DOI: https://doi.org/10.24297/jam.v22i.9279

$\Theta - \textit{Quasi Triple Operator on Hilbert Space}$

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Abstract:

In this paper we given a new class of operators onHilbert space called$\textit{quasi triple operator}$ and $\Theta - \textit{Quasi Triple Operator}$. We study the operator and introduce some properties of it

Keywords: Triple operator , Hilbert space ,quasi operator.

Introduction: Consider B(H) be the algebra of all bounded linear operators on complex Hilbert space H . Anoperator Υ called normal if $\Upsilon^*\Upsilon = \Upsilon\Upsilon^*$. Quasi normal operator introduced by A. Brown in .1953[1].

In [5] E.S.A.Rejab .2016. introduce anew class of operator on Hilbert space said (Triple operators)and defined as $(\Upsilon\Upsilon^*)\Upsilon = \Upsilon(\Upsilon\Upsilon^*)$. In this paper we defied a new class of operators on Hilbert space as $\daleth[(\daleth\daleth^*)\daleth] = [\daleth(\daleth\daleth^*)]\daleth$

called. $\textit{quasi triple operator}$ and we defined $\Theta - \textit{quasi triple operator}$

as $\daleth[(\daleth\daleth^*)\daleth] = \Theta[\daleth(\daleth\daleth^*)]\daleth$,where Θ is a bounded operator. and study the properties of i

.the MainResults

Definition .1.1 Let $\daleth$ be a bounded operator on Hilbert Space H . $\daleth$ is called $\textit{quasi triple operator}$.

if and only if $\daleth[(\daleth\daleth^*)\daleth] = [\daleth(\daleth\daleth^*)]\daleth$

Definition . 1.2 Let $\daleth$ be a bounded operator on Hilbert space H . $\daleth$ is called $\Theta - \textit{quasi triple operator}$

if and only if $\daleth[(\daleth\daleth^*)\daleth] = \Theta[\daleth(\daleth\daleth^*)]\daleth$,where Θ is a bounded operator.

Proposition 1.3 If $\daleth^{-1}$ exist and $\daleth$ is $\Theta - \textit{quasi triple operator on}$ H and $\daleth$ $\textit{is normal operator}$, then, $\daleth^{-1}$ $\textit{is}$ $\Theta - \textit{quasi triple operator}$

Proof: Since $\daleth$ is $\Theta - \textit{quasi triple operator}$, we have $\daleth[(\daleth\daleth^*)\daleth] = \Theta[\daleth.(\daleth.\daleth^*)]\daleth$

$$\daleth^{-1}.\left[\left(\daleth^{-1}.(\daleth^{-1})^*\right)\daleth^{-1}\right] = \daleth^{-1}.\left[\left(\daleth^{-1}.(\daleth^*)^{-1}\right)\daleth^{-1}\right]$$

$$= \daleth^{-1}\left[\left((\daleth^*\daleth)^{-1}\right)\daleth^{-1}\right] = \daleth^{-1}\left[\left(\daleth(\daleth^*\daleth)\right)^{-1}\right]$$

$$= [\daleth((\daleth^*\daleth)\daleth)]^{-1} = [\daleth((\daleth\daleth^*)\daleth)]^{-1} = [\Theta[\daleth(\daleth\daleth^*)]\daleth]^{-1}$$

$$= \Theta.[\daleth^{-1}[\daleth.(\daleth.\daleth^*)]^{-1}] = \Theta.[\daleth^{-1}\left(\daleth^{-1}(\daleth^{-1})^*\right)]\daleth^{-1}$$

Hence, $\daleth^{-1}$ $\textit{is}$ $\Theta - \textit{quasi triple operator}$ on Hilbert space H.

Theorem 1.4 Let $\daleth$ be is $\Theta - quasi\ \ triple\ operator$ on Hilbert space H .Then $\daleth^{k}$ is $\Theta - quasi\ \ triple\ operator$.

Proof : Let $\daleth$ be is $\Theta - quasi\ \ triple\ operator$

By mathematical induction .

The result true for k=1

$$\daleth.\left[\left(\daleth\daleth^{*}\right).\,\daleth\right] = \Theta\left[\daleth.\left(\daleth.\,\daleth^{*}\right)\right].\,\daleth \quad \ldots \qquad (1)$$

Assume that ,the result true when (k=z)

$$[\daleth.\left[\left(\daleth\daleth^{*}\right)\daleth\right]]^{z} = [\Theta\left[\daleth.\left(\daleth\daleth^{*}\right)\right].\,\daleth]^{z} \quad \ldots \qquad (2)$$

We prove the result for k=z+1

$$[\daleth.\left[\left(\daleth\daleth^{*}\right).\,\daleth\right]]^{z+1} = [\daleth.\left[\left(\daleth\daleth^{*}\right).\,\daleth\right]]^{z}\ [\daleth.\left[\left(\daleth\daleth^{*}\right).\,\daleth\right]]$$

$$= [\Theta[\daleth.\left(\daleth\daleth^{*}\right)]\,\daleth]^{z}\ \ [\Theta\left[\daleth.\left(\daleth\daleth^{*}\right)\right].\,\daleth]$$

$$= [\Theta[.\,\daleth\left(\daleth.\,\daleth^{*}\right)]\,\daleth]^{z+1}$$

Then the result true for k=z+1

Hence, $\daleth^{k}$ is $\Theta - quasi\ \ triple\ operator$

Proposition 1.5 If $\daleth\ \ is\,\Theta - quasi\ \ triple\ operator$ and $\left(\daleth\,.\,\daleth^{*}\right) and \left(\daleth\left(\daleth\,.\,\daleth^{*}\right)\right)\ commute\ with\ \daleth\ and \left(\daleth.\left(\daleth\,.\,\daleth^{*}\right)\right)$ then $\daleth^{*}$ $is\,\Theta - quasi\ \ triple\ operator.$

Proof: Since

$\daleth\ \ is\,\Theta - quasi\ \ triple\ operator$, we have $\daleth\left[\left(\daleth\daleth^{*}\right)\daleth\right] = \Theta[\daleth\left(\daleth\daleth^{*}\right)]\,\daleth.$

$$\daleth^{*}.\left[\left(\daleth^{*}\left(\daleth^{*}\right)^{*}\right)\daleth^{*}\right] = \daleth^{*}.\left[\left(\daleth\ \ \daleth^{*}\right)^{*}\daleth^{*}\right] = \daleth^{*}.\left[\daleth\left(\daleth\ \ \daleth^{*}\right)\ \right]^{*} = \left[\left(\daleth.\left(\daleth\ \ \daleth^{*}\right)\right)\ \daleth\right]^{*}$$

$$= \left[\daleth\left(\left(\daleth\,.\,\daleth^{*}\right)\daleth\right)\ \right]^{*} = \left[\Theta[\daleth.\left(\daleth\daleth^{*}\right)]\,\daleth\right]^{*} = \Theta[\daleth^{*}\left(\daleth^{*}.\left(\daleth^{*}\right)^{*}\right)].\,\daleth^{*}$$

$\daleth^{*}$ $is\,\Theta - quasi\ \ triple\ operator$ hence

Theorem 1.6 Let $\mho$ $is\,\Theta - quasi\ \ triple\ operator$ on H then

Theoperator $\delta\mho$ is $is\,\Theta - quasi\ \ triple\ operator$ for every real scalar δ.

Proof:

Let. $\mho$ $is\,\Theta - quasi\ \ triple\ operator\ on\ H$

∂ be a scalar,then $\mho\left[\left(\mho\mho^{*}\right)\mho\right] = \Theta[\mho\left(\mho\mho^{*}\right)]\mho$ $, where\ \Theta\ is\ bounded\ operator.$

Let ∂ be a scalar, hence

$$(\partial\mho)\left[\left((\partial\mho)(\partial\mho)^{*}\right)(\partial\mho)\right] = (\partial\mho)\left[\left((\partial\mho)(\bar{\partial}\mho^{*})\right)(\partial\mho)\right]$$

$$= \partial\partial\bar{\partial}\partial\ \ \mho\left[\left(\mho\mho^{*}\right)\mho\right] = \partial\partial\bar{\partial}\partial\ \ \Theta[\mho\left(\mho\mho^{*}\right)]\mho$$

$= \Theta[(\partial\mho)\big((\partial\mho)(\partial\mho)^{*}\big)](\partial\mho)$

Hence $(\partial\mho) is\ \Theta - quasi\ \ triple\ operator.$

Theorem 1.7 Let $\daleth$ and ε be two $\Theta - quasi\ \ triple\ operator$ on Hilbert space H such that $\daleth^{*}\varepsilon = \daleth\varepsilon^{*} = \daleth^{*}\varepsilon^{*} = \daleth\varepsilon = 0.$Then $\daleth + \varepsilon$ is $\Theta - quasi\ \ triple\ operator$

Proof: Let $\daleth$and ε be two $\Theta - quasi\ \ triple\ operator$

$(\daleth + \varepsilon)\big[\big((.\daleth + \varepsilon).(\daleth + \varepsilon)^{*}\big).(\daleth + \varepsilon)\big]$

$= (\daleth + \varepsilon).\Big[\Big((.\daleth + \varepsilon)(\daleth^{*} + \varepsilon^{*})\Big).(\daleth + \varepsilon)\Big]$

$= (\daleth + \varepsilon).\Big[\Big(\daleth\daleth^{*} + \daleth\varepsilon^{*} + \varepsilon\daleth^{*} + \varepsilon\varepsilon^{*}\Big).(\daleth + \varepsilon)\Big]$

$= (\daleth + \varepsilon)\Big[\Big((\daleth\daleth^{*}).\daleth + (\daleth\varepsilon^{*})\daleth + (\varepsilon\daleth^{*})\daleth + (\varepsilon\varepsilon^{*})\daleth\Big) + \Big((\daleth\daleth^{*})\varepsilon + (\daleth\varepsilon^{*})\varepsilon + (\varepsilon\daleth^{*})\varepsilon + (\varepsilon\varepsilon^{*}).\varepsilon\Big)\Big]$

$= \Big[\Big(\daleth.(\daleth\daleth^{*})\daleth + \daleth(\daleth\varepsilon^{*})\daleth + \daleth(\varepsilon\daleth^{*})\daleth + \daleth(\varepsilon\varepsilon^{*})\daleth\Big) + \Big(\varepsilon(\daleth.\daleth^{*})\varepsilon + \varepsilon(\daleth\varepsilon^{*})\varepsilon + \varepsilon(\varepsilon\daleth^{*})\varepsilon + \varepsilon(\varepsilon\varepsilon^{*})\varepsilon\Big)\Big]$

$= \daleth.\big[(\daleth\daleth^{*}).\daleth\big] + \varepsilon.[(\varepsilon.\varepsilon^{*})]\varepsilon$

since $\daleth$ and ε be two $\Theta - quasi\ \ triple\ operator$,

$= \Theta.\big[\daleth.(\daleth\daleth^{*})\big].\daleth + \Theta\big[\varepsilon(\varepsilon\varepsilon^{*})\big]\varepsilon$

$= \Theta.[\big[\daleth.(\daleth\daleth^{*})\big].\daleth + \big[\varepsilon(\varepsilon\varepsilon^{*})\big]\varepsilon]$

$= \Theta.\Big[\Big(\daleth.(\daleth\daleth^{*}).\daleth + \daleth(\daleth\varepsilon^{*})\daleth + \daleth(\varepsilon\daleth^{*})\daleth + .\daleth(\varepsilon\varepsilon^{*})\daleth\Big) + \Big(\varepsilon(\daleth\daleth^{*})\varepsilon + \varepsilon(\daleth\varepsilon^{*})\varepsilon + \varepsilon(\varepsilon\daleth^{*})\varepsilon + \varepsilon(\varepsilon\varepsilon^{*})\varepsilon\Big)\Big]$

$= \Theta\Big[\Big((\daleth\daleth^{*})\daleth + (\daleth\varepsilon^{*})\daleth + (\varepsilon\Upsilon^{*})\daleth + (\varepsilon\varepsilon^{*})\daleth\Big) + \Big((\daleth\daleth^{*})\varepsilon + (\daleth\varepsilon^{*})\varepsilon + (\varepsilon\daleth^{*})\varepsilon + (\varepsilon\varepsilon^{*})\varepsilon\Big)\Big](\daleth + \varepsilon)$

$= \Theta(\daleth + \varepsilon)\Big[\Big(\daleth\daleth^{*} + \daleth\varepsilon^{*} + \varepsilon\daleth^{*} + \varepsilon\varepsilon^{*}\Big)(\daleth + \varepsilon)\Big]$

$= \Theta\Big[(\daleth + \varepsilon).\Big((\daleth + \varepsilon)(\daleth^{*} + \varepsilon^{*})\Big)\Big].(\daleth + \varepsilon)$

$= \Theta\big[(\daleth + \varepsilon).\big((\daleth + \varepsilon)(\daleth + \varepsilon)^{*}\big)\big].(\daleth + \varepsilon)$

Hence , $(\daleth + \varepsilon)$ is $\Theta - quasi\ \ triple\ operator$

Theorem 1.8 Let ε be $\Theta - quasi\ \ triple\ operator$ and $\daleth$ is $quasi\ \ triple\ operator$ on Hilbert space H such that $\varepsilon^{*}.\daleth = \wp.\daleth^{*}$, $\varepsilon.\daleth^{*} = \daleth^{*}.\varepsilon$, $\varepsilon^{*}.\daleth^{*} = \daleth^{*}\varepsilon^{*} and\ \ \varepsilon\daleth = \daleth\varepsilon$.Then $(\varepsilon\ \daleth)$ is $\Theta - quasi\ \ triple\ operator$ on Hilbert space H

Proof: Let ε and $\daleth$ *is* $\Theta - quasi\ \ triple\ operator$

$(\varepsilon.\daleth)\big[\big((\varepsilon\ \daleth)(\varepsilon.\daleth)^{*}\big)(\varepsilon\ \daleth)\big]$

$= (\varepsilon\ \daleth)\Big[\Big((\varepsilon\ \daleth)(\daleth^{*}\varepsilon^{*})\Big)(\varepsilon\ \daleth)\Big]$

$$= (\varepsilon\, \daleth)\left[\left((\varepsilon\ \daleth)(\daleth^{*}\varepsilon^{*})\right)(\varepsilon\ \daleth)\right]$$

$$= (\varepsilon\, \daleth)(\varepsilon\ \daleth).\,(\daleth^{*}\varepsilon^{*})(\varepsilon\ \daleth)$$

$$= (\varepsilon\, \daleth)(\varepsilon\ \daleth).\,(\varepsilon^{*}\varepsilon\, \daleth^{*}\ \daleth)$$

$$= [\varepsilon[(\varepsilon.\varepsilon^{*})\varepsilon]][\daleth[(\daleth\daleth^{*})\daleth]]$$

$$= (\Theta[\varepsilon(\varepsilon.\varepsilon^{*})]\varepsilon)([\daleth(\daleth\daleth^{*})]\daleth)$$

$$= \Theta\left[(\varepsilon.\ \daleth)\left((\varepsilon\ \daleth)(\varepsilon\ \daleth)^{*}\right)\right].\,(\varepsilon\ \daleth)$$

Hence , $(\varepsilon\,\daleth)$ is $\Theta - quasi\ \ triple\ operator$

Theorem 1.9 The set of all $\Theta - quasi\ \ triple\ operator$on Hilbert Space H is a closed subset of B(H) (the algebra of all bounded linear operators on Hilbert Space H) under scalar multiplication .

Proof: :supposed

$\Finv(H) = \{\daleth \in B(H): \daleth\ is\ \Theta - quasi\ \ triple\ operatoron\ H\ \}$

Let $\daleth \in \Finv(H)$ then we have $\daleth\ is\ \Theta - quasi\ \ triple\ operator\ on\ H$

and $\daleth[(\daleth\daleth^{*})\daleth] = \Theta[\daleth(\daleth\daleth^{*})]\daleth$, $where\ \Theta\ is\ bounded\ operator.$

Let e be a scalar, hence

$$(e.\daleth)[((e\daleth).(e\daleth)^{*}).(e\daleth)] = (e\daleth).\left[((e\daleth).(\overline{e}\daleth^{*})).(e\daleth)\right]$$

$$= ee\overline{e}e\ \daleth[(\daleth\daleth^{*})\daleth] = ee\overline{e}e\ \Theta[\daleth(\daleth\daleth^{*})]\daleth$$

$$= \Theta\left[(e\daleth).\left((e\daleth).(e\daleth)^{*}\right)\right].(e\daleth)$$

Hence $(e\daleth)\in\Finv(H)$.

Let $\daleth_x$ be sequencing $\Finv(H)$ converge to. $\daleth$, then prove that

$$\|[\daleth[(\daleth\daleth^{*})\daleth]] - [\Theta[\daleth(\daleth\daleth^{*})]\daleth]\|$$

$$= \|[\daleth[(\daleth\daleth^{*})\daleth]] - \left[\daleth_x\left[(\daleth_x.\daleth_x^{*})\daleth_x\right]\right] + \left[\Theta\left[\daleth_x(\daleth_x.\daleth_x^{*})\right].\daleth_x\right] - [\Theta[\daleth(\daleth\daleth^{*})]\daleth]\|$$

$$\leq \|[\daleth[(\daleth\daleth^{*})\daleth]] - \left[\daleth_x\left[(\daleth_x.\daleth_x^{*}).\daleth_x\right]\right]\| + \|\left[\Theta.\left[\daleth_x(\daleth_x.\daleth_x^{*})\right].\daleth_x\right] - [.\,\Theta[\daleth(\daleth\daleth^{*})]\daleth]\| \to 0$$

$as\ \ x\to\infty$.

therefore $\daleth \in \Finv(H)$ .Then , $\Finv(H)$ is closed subset.

Definition 1.10 [3]: If A, B be bounded operators on Hilbert Space H. Then A,B are unitary equivalent if there is anisomorphism $U: H\to H$ suchthat $B = UAU^{*}$.

Theorem 1.11 Let $\daleth\ \ is\ \Theta - quasi\ \ triple\ operator$ on Hilbert space H then

If $\Upsilon\in B(H)$ is unitary equivalent to $\wp$ then $\Upsilon\ \ is\ \Theta - quasi\ \ triple\ operator.$

Proof : Since Υ is unitary equivalent to $\daleth$ then $\Upsilon = . U\daleth U^{*}$ and $(U\daleth U^{*})^{n} = U\daleth^{n}U^{*}$ and Since $\daleth$ $is\ \Theta - quasi\ triple\ operator$, then $\daleth[(\daleth\daleth^{*})\daleth] = \Theta[\daleth(\daleth\daleth^{*})]\daleth$, $where\ \Theta\ is\ bounded\ operator$.

$$\Upsilon[(\Upsilon\Upsilon^{*})\Upsilon] = (U\daleth U^{*})\left[\left((U\daleth U^{*})(U\daleth U^{*})^{*}\right)(U\daleth U^{*})\right]$$

$$= (U\daleth U^{*}).\left[\left((U\daleth U^{*}).(U\daleth^{*}U^{*})\right).(U\daleth U^{*})\right]$$

$$= U\,(\daleth[(\daleth\daleth^{*})\daleth])U^{*}, \text{since } \daleth\ is\ \Theta - quasi\ triple\ operator$$

$$= \Theta\left[(U\daleth U^{*}).\left((U\daleth U^{*}).(U\daleth U^{*})^{*}\right)\right].(U\daleth U^{*})$$

$$\Theta[\Upsilon(\Upsilon\Upsilon^{*})]\Upsilon =$$

$hence\ \Upsilon\ \ is\ \Theta - quasi\ triple\ operator$

Theorem 1.12 : Consider $\daleth_1, \daleth_2, \ldots, \daleth_n$ are $\Theta - quasi\ triple\ operator$ .Then the tenser product $(\daleth_1 \otimes \daleth_2 \otimes \ldots \otimes \daleth_n)$ is $\Theta - quasi\ triple\ operator.$

Proof : Since every operator of $\daleth_1, \daleth_2, \ldots, \daleth_n$ are $\Theta - quasi\ triple\ operator$, then

$$\daleth_l.\left[\left(\daleth_l.\daleth_l^{*}\right)\daleth_l\right] = \Theta.[\daleth_l.\left(\daleth_l.\daleth_l^{*}\right)].\daleth_l\ ,\ where\ \Theta\ is\ bounded\ operator\ ,\ \ for\ all\ l = 1, 2, \ldots, n\ .$$

$$(\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n).\left[\left((\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n).(\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n)^{*}\right).(\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n)\right](x_1\otimes x_2\otimes\ \ldots\ \otimes x_n)$$

$$= (\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n)\left[\left((\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n)(\daleth_1^{*}\otimes\daleth_2^{*}\otimes\ldots\otimes\daleth_n^{*})\right)(\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n)\right](x_1\otimes x_2\otimes\ \ldots\ \otimes x_n)$$

$$= \left(\daleth_1\left[\left(\daleth_1\daleth_1^{*}\right)\daleth_1\right]\right)(x_1)\otimes\left(\daleth_2\left[\left(\daleth_2\daleth_2^{*}\right)\daleth_2\right]\right)(x_2)\otimes\ldots.\otimes(\daleth_n\left[\left(\daleth_n\daleth_n^{*}\right)\daleth_n\right](x_n))$$

$$= \left(\Theta[\daleth_1\left(\daleth_1\daleth_1^{*}\right)]\daleth_1\right)(x_1)\otimes\left(\Theta[\daleth_2\left(\daleth_2\daleth_2^{*}\right)]\daleth_2\right)(x_2)\otimes\ldots.\otimes\left(\Theta[\daleth_n\left(\daleth_n\daleth_n^{*}\right)]\daleth_n\right)(x_n)$$

$$= \Theta[\left([\daleth_1\left(\daleth_1\daleth_1^{*}\right)]\daleth_1\right)(x_1)\otimes\left([\daleth_2\left(\daleth_2\daleth_2^{*}\right)]\daleth_2\right)(x_2)\otimes\ldots.\otimes\left([\daleth_n\left(\daleth_n\daleth_n^{*}\right)]\daleth_n\right)(x_n)]$$

$$= \Theta[(\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n).\left((\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n)(\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n)^{*}\right)].(\daleth_1\otimes\daleth_2\otimes\ldots\otimes\daleth_n)(x_1\otimes x_2\otimes\ \ldots\ \otimes x_n)$$

Hence , $(\daleth_1\otimes\daleth_2\otimes\ \ldots\ \otimes\ \daleth_n)$ is $is\ \Theta - quasi\ triple\ operator$

Theorem 1.13 :Consider $\daleth_1, \daleth_2, \ldots, \daleth_n$ are $\Theta - quasi\ triple\ operator$ .Then the direct sum $(\daleth_1\oplus\daleth_2\oplus\ldots\oplus\daleth_n)$ is $is\ \Theta - quasi\ triple\ operator$

Proof : Since every operator of $\daleth_1, \daleth_2, \ldots, \daleth_n$ are $\Theta - quasi\ triple\ operator$, then

$$\daleth_j.\left[\left(.\daleth_j.\daleth_j^{*}\right)\daleth_j\right] = \Theta.[\daleth_j\left(.\daleth_j\daleth_j^{*}\right)]\daleth_j\ ,\ where\ \Theta\ is\ bounded\ operator\ ,\ \ for\ all\ j = 1, 2, \ldots, n\ .$$

$$(\daleth_1\oplus\daleth_2\oplus\ldots\oplus\daleth_n)\left[\left((\daleth_1\oplus\daleth_2\oplus\ldots\oplus\daleth_n)(\daleth_1\oplus\daleth_2\oplus\ldots\oplus\daleth_n)^{*}\right)(\daleth_1\oplus\daleth_2\oplus\ldots\oplus\daleth_n)\right]$$

$$= (\daleth_1 \oplus \daleth_2 \oplus \ldots \oplus \daleth_n)\left[\left((\daleth_1 \oplus \daleth_2 \oplus \ldots \oplus \daleth_n)(\daleth_1^* \oplus \daleth_2^* \oplus \ldots \oplus \daleth_n^*)\right)(\daleth_1 \oplus \daleth_2 \oplus \ldots \oplus \daleth_n)\right]$$

$$= \left(\daleth_1\left[\left(\daleth_1 \daleth_1^*\right)\daleth_1\right]\right) \oplus \left(\daleth_2\left[\left(\daleth_2 \daleth_2^*\right)\daleth_2\right]\right) \oplus \ldots. \oplus \left(\daleth_n\left[\left(\daleth_n \daleth_n^*\right)\daleth_n\right]\right)$$

$$= \left(\Theta[\daleth_1\left(\daleth_1 \daleth_1^*\right)]\daleth_1\right) \oplus \left(\Theta[\daleth_2\left(\daleth_2 \daleth_2^*\right)]\daleth_2\right) \oplus \ldots. \oplus \left(\Theta[\daleth_n\left(\daleth_n \daleth_n^*\right)]\daleth_n\right)$$

$$= \Theta[(\daleth_1 \oplus \daleth_2 \oplus \ldots \oplus \daleth_n).\left((\daleth_1 \oplus \daleth_2 \oplus \ldots \oplus \daleth_n)(\daleth_1 \oplus \daleth_2 \oplus \ldots \oplus \daleth_n)^*\right)](\daleth_1 \oplus \daleth_2 \oplus \ldots \oplus \daleth_n)$$

Hence , $(\daleth_1 \oplus \daleth_2 \oplus \ldots \oplus \daleth_n)$ is $is\ \Theta - quasi\ triple\ operator$

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