh$ . It is observed that Sherwood number increases with increasing in chemical reaction  $\gamma$ , Schmidt number  $Sc$  and Hall currents  $\beta_h$ . likewise, the increase of Eckert number  $Ec$  and thermophoretic parameter  $\tau$  lead to decrease the Sherwood number  $Sh$ .

## Conclusions

The effects of Couple stresses and thermophoresis on free convective heat and mass transfer of viscoelastic fluid with Hall and ion-slip effects have been studied. The behavior of velocities, temperature and concentrations has been noticed at physical values of parameters furthermore, Skin friction, Nusselt and Sherwood numbers have been calculated. It is found that The velocity  $u(y)$  decrease as the Hall parameter increases. the velocity  $u(y)$  increases and induced flow velocity  $w(y)$  in the z-direction decreases with an increase in the Ion slip parameter.

## References

1. Stokes VK. Couple Stresses in fluid. *Phys Fluids*, 1709–15, 1966.
2. R. Piazza, *Soft matter - Royal Society of Chemistry* **4**, 1740 –1744, 2008.
3. K. Das, S. Jana and P.K. Kundu, *Alexandria Engineering Journal* **54**, 35 –44, 2015.
4. Chamkha, M. A. Mansour and A. Aly, *International Journal for Numerical Methods in Fluids* **65**, pages 432–447, 2011.

5. T. Hayat and M. Nawaz. *International Journal for Numerical Methods in Fluids* **66**, 183 –193, 2011.
6. R.C.Chaudhary and P. Jain, *Theoretical and Applied Mechanics***33**, 281 –309, 2006.
7. R. Kumar and K. Chand, *International Journal of Engineering Science and Technology***4**, 2011.
8. S. N. Sahoo, *International Journal of Chemical Engineering* 8 pages 2013.
9. A.K. Jha, K. Choudhary and A. Sharma. *International Journal of Applied Mechanics and Engineering* **19**, pages 79–95, 2014.
10. Kalidas Das, RamPrakashSharma and AmitSarkar, *Journal of Computational Design and Engineering* **7**, 2016
11. K. Das, *African Mathematical Union and Springer***24**, 511 –524, 2013.
12. M.VeeraKrishna and B.V.Swarnalathamma, *IOSR Journal of Mathematics (IOSR-JM)*,**12**, 08 –24, 2016.
13. Nayak and S. Panda, *Mathematical Theory and Modeling* **3**, 38–46(2013).
14. El-dabe, N. T. M., Refaie Ali, A., El-shehkipy, A. A., and Shalaby, G. A., Non-linear heat and mass transfer of second grade fluid ow with hall currents and thermophoresis Effects, *Appl. Math. Inf. Sci.*, vol. 1, no.11, PP. 267-280, 2017.
15. D. Srinivasacharya, K. Kaladhar, Mixed convection flow of couple stress fluid between parallel vertical plates with Hall and Ion-slip effects, *Commun Nonlinear Sci Numer Simulat* **17** 2447-2462, 2012.
16. Refaie, Ahmed & N.T.M.El-Dabe, & El-Shehkipy, Abdelhafeez & Shalaby, G.. (2017). Influence of Thermophoresis on Unsteady MHD Flow of Radiation Absorbing Kuvshinski Fluid with Non-Linear Heat and Mass Transfer. *American Journal of Heat and Mass Transfer*. 4. 10.7726/ajhmt..1010, 2017
17. N.T.M.Eldabe, A.Refaie Ali and G.A.Shalaby. Influence of Couple Stresses and Thermophoresis on Free
18. Convective Heat and Mass Transfer of Viscoelastic Fluid. *Journal of Advances in Physics Vol 17* ISSN: 2347-3487, 2020