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Fusion and Breakup Reactions of ^{17}S + ^{208}Pb and ^{15}C + ^{232}ThHalo Nuclei Systems

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ABSTRACT

In this study the calculations of the total fusion reaction cross sectionσ_{fus} and the fusion barrier distributionD_{fus} have been performed for the systems ^{17}F + ^{208}Pb and ^{15}C + ^{232}Th which involving halo nuclei by using a semiclassicalapproach.Thesemiclassical treatment is comprising the **WKB** approximation to describe the relative motion between target and projectile nuclei, and Continuum Discretized Coupled Channel **(CDCC)** method to describe the intrinsic motion for both target and projectile nuclei.For the sake of comparsion a full quantum mechanical have been preforemd using the (**CCFULL)** code. Our theorticalrestulsare compared with the full quantum mechanical calculations and with the recent experimental data for the total fusion reaction and the fusion barrier distribution. The comparsion with experimental datashows that the full quantum mechanical calculations are shows more stability in the calculations of the total fusion reaction cross section especially around the Coulomb barrier and also for the clalculation of the fusion barrier distribution, therefore the semiclassical approach need to be improved especially in the region around the Coulomb barrier.

Indexing terms/Keywords

Semiclassical treatment; Continuum discretized coupled channel (CDCC); fusion barrier distribution; Halo nuclei.

Academic Discipline And Sub-Disciplines

Physics.

SUBJECT CLASSIFICATION

Theoretical Nuclear Physics

TYPE (METHOD/APPROACH)

Theoretical Study.

1 INTRODUCTION

The interactions between two nuclei can lead to a variety of processes. In a semiclassical picture, it is customary to use the impact parameter, or relative angular momentum, to distinguish between compound nucleus reactions (The fusion reactions) and direct reactions; the latter take place for values of the impact parameter corresponding to grazing trajectories, the former occur for more central interactions. In between, deep inelastic reactions show intermediate behavior. The relative importance of the different mechanisms depends on a number of factors; among these, a very important role is played by the kinetic energy and its value with respect to the amount of Coulomb repulsion between the nuclei for a given trajectory (the 'Coulomb sub-barrier'). At relatively high energies with respect to the barrier it is possible to use geometrical models of the reaction process to provide independent descriptions of individual mechanisms: as, Glauber models [1] for direct reactions, the onedimensionalsub-barrier penetration model for fusion [2].

The kinetic energy is small compared to the Coulomb sub-barrier height this independence no longer holds: the behavior of a particular process can no longer be considered separately from the others. In scattering theory this is expressed by the concept of coupling of the different reaction channels. The total wave function of the scattering problem contains the entrance channel and all possible exit channels; the Hamiltonian connects these states by means of potential terms, for example potentials that can create an excitation. For small kinetic energies, the contributions of these terms become significant in the determination of the wave function of each channel. The effect of the couplings is wellestablished, and visible on both the elastic scattering [3] and fusion reaction cross sections [4,5].

The aim of the presentstudy is to employ a semiclassical approach by adopting Alder and Winther theory originally used to treat the Coulomb excitation of nuclei which is called Continuum-Discretized Coupled Channel (CDCC) method in which Quantum and semiclassical approach have been implemented to calculate the total fusion reaction cross section σ_{fus} and the fusion reaction barrier distribution D_{fus} for the systems involving light halo nuclei ^{17}F + ^{208}Pb and ^{15}C + ^{232}Th. by using the FORTRAN codenamed (SCF) and compare our results with the full quantum mechanical calculations using the coupled channel calculations (CC) with all order coupling using the computer code (CCFULL) and with the available experimental data of complete fusion.

2 THE COUPLING CHANNEL FORMALISM

In general, nuclei participating in a collision may undergo internal excitations and different particle transfer processes that effect their total fusion reaction cross sectionσ_{fus}. These reaction process involves the active participation of several degrees of freedom for its description. Therefore, the fusion approach require the explicit inclusion of the couplings among the different degrees of freedom. This is accomplished by considering into the wave function of the system a number of components equal to the number of intrinsic quantum mechanical states involved[7,8].

Consider the reaction described by the total wave function $\Psi(\mathbf{r},\tau)$,where $\mathbf{r}$stands for the projectile and target nuclei separation vector and $\tau$for the set of intrinsic coordinates of the projectileandtarget nuclei. The dynamics of this reaction is determined by the Hamiltonian,

$$H = H_0 + T + U \dots\dots\dots\dots\dots\dots (1)$$

where$H_0 \equiv H_0(\tau, p_\tau)$is the intrinsic Hamiltonian, $T \equiv -\hbar^2\nabla^2/2\mu$ is the kinetic energy operator of the relative motion between the projectile and target nuclei, and $U \equiv U(\mathbf{r},\tau)$is the interaction potential. The eigenstates of the intrinsic Hamiltonian, $|\eta\rangle$, satisfy the Schrödinger equation [6],

$$(e_\eta - H_0)|\eta\rangle \dots\dots\dots\dots\dots\dots (2)$$

The orthonormality is,

$$\langle\eta'|\eta\rangle = \int d\tau \varphi^*_{\eta'}(\tau)\varphi_\eta(\tau) = \delta_{\eta\eta'} \dots\dots\dots\dots\dots\dots (3)$$

where$\varphi_\eta(\tau)\left(\varphi_{\eta'}(\tau)\right)$is the wave function corresponding to the state $|\eta\rangle(|\eta\rangle)$ in the τ- representation. The interaction potential is split as,

$$U = U' + U'' \dots\dots\dots\dots\dots\dots (4)$$

where$U'$is diagonal in channel space,

$$U' = \sum_\eta |\eta\rangle U'_\eta \langle\eta| \dots\dots\dots\dots\dots\dots (5)$$

$$U'' = \sum_\eta |\eta\rangle U''_{\eta,\eta'} \langle\eta'| \dots\dots\dots\dots\dots\dots (6)$$

where

$$U'_\eta(\mathbf{r}) = \int d\tau |\varphi_\eta(\tau)|^2 U'(\mathbf{r},\tau) \dots\dots\dots\dots\dots\dots (7)$$

$$U''_{\eta,\eta'}(\boldsymbol{r}) = \int d\tau \varphi^*_{\eta'}(\tau) U''(\boldsymbol{r},\tau)\varphi_\eta(\tau) \dots\dots\dots\dots\dots\dots (8)$$

The potential $U'$is arbitrary, except for the condition of being diagonal in channel space. However, once it is chosen, $U''$is given by the relation $U'' = U - U'$. Frequently, it is convenient to choose U'such that $U''$is purely off diagonal. In such cases the components of U''can be written[6],

$$U''_{\eta,\eta'}(\boldsymbol{r}) = \int d\tau \varphi^*_{\eta'}(\tau) U''(\boldsymbol{r},\tau)\varphi_\eta(\tau) - \delta_{\eta\eta'} U'_\eta(\boldsymbol{r}) \dots\dots\dots\dots\dots\dots (9)$$

From the Schrödinger equation, we can start to derive the coupled channel equations,

$$(E - H)|\Psi_\eta(\eta_0 \boldsymbol{k}_0)\rangle = 0 \qquad \dots\dots\dots\dots\dots\dots (10)$$

and the channel-expansion,

$$|\Psi_\eta(\eta_0 \boldsymbol{k}_0)\rangle = \sum_\eta |\psi_\eta(\eta_0 \boldsymbol{k}_0)\rangle|\eta\rangle \dots\dots\dots\dots\dots\dots (11)$$

The notation $|\Psi(\eta_0 \boldsymbol{k}_0)\rangle$indicates that the collision is started in channel η_0, withwave vector $\boldsymbol{k}_0$, and the energy scale is chosen such that $e_{\eta_0} = 0$. Owing to the off diagonal part of the reaction, The Schrödinger equation solution has components $|\Psi_\eta(\eta_0 \boldsymbol{k}_0)\rangle$for both $\eta = \eta_0$and $\eta \neq \eta_0$, The infinite expansion ofEq. (11) is truncated so as to include only the most relevant channels or closed coupling approximation. To account forthe loss of flux through neglected channels, One may include an imaginary part in the channel potentials$U'_\eta(\boldsymbol{r})$.To find the wave function, we must write the Hamiltonian as [6],

$$H = H_0 + H' + U'' \dots\dots\dots\dots\dots\dots (12)$$

where

$$H' = K + U' \dots\dots\dots\dots\dots\dots (13)$$

When we putEqs. (11) and (12) into Eq. (10), and take the scalar product with each intrinsic state $\langle\eta|$,then we get the coupled channel equations,

$$(E_\eta - H'_\eta)|\psi_\eta(\eta_0 \boldsymbol{k}_0)\rangle = \sum_{\eta'} U''_{\eta,\eta'}(\boldsymbol{r})|\psi_{\eta'}(\eta_0 \boldsymbol{k}_0)\rangle \dots\dots\dots\dots\dots\dots (14)$$

Or,

$$\left[E_\eta + \frac{\hbar^2}{2\mu}\Delta - U'_\eta(\boldsymbol{r})\right]\psi_\eta(\boldsymbol{r}) = \sum_{\eta'} U''_{\eta,\eta'}(\boldsymbol{r})\psi_{\eta'}(\boldsymbol{r}) \dots\dots\dots\dots\dots\dots (15)$$

Where,

$$E_\eta = E - e_\eta \dots\dots\dots\dots\dots\dots (16)$$

$E_\eta$is the total energy of the relative motion in channel $\eta$and,

$$H'_\eta = T + U'_\eta \dots\dots\dots\dots\dots\dots (17)$$

The Eq. (15) switched to the more compact notation $|\psi_\eta(\eta_0 \boldsymbol{k}_0)\rangle \longrightarrow \psi_\eta(\boldsymbol{r})$,and the channel potentials are written as,

$$U'_\eta = V_\eta + iW_\eta \dots\dots\dots\dots\dots\dots (18)$$

wherethe flux in channel ηaccounted bythe imaginary part $W_\eta$lost to other channels which were not included in the coupled channel equations. A consequence of the non-Hermitian nature of $H$is that the continuity equation breaks down. In the usual case where the channelcoupling interaction $U''_\eta$is hermitian, the continuity equation is written by the relation [7].

$$\boldsymbol{\nabla} \cdot \sum_\eta \boldsymbol{j}_\eta = \frac{2}{\hbar}\sum_\eta W_\eta(\boldsymbol{r})|\psi_\eta(\boldsymbol{r})|^2 \neq 0 \qquad \dots\dots\dots\dots\dots\dots (19)$$

Where $\boldsymbol{j}_\eta$is the probability current density in channel η. Integrating the above equation inside a large sphere with radius larger than the interaction range and using the definition of the absorption cross section σ_η[9 – 11],

$$\sigma_\eta = \frac{k}{E}\sum_\eta \langle\psi_\eta|W_\eta|\psi_\eta\rangle \dots\dots\dots\dots\dots\dots (20)$$

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If the absorptive potential can be written as,

$$W_\eta = W_\eta^D + W_\eta^F \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

With W_η^D accounting for the flux lost to other direct reaction channels and W_η^F accounting for fusion absorption, the fusion reaction cross section can be written as[9,11],

$$\sigma_F = \frac{k}{E}\sum_\eta \langle \psi_\eta | W_\eta^F | \psi_\eta \rangle \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

There are strong effects in fusion reactions arising from couplings among several channels.

3 FUSION BARRIER DISTRIBUTION

The effect of the coupling of different channels on the fusion reactions has been well recognized for about a 25 years ago. It's most dramatic consequence is the enhancement of the total fusion reaction cross section σ_{fus}atCoulomb sub-barrier energiesV_b, in some cases by several orders of magnitude. the possible way to describe the effect of coupling channels is as a division of the fusion barrier into several, the so-called fusion barrier distributionD_{fus} and given by [6,12],

$$D_{fus}(E) = \frac{d^2G(E)}{dE^2} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

When $G(E)$ is related with the total fusion reaction cross section through,

$$G(E) = E\sigma_{fus}(E) \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

The experimental determination of the fusion reactionbarrierdistribution has lead to significant progress in the understanding,. This comes about because, as already mentioned, the fusion reactionbarrier distribution gives information on the coupling channels appearing in the collision. However, from Eq. (23), we note that, because it must be extracted from the values of the total fusion reaction cross section, it is subject to experimental as well as numerical uncertainties. The usual procedure is to estimate the second derivative appearing in Eq. (23) through a three-point difference method[13,14],

$$D_f(E) \approx \frac{G(E+\Delta E) + G(E-\Delta E) - 2G(E)}{\Delta E^2} \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

where $\Delta E$is the energy interval betweenmeasurementsofthe total fusion reaction cross section.From Eq. (25) one finds that the statistical error associated with the fusion reaction barrier distribution is approximately given by [14],

$$\delta D_f^{stat}(E) \approx \frac{\sqrt{[\delta G(E+\Delta E)]^2 + [\delta G(E-\Delta E)]^2 + 4[\delta G(E)]^2}}{(\Delta E)^2} \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

where$\delta G(E)$is mean the uncertainty in the measurement of the product of the energy by the total fusion reaction cross section at a given value of the collision energy. When the uncertainties are approximately be written as [13],

$$\delta D_f^{stat}(E) \approx \frac{\sqrt{6}\delta G(E)}{(\Delta E)^2} \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

4 RESULTS AND DISCUSSION

The total fusion reaction cross section σ_{fus}, and the fusion reaction barrier distribution D_{fus} have been calculated by using a semiclassical treatment adopted the Coulomb excitations calculations from Alder and Winther (AW), and implemented in a FORTRAN codenamed SCF for the systems ^{17}F + ^{208}Pband ^{15}C + ^{232}Th. For the sake of comparison with other theoretical models using full quantum mechanics with all order coupling channels, we had performed calculations using the famous fusion reaction code CCFULL for the same studiedsystems. The sameAküz-Winther potential parameters, which used in the present calculations for two programs codes,aredisplayed in table 1.

Table 1. The parameters of Aküz-Winther potential along with terms of the Coulomb barrier: height, radius, and curvature, V_b, R_b, and $\hbar\omega$, respectively.

The fusion systems	V_0 (MeV)	a_0 (fm)	r_0 (fm)	V_b (MeV)	R_b (fm)	$\hbar\omega$ (MeV)
^{17}F + ^{208}Pb	80.1	0.66	1.2	83.9	11.91	4.9
^{15}C + ^{232}Th	80.5	0.65	1.21	59.2	12.41	4.3

Our semiclassical and full quantum mechanical calculations for ^{17}F + ^{208}Pbsystem are shown in figure 1. The semiclassical calculations for the total fusion reaction cross section σ_{fus} and the fusion reaction barrier distribution D_{fus} without including the coupling channels effects between the elastic channel and the continuum are represented by the dashed blue curve, while the calculations in case of including the coupling effects are represented by the solid blue curve.The full quantum mechanical calculations using CCFULL code are presented by dashed and solid black curves for the case of no-coupling and coupling included, respectively. The full quantum mechanical coupled channel calculations performed by considering vibrational deformations for target nucleus with deformation parameter$\beta_0 = 0.157$, adopted from Ref. [15]withconsidering double phonon excitation, and the projectile nucleus taken to be inert.

The arrow in figure 1 and 2 represents the position of the Coulomb sub-barrier V_b. In the case of no-coupling both semiclassical and full quantum mechanical calculations underestimate the experimental data of complete fusion reaction cross section below the Coulomb sub-barrier, the inclusion of the coupling in both calculations shows that the the full quantum mechanical are more closer than semiclassical treatmentones in comparison with the experimental data of complete fusionbelow the Coulomb sub-barrier. The comparison of the calculated fusion reaction barrier distribution D_{fus} for both semiclassical and full quantal mechanical ones along with the experimental data of complete fusionextracted using thethree-point difference method is shown in panel (b) in Figure 1.

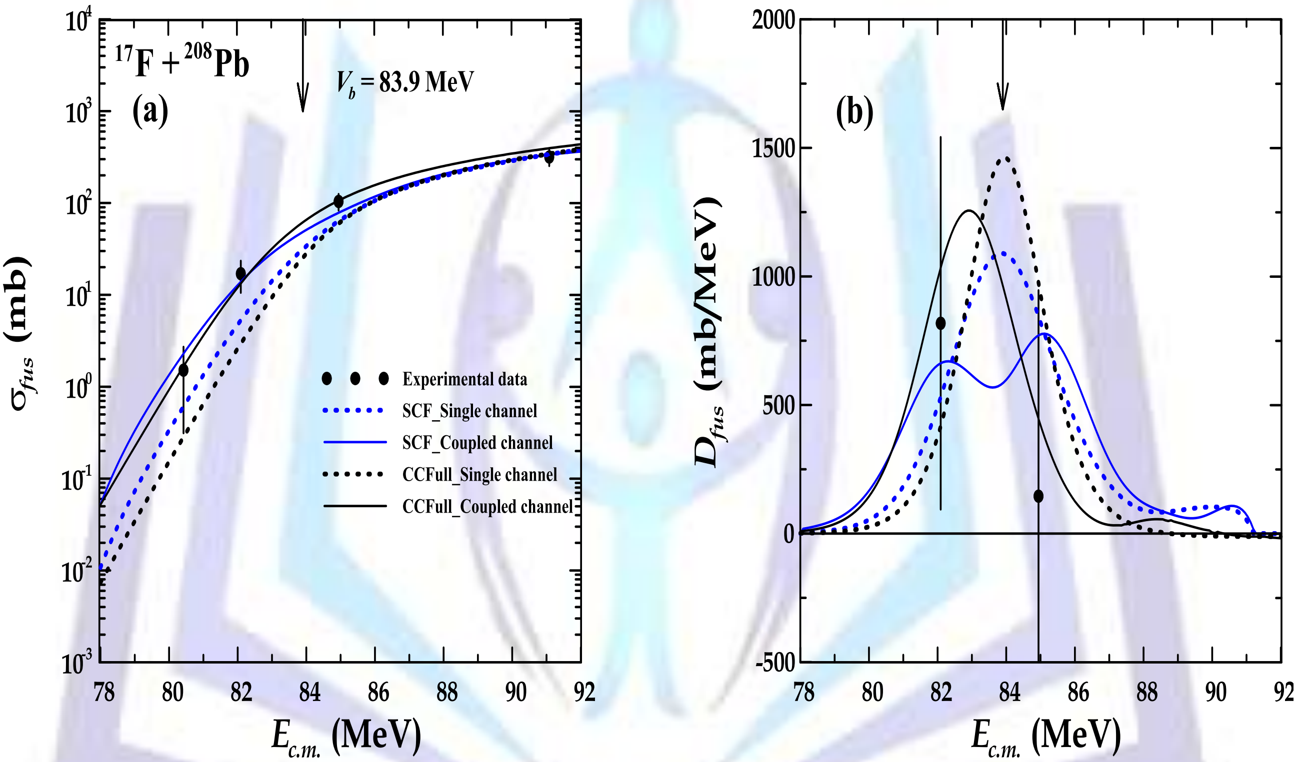

Fig 1: The comparison of the coupled channels calculations of semiclassical treatment(blue curves) andfullqauntum mechanical (black curves) with the experimental data of complete fusion (black filled circles) [16] for ^{17}F + ^{208}Pb system. Panel (a) for the total fusion reaction cross section σ_{fus}(mb), and Panel (b) for the fusion reaction barrier distribution D_{fus} (mb/MeV), and the arrow indicate the position of the Coulomb barrier V_b.

The semiclassical and full quantum mechanical calculations for the total fusion reaction cross section σ_{fus} and the fusion reaction barrier distribution D_{fus} for ^{15}C + ^{232}Th system both with and without including the coupling effects is presented in Figure 2.The semiclassical and fullquantum mechanical calculations without introducing the coupling effectsrepresented by the dashed curves in blue and black, respectively.While,the calculations taking into consideration the coupling effects represented by the solid blue and black curves, respectively. The coupling effect is taken between the elastic channel and the continuum in our semiclassical calculations, while the coupling is considered as rotational deformation in target nucleus with deformations parameters $\beta_2 = 0.207$, and $\beta_4 = 0.108$adopted from Ref. [15], and considered inert projectile nucleus. The semiclassical calculations including the coupling effects enhanced and brings the calculated σ_{fus} to the experimental values of complete fusion below the Coulombsub-barrier marked by the arrow. The semiclassical method shows unstability around the Coulomb barrier even it reproduce the experiment, but still the CCFULL calclations are more stable and reproduce the data better than the semiclassical approach.

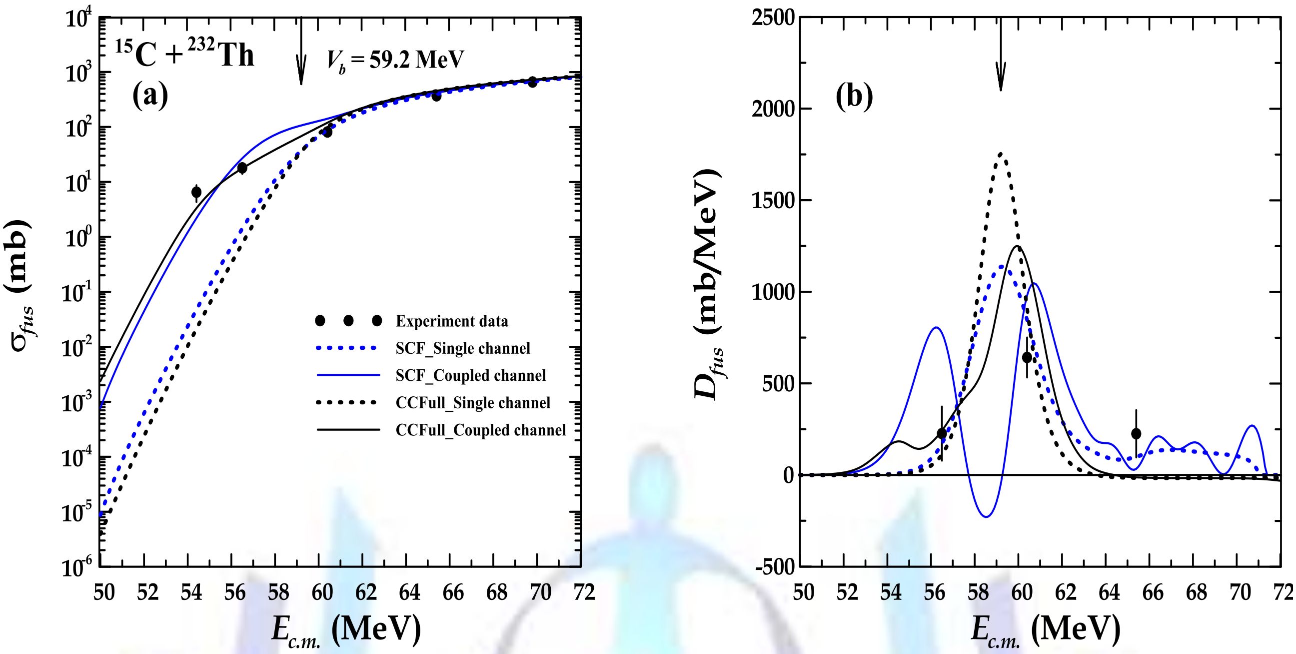

Fig 2: The comparison of the coupled channels calculations of semiclassical treatment(blue curves) andfullqaantum mechanical (black curves) with the experimental data of complete fusion (black filled circles) [17] for ^{15}C + ^{232}Th system. Panel (a) for the total fusion reaction cross section σ_{fus}(mb), and Panel (b) for the fusion reaction barrier distribution D_{fus} (mb/MeV), and the arrow indicate the position of the Coulomb barrier V_b.

5 CONCLUSIONS

The coupled channel effect between the elastic channel and the continuum is found to be very essential in the semiclassical calculations which leads to improvement in the total fusion reaction cross section σ_{fus} and the fusion reaction barrier distribution D_{fus} around and below the Coulomb sub-barrier and brings the theoretical results closer to the experimental data of complete fusion. The inclusion of the coupling channel effects by considering the target hasrotationaldeformationin ^{17}F + ^{208}Pbsystem, and vibrationaldeformation with douple phonon excitation in ^{15}C + ^{232}Thsystem,enhances the full quantum mechanical calculations around and below the Coulomb sub-barrier. The semiclassicaltreatment used in the present work shows unstability in the calculations around the Coulomb barrier, even if it is able to reproduce the experimental data and still the semiclassical approach need more improvement especially around the Coulomb barrier. This work can be extended to study more systems involving halo nuclei and medium and heavy system to confirm it's validity to fusion reaction calculations using our semiclassical approach.

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