DOI: https://doi.org/10.24297/jam.v21i.9357

Generalized Higher Left Centralizer of Prime Γ-Rings

Salah Mehdi Salih[1] Mazen O. Karim[2] Claus Haetinger[3]

[1] *Depart. of Mathematical, Collage of Education Al- Mustansiryh University Baghdad, Iraq.*
[2]*Department of mathematics College of Educations .University of Al-Qadisiyah , Iraq.*
[3]*Center of Exact and Technological Sciencess, UNIVATEES University Center, Brazil*
E-mail: dr.salahms2014@gmail.com

Abstract

In this paper we introduce the concepts of generalized higher left centralizer and generalized Jordan higher left centralizer of Γ-rings M as well as we proved that every generalized Jordan higher left centralizer of certain Γ-ring M is generalized higher left centralizer of M and we prove every Jordan generalized higher left centralizer of certain Γ-ring M is Jordan generalized triple higher left centralizer of M.

2010 mathematics subject classification : 06B35 , 06B99 , 16B70 , 16B99 .

Keywords: Lattices , Modular lattices , Distributive Lattices , Symmetric(f, g)- bi derivation , generalized symmetric (f, g)- biderivation

1. Introduction

Let M and Γ be additive abeliane groups. M is called a Γ-ring if there exists a mapping M×Γ×M into M (sending (a,α,b) into aαb where a,b ∈ M and α ∈ Γ, such that for all a,b,c ∈ M and α,β ∈ Γ

i) (a+b)αc = aαc + bαc
a(α+β)b = aαb + aβb
aα(b+c) = aαb + aαc

ii) (aαb)βc = aα(bβc)

In 1964 Nobusawa [6] presented the notion of a Γ- ring. In 1966 Barnes [1] generalized the concept of Γ-ring the above definition is dual to Barnes, this concept is more general than concept of a ring.

M is called prime Γ-ring if aΓMΓb = (0) with a,b ∈ M implies a=0 or b=0, M is called semiprime Γ-ring if aΓMΓa = (0) with a ∈ M implies a=0 and M is n-torsion free if na = 0 , for a ∈ M implies a= 0 , where n is an positive integer. as usual $[a,b]_\alpha$ denotes the commutater aαb-bαa, M is commutative if $[a,b]_\alpha$= 0 for all a,b ∈ M , α ∈ Γ.

In [5] J.Jing defined a derivation on ΓΓ-ring as follows, an additive map d from M into itself is said to be a derivation of M if d(aαb) = d(a)αb + aαd(b) for all a,b ∈ M and α ∈ Γ. In [7] M. Sapanci and A. Nakajima defined a Jordan derivation on Γ-ring as follows, an additive map d from M into itself is said to be a Jordan derivation of Γ-ring M if d(aαa) = d(a)αa + aαd(a) for all a ∈ M and α ∈ Γ. It is clear that every derivation of M is Jordan derivation of M. M. Sappanci and A. Nakajima [7], proved that a Jordan derivation on a certain type of completely prime Γ-ring is a derivation. In [2] Y. Ceven and M. Ozturk define a generalized derivation and a Jordan generalized derivation on ΓΓ-ring and proved that Let M be a 2-torsion free prime Γ-ring, for all a,b,c ∈ M so that for any α,β ∈ Γ α, β∈Γwe have $c\beta x\beta[a,b]_\alpha = 0$ or $[a,b]_\alpha\beta x\beta c = 0$cβxβ$[a,b]_\alpha$=0 or $[a,b]_\alpha$βxβc = 0 implies c=0 for all x ∈ M and β∈Γβ ∈ Γ. Then every Jordan generalized derivation of MM is generalized derivation of MM.

In [12] B. Zalar defined a left(resp. right) centralizer and left(resp. right) Jordan centralizer of semiprime ring as follows a left (resp. right) centralizer of RR is an additive mapping T: R→RT from M into itself which satisfies $T(ab) = T(a)b(r\,es\,p.\ T(ab) = aT(b))$T(ab) =T(a)b (resp. T(ab) =at(b)) for all a,b ∈ R. A centralizer of a ring R is both left and right centralizer, and a left (resp. right) Jordan centralizer of a ring R is an additive mapping$T$: R→R T from M into itself which satisfies T(aa) = T(a)a (resp. T(aa) = aT(a)), a Jordan centralizer of a ring R is both left and right Jordan centralizer. It is also proved that every Jordan centralizers of semiprime ring R of characteristic not equal two is a centralizer on a ring R. Joso Vukman [9],[10] and [11] developed some results by using centralizers on prime and semiprime rings.

W.Cortes and G. Haetinger [3] defined σ-centralizer and Jordan σ-centralizer on ring and proved that any left (resp. right) Jordanσ σ-centralizer of 2-torsion free ring is a left (resp. right) σ-σcentralizer.

Md. Fazlud Hoque and A. C. Paul [4] defined a centralizer on ΓΓ-ring as follows , an additive map T from M into itself is said to be a centralizer of MM if T(αab) = T(a)αb + aαT(b)

for all a,b ∈ M and α ∈ Γ, and proved that every Jordan centralizer of a 2-torsion free semiprime Γ-ring is a centralizer. Z.Ullah and M.A. Chaudhary [8] defined a left(resp. right) K-centralizer and Jordan left(resp. right) K-centralizer of semiprime Γ-ring as follows:-

Let MM be a ΓΓ-ring and $K: M \to M$K from M into M an outomorphism such that K(aαb) = K(a)αK(b) for all a,b ∈ M and α ∈ Γ. An additive mapping T from M into itself is a left(resp. right) K-centralizer if T(aαb) = T(a)αK(b) (resp. T(aαb)= K(a)αT(b)) holds for all a,b ∈ M and α ∈ Γ$T(x\alpha y) = T(x)\alpha K(y)$ $(resp. T(x\alpha y) = K(x)\alpha T(y))$.$T$T is called a K-centralizer if it is both a left and a right K-centralizer. And defined that an additive mapping T from M into itself $T: M \to M$is called a Jordan left(resp. right) K-centralizer if T(aαa)= T(a)αK(a) (resp. T(aαa) = K(a)αT(a) holds for all a ∈ M a$x \in M$nd α ∈ Γ α∈Γ. It is also proved that every Jordan left K-centralizer on MMis a left K-centralizer on MM.

In this paper we present and study the concept of higher left centralizers ,Jordan higher left centralizers and Jordan triple higher left centralizer we study the relation among them also we introduce generalized higher left centralizer, Jordan generalized higher left centralizer Jordan triple generalized higher left centralizer and we study the relation among them.

2. Jordan Higher Left Centralizer on Γ-Rings

In this section we present the concepts of higher left centralizer , Jordan higher left centralizer of a Γ-ring M also we study the properties of them. We begin with the following definition:

Definition 2.1: Let T= $(t_i)_{i \in N}$ be a family of additive mappings of a Γ- ring M into itself. Then T is called higher left centralizer if for every n ∈ N

$$t_n(a\alpha b) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b) \quad \text{for all } a,b \in M, \alpha \in \Gamma \qquad ...(1)$$

And T is called Jordan higher left centralizer of R if for every n ∈ N

$$t_n(a\alpha a) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(a) \quad \text{for all } a \in M, \alpha \in \Gamma \qquad ...(2)$$

T is called Jordan triple higher left centralizer of R if for every n ∈ N

$$t_n(a\alpha b\beta a) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) \quad \text{for all } a,b \in M, \alpha,\beta \in \Gamma \qquad ...(3)$$

Lemma 1: Let T= $(t_i)_{i \in N}$ be Jordan higher left centralizer of a Γ- ring M into itself then for all a,b,c ∈ M , α,β ∈ Γ and n ∈ N

1) $t_n(a\alpha b+b\alpha a) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b) + t_i(b)\alpha t_{i-1}(a)$

2) $t_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a)$

In particular if M is 2-tortion free commutative Γ-ring

3) $t_n(a\alpha b\beta c) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c)$

Proof:

1)

$$t_n((a+b)\alpha(a+b)) = \sum_{i=1}^{n} t_i(a + b)\alpha t_{i-1}(a + b)$$

$$= \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(a) + t_i(a)\alpha t_{i-1}(b) + t_i(b)\alpha t_{i-1}(a) + t_i(b)\alpha t_{i-1}(b)$$

...(1)

On the other hand

$t_n((a+b)\alpha(a+b)) = t_n(a\alpha a + a\alpha b + b\alpha a + b\alpha b)$

$$= \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(a) + t_i(b)\alpha t_{i-1}(b) + t_n(a\alpha b + b\alpha a) \quad ...(2)$$

Comparing (1) and (2) we get

$$t_n(a\alpha b + b\alpha a) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b) + t_i(b)\alpha t_{i-1}(a)$$

2) Replace a+c for a in Definition 2.1 (iii)

$$t_n((a+c)\alpha b\beta(a+c)) = \sum_{i=1}^{n} t_i(a+c)\alpha t_{i-1}(b)\beta t_{i-1}(a+c)$$

$$= \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) + t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c)$$

$$+ t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(c) \quad ...(1)$$

On the other hand

$t_n((a+c)\alpha b\beta(a+c)) = t_n(a\alpha b\beta a + a\alpha b\beta c + c\alpha b\beta a + c\alpha b\beta c)$

$$= \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_n(a\alpha b\beta c + c\alpha b\beta a) \quad ...(2)$$

Comparing (1) and (2) we get

$$t_n(a\alpha b\beta c + c\alpha b\beta a) = = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a)$$

3) Since M is commutative and from (2) we get

$$2\, t_n(a\alpha b\beta c) = 2\sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c)$$

Since M is 2-torsion free we get the require result.

Definition 2.2: Let $T=(t_i)_{i \in N}$ be a family of higher Jordan left centralizer of a Γ- ring M and n ∈ N

$$\delta_n(a,b) = t_n(a\alpha b) - \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)$$

Lemma 2: Let $T=(t_i)_{i \in N}$ be a family of higher Jordan left centralizer of a Γ-ring M then for all a,b,c ∈ M, α,β ∈ Γ and n ∈ N

1) $\delta_n(a+b,c)_\alpha = \delta_n(a,c)_\alpha + \delta_n(b,c)_\alpha$

2) $\delta_n(a,b+c)_\alpha = \delta_n(a,b)_\alpha + \delta_n(a,c)_\alpha$

3) $\delta_n(a,b)_\alpha =- \delta_n(b,a)_\alpha$

4) $\delta_n(a,b)_{\alpha+\beta} = \delta_n(a,b)_\alpha + \delta_n(a,b)_\beta$

Proof :

$$1)\delta_n(a+b,c) = t_n((a+b)\alpha b) - \sum_{i=1}^{n} t_i(a+b)\alpha t_{i-1}(c)$$

$$= t_n(a\alpha c + b\alpha c) - \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(c) + t_i(b)\alpha t_{i-1}(c)$$

$$= t_n(a\alpha c) - \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(c) + t_n(a\alpha c) - \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(c)$$

$$= \delta_n(a,c)_\alpha + \delta_n(b,c)_\alpha$$

2) As the same way of (1)

3) By Lemma 2.1 (i)

$$t_n(a\alpha b+b\alpha a) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b) + t_i(b)\alpha t_{i-1}(a)$$

$$t_n(a\alpha b) - \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b) = -t_n(b\alpha a) + \sum_{i=1}^{n} t_i(b)\alpha t_{i-1}(a)$$

$$\delta_n(a,b)_\alpha = -\delta_n(b,a)_\alpha$$

Remark 2.3: Not that $T=(t_i)_{i\in N}$ is higher left centralizer of a Γ- ring M, iff $\delta_n(a,b)_\alpha = 0.$

Lemma 3: Let $T=(t_i)_{i\in N}$ be a family of higher Jordan left centralizer of 2-torsion free prime Γ-ring M then for all a,b ∈ M ,α,β ∈ Γ and n ∈ N

$$\delta_n(a,b)_\alpha\beta[t_{n-1}(a),\ t_{n-1}(b)]_\alpha = 0$$

Proof: we prove by induction on n ∈ N.

If n=1 ,

Let w= aαbβmβbαa + bαaβmβaαb

t(w) = t(aα(bβmβb)αa + bα(aβmβa)αb)

= t(a)α(bβmβb)αa t(b)α(aβmβa)αb ...(1)

On the other hand

t(w) = t((aαb)βmβ(bαa) + (bαa)βmβ(aαb)

= t(aαb)βmβ(bαa) + t(bαa)βmβ(aαb) ...(2)

Compare (1) and (2) we get

0 = (t(aαb) − t(a)α b)βm β(bαa) + (t(bαa) − t(b)αa) βmβ (aαb)

$$= \delta(a,b)_\alpha\beta m\beta b\alpha a + \delta(b,a)_\alpha\beta m\beta a\alpha b$$

$$= \delta(a,b)_\alpha\beta m\beta b\alpha a - \delta(a,b)_\alpha\beta m\beta a\alpha b$$

$$= \delta(a,b)_\alpha\beta m\beta(b\alpha a - a\alpha b)$$

$= \delta(a,b)_\alpha\beta m\beta[a,b]_\alpha$ for all a,b,m ∈ R

Then we can assume that

$$\delta_n(a,b)_\alpha\beta[t_{n-1}(a),\ t_{n-1}(b)]_\alpha = 0$$

for all a,b,m $\in$ R and n,s $\in$ N, s < n.

Now,

$t_n(w) = t_n\,(a\alpha(b\beta m\beta b)\alpha a + b\alpha(a\beta m\beta a)\alpha b)$

$$= \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b\beta m\beta b)\alpha t_{i-1}(a) + t_i(b)\alpha t_i(a\beta m\beta a)\alpha t_{i-1}(b)$$

$$= \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(b)\beta t_{i-1}(b)\alpha t_{i-1}(a) + t_i(b)\alpha t_i(a)\beta t_{i-1}(m)\beta t_{i-1}(a)\alpha t_{i-1}(b)$$

$$= (\sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b))\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a)$$

$$+ t_n(b)\alpha t_{n-1}(a)\beta t_{n-1}(m)\beta \sum_{i=1}^{n} t_{i-1}(a)\alpha t_{i-1}(b)$$

$$+ (\sum_{i=1}^{n} t_i(b)\alpha t_{i-1}(a))\beta t_n(m)\beta t_{n-1}(a)\alpha t_{n-1}(b)$$

$$+ t_n(b)\alpha t_{n-1}(a)\beta t_{n-1}(m)\beta \sum_{i=1}^{n} t_{i-1}(a)\alpha t_{i-1}(b) \qquad ...(1)$$

On the other hand

$t_n(w) = t_n(\,(a\alpha b)\beta m\beta(b\alpha a) + (b\alpha a)\beta m\beta(a\alpha b))$

$$= \sum_{i=1}^{n} t_i(a\alpha b)\beta t_{i-1}(m)\beta t_{i-1}(b\alpha a) + t_i(b\alpha a)\beta t_{i-1}(m)\beta t_{i-1}(a\alpha b)$$

$$= t_n(a\alpha b)\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a) + \sum_{i=1}^{n-1} t_i(a\alpha b)\beta t_{i-1}(m)\beta t_{i-1}(b)\alpha t_{i-1}(a)$$

$$+ t_n(b\alpha a)\beta t_{n-1}(m)\beta t_{n-1}(a)\alpha t_{n-1}(b) + \sum_{i=1}^{n-1} t_i(b\alpha a)\beta t_{i-1}(m)\beta t_{i-1}(a)\alpha t_{i-1}(b) \qquad ...(2)$$

Compare (1) and (2) we get

$$t_n(a\alpha b) - \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b))\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a) + (t_n(b\alpha a) - \sum_{i=1}^{n} t_i(b)\alpha t_{i-1}(a))\beta t_{n-1}(m)\beta t_{n-1}(a)\alpha t_{n-1}(b)$$

$$+ \; t_n(a)\alpha t_{n-1}(b)\beta \sum_{i=1}^{n-1} t_{i-1}(m)\beta t_{i-1}(b)\alpha t_{i-1}(a) + t_n(b)\alpha t_n(a)\beta \sum_{i=1}^{n-1} t_i(m)\beta t_{i-1}(a)\alpha t_{i-1}(b) \; +$$

$$\sum_{i=1}^{n-1} t_i(a\alpha b)\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a) + \sum_{i=1}^{n-1} t_i(b\alpha a)\beta t_{i-1}(m)\beta t_{i-1}(a)\alpha t_{i-1}(b) = 0$$

On our hypothesis, we have

$$0 = \delta_n(a,b)_\alpha\, \beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a) + \delta_n(b,a)_\alpha\, \beta t_{n-1}(m)\beta t_{n-1}(a)\alpha t_{n-1}(b)$$

$$= \delta_n(a,b)_\alpha\, \beta t_{n-1}(m)\beta[t_{n-1}(a), t_{n-1}(b)]_\alpha$$

Theorem 4: Let T=(t_i) $_{i\in N}$ be a Jordan higher left centralize of a prime Γ-ring M then for all a,b,c,d,m $\in$ M ,α,β $\in$ Γ and n $\in$ N

$$\delta_n(a,b)_\alpha\, \beta t_{n-1}(m)\beta[t_{n-1}(c), t_{n-1}(d)]_\alpha = 0$$

Proof: Replacing a+c for a in Lemma 3

$$\delta_n(a+c,b)t_{n-1}(m)[t_{n-1}(a+c),t_{n-1}(b)]=0$$

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]$ + $\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]$ + $\delta_n(c,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]$ + $\delta_n(c,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]$ =0

By Lemma2.3 we get

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha$ + $\delta_n(c,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ =0

Therefore, we get

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha\ \beta\ t_{n-1}(m)\ \beta\ \delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha$ =0

_ $\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha\ \beta\ t_{n-1}(m)\ \beta\ \delta_n(c,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ =0

Hence by primeness of M

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha$ =0 ... (1)

Now, replace b+d for b in Lemma 3, we get

$$\delta_n(a,b+d)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b+d)]_\alpha=0$$

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ + $\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$

$\delta_n(a,d)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ + $\delta_n(a,d)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$ =0

By Lemma3 we get

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$ + $\delta_n(a,d)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ = 0

Then we get

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha\ \beta\ t_{n-1}(m)\ \beta\ \delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$ =0

_ $\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha\ \beta\ t_{n-1}(m)\ \beta\ \delta_n(a,d)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ =0

Since M is prime then

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$ =0 ...(2)

Thus

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a+c),t_{n-1}(b+d)]_\alpha$ =0

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ + $\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$ +

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha$ + $\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(d)]_\alpha$ =0

By (1) and (2) and Lemma 3, we get

$\delta_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(d)]_\alpha$ =0

Theorem 5: Every Jordan higher left centralizer of 2-torsion free prime Γ-ring M is higher left centralizer of M.

Proof: Let $T=(t_i)_{i\in N}$ be Jordan higher left centralizer of prime Γ-ring M.

Since M is prime, we get from Theorem 4 , either $\delta_n(a,b)_\alpha = 0$ or $[t_{n-1}(c), t_{n-1}(d)]_\alpha=0$ for all a,b,c,d ∈ M, α ∈ Γ and n ∈ N.

If $[t_{n-1}(c), t_{n-1}(d)]_\alpha \neq 0$ for all c,d ∈ M, α ∈ Γ, n ∈ N then $\delta_n(a,b)_\alpha = 0$ by Remark 2.3 we get T is higher left centralizer of M.

If $[t_{n-1}(c), t_{n-1}(d)]_\alpha=0$ for all c,d ∈ M, α ∈ Γ, n ∈ N then M is commutative Γ-ring and by Lemma 1 we get

$$t_n(2a\alpha b) = 2\sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)$$

Since M is 2-torsion free, we obtain T is a higher left centralizer of M.

Proposition 6 : Let $T=(t_i)_{i\in N}$ be Jordan higher left centralizer of 2-torsion free Γ-ring M, such that aαbβc = aβbαc for all a,b,c ∈ M, α,β ∈ Γ then T is Jordan triple higher left centralizer of M.

Proof : Replace b by aβb + bβa in Definition 2.1, then :

$$t_n(a\alpha(a\beta b+b\beta a) + (a\beta b+b\beta a)\alpha a) = \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(a\beta b+b\beta a) + t_i(a\beta b+b\beta a)\alpha t_{i-1}(a) =$$

$$\sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(a)\beta t_{i-1}(b) + t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) + t_i(a)\beta t_{i-1}(b)\alpha t_{i-1}(a) + t_i(b)\beta t_{i-1}(a)\alpha t_{i-1}(a)$$

$$= \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(a)\beta t_{i-1}(b) + t_{i-1}(b)\beta t_{i-1}(a)\alpha t_{i-1}(a) + 2\sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) \quad ...(1)$$

On the other hand :

$t_n(a\alpha(a\beta b+b\beta a) + (a\beta b+b\beta a)\alpha a) = t_n(a\alpha a\beta b + a\alpha b\beta a + a\beta b\alpha a + b\beta a\alpha a)$

$= t_n(a\alpha a\beta b + b\beta a\alpha a) + 2t_n(a\alpha b\beta a)$

$$= \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(a)\beta t_{i-1}(b) + t_i(b)\beta t_{i-1}(a)\alpha t_{i-1}(a) + 2t_n(a\alpha b\beta a) \quad ...(2)$$

Now, compare (1) and (2) we get

$$2t_n(a\alpha b\beta a) = 2\sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a)$$

Since M is 2-torsion free we obtain that T is Jordan triple higher left centralizer of M.

3. Generalized Higher Left Centralizer on Γ-Rings

In this section we present the concepts of generalized higher left centralizer and generalized Jordan higher left centralizer of Γ-rings also we present some properties of them.

Definition 3.1: Let $F=(f_i)_{i\in N}$ be a family of additive mappings of a Γ- ring M into itself. F is called generalized higher left centralizer of M if there exists a higher left centralizer $T=(t_i)_{i\in N}$ of M such that for every n ∈ N we have

$$f_n(a\alpha b) = \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b) \quad ...(1)$$

for all a,b ∈ M and α ∈ Γ where T is called the relating higher left centralizer.

F is called Jordan generalized higher left centralizer of M if there exists a Jordan higher left centralizer of M if for every n ∈ N

$f_n(a\alpha a) = \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(a)$ ...(2)

for all a ∈ M and α ∈ Γ where T is called the relating Jordan left centralizer.

F is called Jordan generalized triple higher left centralizer of M if there exists a Jordan triple higher left centralizer of M if for every n ∈ N

$f_n(a\alpha b\beta a) = \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a)$ ...(3)

for all a,b ∈ M and α,β ∈ Γ where T is called the relating Jordan triple left centralizer,

Lemma 7: Let F= $(f_i)_{i \in N}$ be Jordan generalized higher left centralizer of a Γ-ring M into itself then for all a,b,c ∈ M ,α,β ∈ Γ and n ∈ N

1) $f_n(a\alpha b+b\alpha a) = \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b) + f_i(b)\alpha t_{i-1}(a)$

2) $f_n(abc + cba) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(c) + f_i(c)t_{i-1}(b)t_{i-1}(a)$

In particular if M is 2-tortion free commutative ring

3) $f_n(a\alpha b\beta c) = \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c)$

Proof: 1) $f_n((a+b)\alpha(a+b)) = \sum_{i=1}^{n} f_i(a+b)\alpha t_{i-1}(a+b)$

$= \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(a) + f_i(a)\alpha t_{i-1}(b) + f_i(b)\alpha t_{i-1}(a) + f_i(b)\alpha t_{i-1}(b)$ ...(1)

On the other hand

$f_n((a+b)\alpha(a+b)) = f_n(a\alpha a + a\alpha b + b\alpha a + b\alpha b)$

$= \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(a) + f_i(b)\alpha t_{i-1}(b) + f_n(a\alpha b + b\alpha a)$ ...(2)

Comparing (1) and (2) we get

$f_n(a\alpha b+b\alpha a) = \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b) + f_i(b)\alpha t_{i-1}(a)$

2) Replace a+c for a in Definition 2.1 (iii)

$f_n((a+c)\alpha b\beta(a+c)) = \sum_{i=1}^{n} f_i(a+c)\alpha t_{i-1}(b)\beta t_{i-1}(a+c)$

$= \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) + f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + f_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a) + f_{i-1}(c)\alpha t_{i-1}(b)\beta t_{i-1}(c)$

...(1)

On the other hand

$f_n((a+c)\alpha b\beta(a+c)) = f_n(a\alpha b\beta a + a\alpha b\beta c + c\alpha b\beta a + c\alpha b\beta c)$

$= \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) + ft_{i-1}(c)\alpha t_{i-1}(b)\beta t_{i-1}(c) + f_n(a\alpha b\beta c + c\alpha b\beta a)$ ...(2)

Comparing (1) and (2) we get

$f_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + f_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a)$

3) Since M is 2-torsion free commutative ring and from (2) we get the require result.

Definition 3.2: Let $F=(f_i)_{i\in N}$ be a family of Jordan generalized higher left centralizer of aΓ- ring M with relating Jordan higher left centralizer $T=(t_i)_{i\in N}$ of M , then for all a,b $\in$ M, $\alpha \in \Gamma$ and n $\in$ N,we define

$$\Phi_n(a,b)_\alpha = f_n(a\alpha b) - \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)$$

Lemma 8: Let $F=(f_i)_{i\in N}$ be a family of higher Jordan left centralizer of a Γ- ring M with relating Jordan higher left centralizer $T=(t_i)_{i\in N}$ of M then for all a,b,c $\in$ M, $\alpha \in \Gamma$ and n $\in$ N

1) $\Phi_n(a+b,c)_\alpha = \Phi_n(a,c)_\alpha + \Phi_n(b,c)_\alpha$

2) $\Phi_n(a,b+c)_\alpha = \Phi_n(a,b)_\alpha + \Phi_n(b,c)_\alpha$

3) $\Phi_n(a,b)_\alpha = -\Phi_n(b,a)_\alpha$

Proof :

1) $\Phi_n(a+b,c)_\alpha = f_n((a+b)c)_\alpha - \sum_{i=1}^{n} f_i(a+b)\alpha t_{i-1}(c)$

$= f_n(a\alpha c + b\alpha c) - \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(c) + f_i(b)\alpha t_{i-1}(c)$

$= f_n(a\alpha c) - \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(c) + f_n(b\alpha c) - \sum_{i=1}^{n} f_i(b)\alpha t_{i-1}(c)$

$= \Phi_n(a,c)_\alpha + \Phi_n(b,c)_\alpha$

2) As the same way of (1)

3) By Lemma 7 (i)

$f_n(a\alpha b + b\alpha a) = \sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b) + f_i(b)\alpha t_{i-1}(a)$

$$f_n(a\alpha b)-\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)=-f_n(b\alpha a)+\sum_{i=1}^{n} f_i(b)\alpha t_{i-1}(a)$$

$$\Phi_n(a,b)_\alpha=-\Phi_n(b,a)_\alpha$$

Remark 3.3: Not that F=(f_i)$_{i\in N}$ is generalized higher left centralizer of a Γ- ring M with relating higher left centralizer T=(t_i)$_{i\in N}$ of M , iff $\Phi_n(a,b)_\alpha=0.$

Lemma 9: Let F=(f_i)$_{i\in N}$ be a family of Jordan generalized higher left centralizer of 2-torsion free prime Γ-ring M then for all a,b ∈ M, α,β ∈ Γ and n ∈ N

$$\Phi_n(a,b)_\alpha\beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha=0$$

Proof: we prove by induction on n ∈ N.

If n=1 ,

Let w= aαbβmβbαa + bαaβmβaαb

f(w) = f(aα(bβmβb)αa + bα(aβmβa)αb)

= f(a)α(bβmβb)αa+ t(b)α(aβmβa)αb ...(1)

On the other hand

f(w) = f((aαb)βmβ(bαa) + (bαa)βmβ(aαb)

=f(aαb)βmβ(bαa) + f(bαa)βmβ(aαb) ...(2)

Compare (1) and (2) we get

0= (f(aαb) –f(a)α b)βmβ(bαa) + (f(bαa) – f(b)αa)β mβ (aαb)

$= \Phi(a,b)_\alpha\beta m\beta b\alpha a+\Phi(b,a)_\alpha\beta m\beta a\alpha b$

$= \Phi(a,b)_\alpha\beta m\beta b\alpha a-\Phi(a,b)_\alpha\beta m\beta a\alpha b$

$= \Phi(a,b)_\alpha\beta m\beta(b\alpha a-a\alpha b)$

$= \Phi(a,b)_\alpha\beta m\beta[a,b]_\alpha$ for all a,b,m ∈ M and α,β ∈ Γ

Then we can assume that

$\Phi_s(a,b)t_{s-1}(m)[t_{s-1}(a),t_{s-1}(b)]=0$ for all a,b,m ∈ R and n,s ∈ N, s < n.

Now,

f_n(w) = f_n (aα(bβmβbαa+ bα(aβmβa)αb)

$$=\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b\beta m\beta b)\alpha t_{i-1}(a)+f_i(b)\alpha t_i(a\beta m\beta a)\alpha t_{i-1}(b)$$

$$=\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(m)\beta t_{i-1}(b)\alpha t_{i-1}(a)+f_i(b)\alpha t_i(a)\beta t_{i-1}(m)\beta t_{i-1}(a)\alpha t_{i-1}(b)$$

$$=(\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b))\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a)+f_n(b)\beta t_{n-1}(a)\beta t_{n-1}(m)\beta\sum_{i=1}^{n} t_{i-1}(a)\alpha t_{i-1}(b)$$

$$+ (\sum_{i=1}^{n} f_i(b)\alpha t_{i-1}(a))\beta t_n(m)\beta t_{n-1}(a)\alpha t_{n-1}(b) + f_n(b)\alpha t_{n-1}(a)\beta \sum_{i=1}^{n} t_{i-1}(a)\alpha t_{i-1}(b)\alpha t_{i-1}(a)$$

...(1)

On the other hand

$f_n(w)= f_n((a\alpha b)\beta m\beta(b\alpha a) + (b\alpha a)\beta m\beta(a\alpha b))$

$$= \sum_{i=1}^{n} f_i(a\alpha b)\beta t_{i-1}(m)\beta t_{i-1}(b\alpha a) + f_i(b\alpha a)\beta t_{i-1}(m)\beta t_{i-1}(a\alpha b)$$

$$= f_n(a\alpha b)\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a) + \sum_{i=1}^{n-1} f_i(a\alpha b)\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a)$$

$$+ f_n(b\alpha a)\beta t_{n-1}(m)\beta t_{n-1}(a\alpha b) + \sum_{i=1}^{n-1} f_i(b\alpha a)\beta t_{i-1}(m)\beta t_{i-1}(a)\alpha t_{i-1}(b)$$ ...(2)

Compare (1) and (2) we get

$$0= (f_n(a\alpha b) - \sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b))\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a) + (f_n(b\alpha a) - \sum_{i=1}^{n} t_i(b)\alpha t_{i-1}(a))\beta t_{n-1}(m)\beta t_{n-1}(a)\alpha t_{n-1}(b)$$

$$+ f_n(a)\alpha t_{n-1}(b)\beta \sum_{i=1}^{n-1} t_{i-1}(m)\beta t_{i-1}(b)\alpha t_{i-1}(a) + f_n(b)\alpha t_n(a)\beta \sum_{i=1}^{n-1} t_i(m)\beta t_{i-1}(a)\alpha t_{i-1}(b) +$$

$$\sum_{i=1}^{n-1} f_i(a\alpha b)\beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a) + \sum_{i=1}^{n-1} f_i(b\alpha a)\beta t_{i-1}(m)\beta t_{i-1}(a)\alpha t_{i-1}(b)$$

On our hypothesis, we have

$$0= \Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta t_{n-1}(b)\alpha t_{n-1}(a) + \Phi_n(b,a)_\alpha \beta t_{n-1}(m)\beta t_{n-1}(a)\alpha t_{n-1}(b)$$

$$= \Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a), t_{n-1}(b)]_\alpha$$

Theorem 10: Let $F=(f_i)_{i \in N}$ be a Jordan generalized higher left centralize of a prime Γ-ring M then for all a,b,c,d,m ∈ M, α,β ∈ Γ and n ∈ N

$$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c), t_{n-1}(d)]_\alpha = 0$$

Proof: Replacing a+c for a in Lemma 9

$$\Phi_n(a+c,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a+c), t_{n-1}(b)]_\alpha = 0$$

$$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a), t_{n-1}(b)]_\alpha + \Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c), t_{n-1}(b)]_\alpha +$$

$$\Phi_n(c,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a), t_{n-1}(b)]_\alpha + \Phi_n(c,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c), t_{n-1}(b)]_\alpha = 0$$

By Lemma 9 we get

$$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c), t_{n-1}(b)]_\alpha + \Phi_n(c,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a), t_{n-1}(b)]_\alpha = 0$$

Therefore, we get

$$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c), t_{n-1}(b)]_\alpha \, \beta \, t_{n-1}(m) \, \beta \, \Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c), t_{n-1}(b)]_\alpha = 0$$

$_-\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha \, \beta \, t_{n-1}(m) \, \beta \, \Phi_n(c,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ =0

Hence by primeness of M

$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha$ =0 ... (1)

Now, replace b+d for b in Lemma 9, we get

$\Phi_n(a,b+d)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b+d)]_\alpha = 0$

$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha + \Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]$

$\delta_n(a,d)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha + \Phi_n(a,d)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$ =0

By Lemma 9 we get

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)] + \Phi_n(a,d)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ = 0

Then we get

$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha \, \beta \, t_{n-1}(m) \, \beta \, \Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$ =0

$_-\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha \, \beta \, t_{n-1}(m) \, \beta \, \Phi_n(a,d)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha$ =0

Since R is prime then

$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha$ =0 ...(2)

Thus

$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a+c),t_{n-1}(b+d)]_\alpha$ =0

$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(b)]_\alpha + \Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(a),t_{n-1}(d)]_\alpha +$

$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(b)]_\alpha + \Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(d)]_\alpha$ =0

By (1) and (2) and Lemma 2.3, we get

$\Phi_n(a,b)_\alpha \beta t_{n-1}(m)\beta[t_{n-1}(c),t_{n-1}(d)]_\alpha$ = 0

Theorem 11: Every Jordan generalized higher left centralizer of 2-torsion free prime Γ-ring M is generalized higher left centralizer of M.

Proof: Let F= (f_i) $_{i \in N}$ be Jordan generalized higher left centralizer of prime Γ-ring M.

Since M is prime, we get from Theorem10 , either $\Phi_n(a,b)_\alpha = 0$ or $[t_{n-1}(c), t_{n-1}(d)]_\alpha$=0 for all a,b,c,d ∈ M, α ∈ Γ and n ∈ N.

If $[t_{n-1}(c), t_{n-1}(d)]_\alpha \neq$ 0 for all c,d ∈ M, α ∈ Γand n ∈ N then $\Phi_n(a,b)_\alpha = 0$ by Remark 2.3 we get T is higher left centralizer of M.

If $[t_{n-1}(c), t_{n-1}(d)]_\alpha$=0 for all c,d ∈ M, α ∈ Γ and n ∈ N then M is commutative ring and by Lemma 7 we get

$$t_n(2a\alpha b) = 2\sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)$$

Since M is 2-torsion free, we obtain T is a higher left centralizer of M.

Proposition 12: Let F= (f_i) $_{i \in N}$ be Jordan generalized higher left centralizer of 2-torsion free Γ-ring M, such that aαbβc = aβbαc, for all a,b,c ∈ M and α,β ∈ Γ then T is Jordan generalized triple higher left centralizer of M.

Proof : Replace b by aβb + bβa in Definition 3.1, then :

f_n(aα(aβb+bβa) + (aβb+bβa)αa) = $\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(a\beta b + b\beta a) + f_i(a\beta b + b\beta a)\alpha t_{i-1}(a)$

= $\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(a)\beta t_{i-1}(b) + f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) + f_i(a)\beta t_{i-1}(b)\alpha t_{i-1}(a) + f_i(b)\beta t_{i-1}(a)\alpha t_{i-1}(a)$

= $\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(a)\beta t_{i-1}(b) + f_i(b)\beta t_{i-1}(a)\alpha t_{i-1}(a) + 2\sum_{i=1}^{n} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a)$ ...(1)

On the other hand :

f_n(aα(aβb+bβa) + (aβb+bβa)αa) = f_n(aαaβb + aαbβa + aβbαa+ bβaαa)

= f_n (aαaβb + bβaαa) + 2f_n(aαbβa)

= $\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(a)\beta t_{i-1}(b) + f_i(b)\beta t_{i-1}(a)\alpha t_{i-1}(a) + 2f_n(a\alpha b\beta a)$ ...(2)

Now, compare (1) and (2) we get

$$2f_n(a\alpha b\beta a) = 2\sum_{i=1}^{n} f_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a)$$

Since M is 2-torsion free we obtain that F is Jordan generalized triple higher left centralizer of M.

References

[1] W.E.Barnes, "On the Γ-Rings of Nobusawa", Pacific J.Math., Vol.18, pp.411-422, 1966.

[2] Y.Ceven and M.A.Özturk, "On Jordan Generalized Derivations in Gamma Rings", Hacettepe J. of Mathematics and Statistics, Vol.33, pp.11-14, 2004.

[3] W.Cortes and C.Haetinger, "On Lie Ideals Left Jordan σ-Centralizer of 2-torsion Free Rings", Math. J .Okayama Univ., Vol.51, No.1, pp.111-119, 2009.

[4] Md.Fazlud Hoque and A.C.Paul, "On Centralizers of Semiprime Gamma Rings", International Mathematical Forum, Vol.6, No.13, pp.627-638, 2011.

[5]] F.J.Jing, "On Derivations of Γ-Ring", QU FU Shi Fun Daxue Xuebeu Ziran Kexue Ban, Vol.13, No.4, pp.159-161, 1987.

[6] N.Nobusawa , "On a Generalization of the Ring Theory", Osaka J.Math. , pp.81-89, 1964.

[7] M.Sapanci and A.Nakajima, "Jordan Derivations on Completely Prime Gamma Rings", Math.Japonica, Vol.46, No.1, pp.47-51, 1997.

[8] Z.Ullah and M.A.Chaudhary, "On K-Centralizers of Semiprime Gamma Rings", International Journal of Algebra, Vol.6, No.21, pp.1001-1010, 2012.

[9] J.Vakman,"Centralizers in Prime and Semiprime Rings", Comment. Math. Univ.Carolinae , pp.231-240, 38 (1997).

[10] J.Vakman, "An Identity Related to Centralizers in Semiprime Rings", Comment. Math. Univ. Carolinaae, 40, pp.446- 456, 3 (1999).

[11] J.Vakman, "Centralizers on Semiprime Rings", Comment. Math. Univ. Carolinae, 42, pp.237-245, 2 (2001).