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Linear Preserves of BP-quasi invertible elements in JB*-algebras

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Abstract

In this note, we study one of the main outcomes of the Russo-Dye Theorem of JB*-algebra: a linear operator that preserves Brown-Pedersen-quasi invertible elements between two JB*-algebras is characterized by a Jordan $*$-homomorphism. Earlier, in C*-setting of algebras, Russo and Dye gave a characterization of any linear operator that maps unitary elements into unitary elements; namely a Jordan $*$-homomorphism. Special sorts of linear preservers between C*-algebras and between JB*-triples were introduced by Burgos et al. As a result, if G is a linear operator between two JB*-algebras having non-empty sets of extreme points of the closed unit sphere that preserves extreme points, then there exists a Jordan $*$-homomorphism **Φ** which also preserves extreme points and characterizes the linear operator G. We also explore the connection between linear operators that strongly preserve Brown-Pedersen-quasi invertible elements between two JB*-triples and the λ-property of both JB*-triples. Other geometric properties, such as extremally richness and the Bade property of two JB*-algebras or triples under linear preservers, are to be elaborated on in forthcoming research.

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Introduction

In [4], Burgos et al. studied linear operators strongly preserving Brown-Pedersen quasi invertibility between C*-algebras considered as JB*-triples and they proved that it is a triple homomorphism. They discussed a consequence of this result that concerns only C*-algebras; if G is a linear operator strongly preserving Brown-Pedersen-quasi invertible elements (BP-quasi invertible, for short) between two unital C*-algebras A and B, authors proved that there is a Jordan $*$-homomorphism ϕ: $A \to B$ that satisfies $G(a) = G(e)\phi(a)$ for every $a \in A$ where e is the unit of A. They also explored other types of linear operators between some Jordan algebra structures that preserve; Bergmann-zero pairs, BP-quasi invertible elements and extreme points [4].

In this note, we studied a linear operator between JB*-algebras mapping a fixed extreme point of the closed unit sphere of one JB*-triple onto a fixed extreme point of the other, and we deduced analogous of Burgos et al. conclusion.

The set, A_q^{-1} of all BP-quasi invertible elements in a unital C*-algebra A was originally initiated by L. G. Brown and G. K. Pedersen. Several equivalent conditions were given [2, Theorem 1.1] so that an element is BP-quasi invertible. In particular, they demonstrated that such elements are obtained using invertibility notion by the form $A_q^{-1} = A^{-1}\ ext\left(A_1\right) A^{-1}$), where $ext\left(A_1\right)$ is the class of extreme points of the closed unit sphere of A. Further, $a \in A$ is BP-quasi invertible if and only if, the binary operator $B(a,\ b)$ (defined in section 2) vanishes for some $b \in A$ [8, Theorem 11].

Authors in [15, Theorem 6] expanded this special invertibility notion to any JB*-triple. They implemented the known Bergmann operator so that an element a in a JB*-triple J is BP-quasi invertible if there is some element $b \in J$, such that $B\ (a,\ b) = 0$. Note that, whenever B $(a,\ b)$ vanishes for some $a,\ b \in J$, $B\ (a,\ Q(b)(a))$ also vanishes. Therefore, for any BP- quasi inverse b of a is not the only one in general and, $Q(b)(a)$ is another BP-quasi inverse of a.

Another characterization of this notion stated in [15, Theorems 6 and 11], using the von Neumann regularity and the range tripotent $r(a)$ obtained from any element, a in a JB*-triple J so that a must be von Neumann regular element and $r(a)$ is in fact an extreme point of the closed unit sphere of J. Every von Neumann regular element a in J has a unique commuting normalized generalized inverse symbolized by $a^{\wedge}$. Among others, the set, J_q^{-1}, of all BP-quasi invertible elements in J properly includes the family of all regular (invertible and von Neumann regular) elements and the class of all extremes, $ext\left(J_1\right)$.

In section 3, we established that a strongly preserving BP-quasi invertibility linear operator, $G: J \to H$ between two JB*-triples J and H with $ext(J_1) \neq \emptyset$, and $u \in J$ is a unitary element (thus, J is a JB*-algebra), then there exists a Jordan $*$-homomorphism $\Phi : J \to H$ such that $G(a) = G(u)\Phi(a),\ \forall\, a \in J$.

Preliminaries

In this section, we scan the main concepts used in this note. To begin with, a commutative algebra J (which is in general not associative) with a binary product $\circ$, defined on a scalar field of characteristic other than 2 and satisfying the identity $a^2 \circ (a \circ b) = (a^2 \circ b) \circ a$ for all $a, b \in J$, where a^2 means $a \circ a$, is called a Jordan algebra.

The binary product $a \circ b = ½(ab + ba)$ induced from the associative product ab, between elements a and b in any algebra A, defines the special Jordan algebra A^+, with the same linear space structure A (cf. [5]). If $(J, \circ)$ is any Jordan algebra, then we can define Jordan triple product $\{a, b, c\} = (a \circ b) \circ c + (c \circ b) \circ a - (a \circ c) \circ b$ on J so that it is linear symmetric in a, c and linear or anti-linear in the variable b. If one of the three variables is the unit e, this triple product reduces to the original binary Jordan product (see [5]).

On any Jordan algebra, we have the following fundamental operators: $V_{a,b}(x) = \{a, b, x\}$ and $U_{a,b}(x) = \{a, x, b\} = V_{a,x}(b)$. The short symbol U_a is used for the operator $U_{a,a}$. An element a in a Jordan algebra J (with unit e) is invertible if it satisfies that $a \circ a^{-1} = e$ and $a^2 \circ a^{-1} = a$ for some element $a^{-1} \in J$. Equivalently, a is invertible $\Leftrightarrow$ U_a is invertible and $U_a^{-1} a = a^{-1}$ [6, *Theorem* 13].

The involution map $*: J \to J$ is defined on Jordan algebra J such that for any $a, b \in J$ and ever $\lambda, \mu \in C$, this map satisfies, $(\lambda a + \mu b)^* = \overline{\lambda} a^* + \overline{\mu} b^*$; $a^{**} = a$ and $(a \circ b)^* = b^* \circ a^*$ where, a^* symbolizes the image of a under $*$. Moreover, we say that $a \in J$ is self-adjoint if $a^* = a$.

The Jordan algebra $J_{[x]}$ (the x-homotope of a *Jordan algebra* J) is formed from the same elements of J but with a special product "$\cdot_x$" given by $a \cdot_x b = \{a, x, b\}$ for every $a, b \in J$. If we take an invertible element x in J, then $J^{[x]}$ denotes the x-isotope of J which is nothing but the x^{-1}-homotope of J.

A Banach Jordan algebra is a Jordan algebra J over real or complex scalar field with a complete norm $\|.\|$ and $\|a \circ b\| \leq \|a\|\|b\|$ for all $a, b \in J$. Moreover, if J has a unit element e with $\|e\| = 1$, then we say that this Banach Jordan algebra is unital. A C^*-algebra A is an evolutive complex Banach algebra satisfying that $\|aa^*\| = \|a\|^2$ for all $a \in A$ (cf. [16]).

The main literature for the algebraic structure known as a JB-algebra is stated in Hanche-Olsen and Størmer' book [5].

An evolutive complex Banach Jordan algebra $(J, \circ, *)$ is called a JB*-algebra if the norm defined on J satisfies $\|a^*\| = \|a\|$ and $\|\{a, a^*, a\}\| = \|a\|^3$ for all a, $b \in J$.

The condition $\|a^*\| = \|a\|$, was originally stated by J. D. M. Wright in the first article of the area [17], and he showed that this condition is redundant. If J has a unit e with $\|e\| = 1$ then J is also unital.

In 1976, I. Kaplansky introduced a generalization of a C^*-algebras and he initially called it a Jordan C^*-algebra [18]. Later, it became a JB*-algebra, and it has been studied extensively after that (see for example [13]). The self-adjoint part of a JB*-algebra J, is in fact a JB-algebra, say A, so that $J = A + iA$. On the other hand, the complex analogs of JB-algebras are the JB*-algebras [18, p. 292].

Recall that [12, p. 339] an element p in a unital JB-algebra A, such that $p^2 = p$ is called a projection. The class of all projections in A includes the set $ext(A_1)$ [12, Lemma 1.2]. A central projection p in a JB-algebra A commutes with every element of A. Isidro and Rodrguez [12] showed that central projections are precisely the isolated projections, and those are preserved by any surjective isometry of A.

Authors in [18, Theorem 6], showed that any unital surjective linear isometry between two unital JB*-algebras is indeed a Jordan $*$-isomorphism. Later, in 1995, J. M. Isidro and A. Rodriguez [12, *Theorem* 1.9] concluded that, if T is a surjective algebra isomorphism between two JB-algebras and ϕ is a surjective linear isometry, then $\phi(a) = bT(a)$, where b is a central projection in the algebra of multipliers of the range JB-algebra and a

in the domain JB-algebra. Moreover, if the above map ϕ is one-to-one, then it is an isometry if and only if ϕ is a triple-isomorphism [12, $Theorem\ 1.9$].

An element u in a unital *JB*-*algebra J *is unitary* if $u \in J^{-1}$ and $u^{-1} = u^*$. Let $U(J)$ be the set of all unitaries in J. As usual, a self-adjoint element $a \in J$ is called positive if its spectrum $\sigma(a)$ is non-negative, where $\sigma(a) := \{\lambda \in C : \lambda e - a \text{ is not invertible}\}$.

In a C*-algebra A, every invertible element a has a unique polar decomposition in the form $a = up$, where u is unitary and p is positive in A [13]. Using this fact, along with some other tools, A. A. Siddiqui proved that each invertible a in a *JB**-algebra J has a unique associated unitary, u in J such that the unitary isotope, $J^{[u]}$ contains a as a positive invertible element. [14, Theorem 4.12].

The system of Jordan triples is a more general notion of Jordan structures. If a Jordan algebra J with a triple product {.,.,.}: $J \times J \times J \longrightarrow J$ that it is linear and symmetric in the outer variables and linear or anti-linear in the inner variable and satisfying the Jordan triple identity,

$$\{a, u, \{b, v, c\}\} + \{\{a, v, b\}, u, c\} - \{b, v, \{a, u, c\}\} = \{a, \{u, b, v\}, c\},$$

for all $u, v, a, b, c \in J$, then J is called a Jordan triple. further, if the triple product is continuous and J is Banach, then J becomes a Banach Jordan triple (cf. [13]).

An extensively studied subclass of Banach Jordan triples called the *JB**-triples, is of main interest in this work and was originally initiated by W. Kaup [9]. A JB*-triple (cf. [9, p. 504] or [13, page 336]) is a complex Banach Jordan space J jointly with a continuous, sesquilinear operator defined by $L(a, b)c := \{a, b, c\}$, on J making it a Banach Jordan triple system that satisfies:

1. $L(a, a)c$ consummates the Jordan triple identity.
2. $L(a, a)$ is a positive Hermitian operator on J.
3. $\|\{a, a^*, a\}\| = \|a\|^3$ for all $a \in J$.

A subtriple F is a linear subspace of J such that $\{F, F, F\} \subseteq F$. Moreover, if a subtriple is norm closed in J then this subtriple turn out to be a JB*-triple. For any elements a, b, c in a JB*-triple J,we have the basic operators, $Q(a)c := \{a, c, a\}$ and $L(a, b)c := \{a, b, c\}$ which are the JB*-triple analogues of JB*-algebra operators, $U_a c^* = \{a, c^*, a\} = Q(a)c$ and $V_{a,b^*} c = \{a, b^*, c\} = L(a, b)$ for all c $\in J$. For any two elements a, $b \in J$, there is another basic operator, called the Bergmann operator, defined on J by

$$B(a, b) := I - 2L(a, b) + Q(a)Q(b),$$

where I is the identity operator on J.

A Jordan homomorphism ψ is a linear operator $\psi: A \rightarrow B$ between two Jordan algebras such that $\psi(a \circ b) = \psi(a) \circ \psi(b)\ \forall\ a, b \in A$. If, in addition, ψ is one-to-one and onto B, then ψ is a Jordan isomorphism; in this case, A and B are isomorphic to each other. A Jordan homomorphism ψ between JB*-algebras such that $\psi(a^*) = (\psi(a))^*$, for every $a \in A$, is called symmetric. In particular, Jordan ∗-homomorphisms are symmetric Jordan homomorphisms. Further, if ψ is injective and $\psi\{a, b, c\} = \{\psi(a), \psi(b)^*, \psi(c)\}\ \forall\ a, b, c \in A$, then ψ is JB*-algebra isomorphism.

In a JB*-triple J, every von Neumann regular a has a unique commuting normalized generalized inverse $\hat{a} \in J$, $satisfying$ $Q(a)\hat{a} = a$, $Q(a)\hat{a} = a$, $Q(\hat{a})a = \hat{a}$ and $Q(a)Q(\hat{a}) = Q(\hat{a})Q(a)$. Observe that a tripotent v in J satisfies; $Q(v)(v) = \{v, v, v\} = v$, so it is von Neumann regular with self-generalized inverse. The class of von Neumann regular elements in JB*-algebras/triples symbolized by $\hat{J}$, has been intensely studied in [11] and [3]. If v is a tripotent in a JB*-triple J, the operator $L(v, v)$ has the eigenvalues 0, $\frac{1}{2}$, 1 and J splits into a direct topological sum of the corresponding eigenspaces (the Peirce decomposition corresponding to *v); $J = J_0(v) \oplus J_{\frac{1}{2}}(v) \oplus J_1(v)$, where each summand is a JB*-sub triples of J (cf. [13]). It is well known that the Peirce 1-space, $J_1(v)$ is a JB*-algebra with Jordan product given by $a \bullet_v b =: \{a, v^*, b\}$ and involution $a^{*_v} = \{v, a^*, v\}$; obviously, *v* is a unit in $J_1(v)$.

Burgos et al. in [4] studied some new linear preservers between JB*-triples. If $G: J \to H$ is a linear operator between JB*-triples and satisfies that $ext\,(J_1) \subseteq ext\,(H_1)$, then G preserves extreme points. [4, Definition 5.4]. If $G(u^{\wedge}) = G(u)^{\wedge}\ \forall u \in J^{\wedge}$, then we say that the linear operator G *strongly preserves regularity.* Obviously, every triple homomorphism $G: J \to H$ between JB*-triples is strongly preserving regularity linear Operator.

Linear Preservers on JB*-triples

Let's recall that a non-zero von Neumann regular element u in a JB*-triple with range tripotent $r(u)$ satisfies,

$$L(u, u^{\wedge}) = L(u^{\wedge}, u) = L(r(u), r(u)), \text{ (cf. [3, p. 198])}$$

Proposition 3.1. *Let J and H be two JB*-algebras, such that J contains a unitary element u. If $G: J \to H$ is a bijective linear operator preserving extreme points, then there is a Jordan $*$-homomorphism $\Phi: J \to H$ such that,*

$$G(x) = G(u)\Phi(x),\ \forall\ x \in J.$$

Proof. First, recall that there is a natural bijective correspondence between JB*-algebras (unital) and nonzero JB*-triples, each with a distinguished unitary element (cf. [17]). The linear operator G in the theorem is a triple isomorphism, since G is a bijective linear operator preserving extreme points, where $u \in U(J) \subseteq ext\,(J_1)$ [2, Theorem 3.2]. Since G is also surjective, there corresponds $a \in J$ with every $b \in H$ such that $y = G(x)$. Also, $\forall\, b \in H$, $L(G(u), G(u))b = G\,L\,(u, u)(a) = G\,I_J\,(a) = G(a) = b = I_H(b)$, hence G$(u)$ is unitary in H. Associated with u and G(u), there correspond two JB*-algebra isotopes $J^{[u]}$ with unit u, and $H^{[G(u)]}$. Let G $(u) = v$ and let G: $J^{[u]} \to H^{[v]}$ be defined on $J^{[u]}$ in the same way as on J. Hence, G is a bijective linear triple isomorphism between the two unital JB*-algebras $J^{[u]}$ and $H^{[v]}$ and it maps unit onto unit. The Jordan triple product $\{x, y^{*_u}, z\}_u$ defined on the isotope $J^{[u]}$ relative to the Jordan product, $\bullet_u$ coincides with the original triple product $\{x, y^*, z\}$, for all $x, y, z \in J$. Being units, u and v are self-adjoint in $J^{[u]}$ and $H^{[v]}$, respectively. By Lemma 5 in [18], G$(x^*) = (G(x))^*$ for all $x \in J^{[u]}$, hence; G maps self-adjoint elements onto self-adjoint elements. Let $A = \{x \in J^{[u]}: x = x^*\}$ be the self-adjoint part of $J^{[u]}$. Since $\|x\| = \|x^*\|$ for any element x in a JB*-algebra, then A is a closed (real) subspace of the unital JB*-algebra $J^{[u]}$, that is; A is a JB-algebra such that $J^{[u]} = A \oplus iA$ which is called the complexification of A [17, Theorem 2.8].

Similarly, for $B = \{x \in H^{[v]}: x = x^*\}$, we have $H^{[v]} = B \oplus iB$, hence both A and B are JB-algebras. Let $G_1: A \to B$ be the restriction of the bijective linear triple isomorphism G which maps self-adjoint elements onto self-adjoint elements, hence G_1 is a bijective linear triple isomorphism. Using [12, Theorem 1.9] that G_1 is also an isometry between A and B. By [12, Theorem 1.9] again, there is a bijective linear isomorphism $\phi: A \to B$ that characterizes G_1 by the relation, $G_1(x) = G_1(u)\phi(x)$, for all $x \in A$. Note that $G_1(u) = G(u) = v$, by definition of the restriction operator G_1. Since any surjective linear isometric between JB- algebras extends to a surjective linear isometric of associated JB*-complexifications [12, Theorem 1.9 and Corollary 1.11], the linear operator $G: J^{[u]} \to H^{[v]}$ which is defined by $G\,(a + ib) = G_1(a) + iG_1(b)$ for all self-adjoint elements $a, b \in A$. Thus, G is a bijective linear isometry. Finally, define $\Phi: J^{[u]} \to H^{[v]}$ by $\Phi(c) = \Phi\,(a + ib) = \phi(a) + i\phi(b)\ \forall\, a, b \in A$ and $c \in J^{[u]}$, which is a bijective linear isomorphism. Thus, $G(c) = G\,(a + ib) = G(u)(\phi(a) + i\phi(b)) = v\,\Phi\,(a + ib) = v\,\Phi(c), \forall c \in J^{[u]}$. Since ϕ is a linear isomorphism, the operator Φ defined above a Jordan homomorphism. Moreover, $\Phi(c^*) = \Phi\,(a - ib) = \phi(a) - i\phi(b) = (\phi(a) + i\phi(b))^* = (\Phi(c))^*$, hence Φ is a Jordan $*$-homomorphism. ☐

By definition, a linear operator between JB*-triples that is strongly preserves BP-quasi invertible elements must also preserves extreme points [4, p. 557], hence we have the corollary.

Corollary 3.2. A bijective linear operator that strongly preserves *BP-quasi invertible elements between two unital JB*(or C*)-algebras is characterized by some Jordan $*$-homomorphism.*

If C*-algebras, A and B are considered as JB*-triples in Proposition 3.1, then [4, Proposition 5.5] follows as a corollary.

Next, we discuss the invariant of the geometric λ– *property* of JB*-triples under linear operators.

Let (λ_k) be a sequence of real numbers with $\lambda_k \geq 0\ \forall\ k \in N$ and $\sum_{k=1}^{\infty} \lambda_k = 1$. If A is a normed space such that for every $a \in A_1$ there correspond two sequences, (λ_k) as described above and $(e_k) \in ext\,(A_1)$ such that a has convex series expansion given by $a = \sum_{k=1}^{\infty} \lambda_k e_k$, then A is said to have **the convex series representation property,**

The geometric λ-property of a normed space A (which is closely related to convex series representation property) was originally studied by Aron and Lohman [1] and they defined the **uniform λ-property** [1, Theorem 3.1 and Remark 3.2] when the sequences of partial sums of those series converge uniformly.

Recall that [8, Definition 2.1] if the set J_q^{-1}, of BP-quasi invertible elements in a JB^*-triple J, is dense in J, then we say that J is extremally rich.

Proposition 3.3. *Let J and H be JB*-triples and let $G: J \to H$ be a non-zero bijective linear operator that strongly preserves BP-quasi invertible elements, then if J has (uniform) λ– property, then so does H.*

Proof. If J has (uniform) λ– property, then as noted before the proposition, J has the convex series representation property. So, for each a in the closed unit sphere of J_1 there is a sequence $(e_k) \in ext(J_1)$ for which $a = \sum_{k=1}^{\infty} \lambda_k e_k$. It is clear that, any linear operator that strongly preserves von Neumann regular elements, obviously strongly preserves BP-quasi invertible elements. Moreover, it was shown in (Theorem 5.11 [4]) that this operator between JB*-triples with $ext\,(J_1) \neq \emptyset$, is indeed a triple homomorphism which means that it preserves triple products. Since the class of extreme points of a JB*-triple is included in the class of BP-quasi invertible elements of JB*-triples. Thus, G also preserves extreme points (cf. [4]). Therefore, $G(a) = \sum_{k=1}^{\infty} \lambda_k G(e_k)$ is a convex series representation of $G(a)$, where $(G(e_k))$ is a sequence in $ext\,(H_1)$.

It follows from *Kaup-Banach-Stone* theorem [10, Proposition 5.5] that the triple isomorphism G between JB*-triples is a linear surjection isometry. Hence, $\|G(a)\| = \|a\| \leq 1$ for all $a \in H_1$, and therefore G maps J_1 onto H_1. So, H has the convex series representation property and hence, it has the (uniform) λ– property. ☐

Remark 3.4. From the proof of Proposition 3.3. above, if G as in the proposition, and if J is extremally rich, then H is also extremally rich.

Conclusions

To sum up, a linear mapping preserving Brown-Pedersen quasi invertible elements between two JB*-algebras, is characterized by a Jordan ∗-homomorphism. This result is a generalization of a similar result of C*-algebars [7]. So, given two JB*-algebras J and H with a non-empty set of extreme points of the closed unit ball of J, if G: $J \to H$ is a linear map strongly preserving BP-quasi invertibility and u is a unitary element in J, then there exists a Jordan ∗-homomorphism Φ: $J \to H$ such that $G(x) = G(u)\Phi(x)$, for every $x \in J$. Other linear operators preservers between JB*-algebras, namely, Bergmann-zero pairs' preservers and extreme points preservers are more challenging cases to be considered. We also deduced that linear operators strongly preserving BP-Pedersen quasi invertible elements between two JB*-triples also preserve the λ-property of both JB*-triples. Other geometric properties such as Bade property or MP-invertibility notion of two JB*-algebras/triples under linear preservers are to be elaborated in forthcoming research.

Conflicts of Interest

The authors declare no conflict of interest.

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