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SOME RESULTS OF A QUARTER SYMMETRIC METRIC CONNECTION ON A P-SASAKIAN MANIFOLD

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ABSTRACT

In the present paper we have a study on some properties of quarter symmetric metric connection in P-sasakian manifold and some results on quarter symmetric metric connection, m-projective curvature tensor, con-harmonic curvature tensor and pseudo projective curvature tensor.

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1 Introduction

The idea of quarter symmetric linear connection was introduced by Golabh S. [14] in 1975. Various properties of quarter symmetric connection is studied by Mishra R.S. and Pandey S.N. [10], Rastogi S.C. [13],. Mondal A.K and De U.C. [2], Sengupta J. and Biswas B. [6] and many others. In 2003 Sengupta J. and Biswas B. studied quarter symmetric non metric connection on a sasakian manifold. Adati and Matsumoto [18] defined para sasakian Manifold and special para sasakian manifold which are special classes of an almost para contact manifold introduced by Sato [5]. Para sasakian manifold has been studied by Aditi T., Miyazawa D. [19] and others. In this paper we have studied some results on a quarter symmetric connection on P-sasakian Manifold. We have also studied some properties of quarter symmetric metric connection in $P-$sasakian manifold, W_2-curvature tensor, Pseduo projective curvature tensor in Sasakian manifold. A linear connection D in an n- dimensional differentiable manifold M (where n is odd) is said to be semi-symmetric connection if its torsion tensor T of type (1, 2) is defined as

$$T(X,Y) = D_X Y - D_Y X - [X,Y]$$

$$= \eta(Y)X - \eta(X)Y. \tag{1}$$

for arbitrary vector fields X and Y, where η is 1-form. If T vanishes then the manifold M becomes torsion free. The connection D on the manifold is a metric connection, if there is a Riemannian metric g in M such that $Dg = 0$, otherwise it is non metric. Section 2 is devoted to preliminaries and some definitions. In section 3 we have studied covaraiant derivative of Riemannian curvature with respect to quarter symmetric metric connection. Also we have proved the Bianchi's identity with respect to quarter symmetric metric connection. In section 4 we have obtained some results on pseudo projective curvature tensor, con-circular curvature tensor and Weyl cutvature tensor. In this section we have also proved that covariant derivative of W_2-curvature is equal to the Riemannian curvature with respect to quarter symmetric metric connection.

2 Preliminaries

An n-dimensional (where n is odd) differentiable manifold M is called an almost paracontact manifold if it admits almost paracontact structure (ϕ, ξ, η) consisting of a (1,1) tensor field ϕ, a vector field ξ and 1-form η satisfying

$$\phi^2 X = X - \eta(X)\xi \tag{2}$$

$$\eta(\xi) = 1, \tag{3}$$

$$\phi o \xi = 0 \tag{4}$$

and

$$\eta o \phi = 0. \tag{5}$$

The structure (ϕ, ξ, η) has a compatible Riemannian metric g such that

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{6}$$

or equivalently

$$g(\phi X, Y) = g(X, \phi Y) \tag{7}$$

and

$$g(X,\xi) = \eta(X) \tag{8}$$

for all $X \,\&\, Y \in TM$, then M is said to be an almost paracontact Riemannian manifold equipped with an almost paracontact Rimannian structure (ϕ, ξ, η, g). An almost para contact Riemannian manifold is called a P-sasakian manifold if it satisfies

$$(\nabla_X \phi)Y = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi \tag{9}$$

$$\forall X, Y \in TM,$$

where ∇ is Levi Civita connection of the Riemannian metric [8].

From above equations we have

$$\nabla_X \xi = \phi X \quad (10)$$

$$(\nabla_X \eta)Y = (\nabla_Y \eta)X = g(X, \phi Y), \forall X, Y \in TM \quad (11)$$

In an n-dimensional (where n is odd) P-sasakian manifold M, the curvature tensor R, Ricci operator Q and Ricci tensor S satisfy [18]

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \quad (12)$$

$$R(\xi, X)Y = \eta(Y)X - g(X,Y)\xi, \quad (13)$$

$$S(X,\xi) = -(n-1)\eta(X), \quad (14)$$

$$S(\phi(X), \phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \quad (15)$$

$$\eta(R(X,Y)Z) = g(R(X,Y)Z,\xi) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \quad (16)$$

$$\eta(R(\xi, X)Y) = \eta(X)\eta(Y) = g(X,Y) \quad (17)$$

A linear connection $\tilde{\nabla}$ in a Riemannian manifold is said to be quarter symmetric metric connection [17], if its torsion tensor satisfies

$$T(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y] \quad (18)$$

$$= \rho(Y)\phi X - \rho(X)\phi Y. \quad (19)$$

A quarter symmetric metric connection $\tilde{\nabla}$ is given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X \quad (20)$$

If $\phi(X) = X$, then quarter symmetric connection becomes semi symmetric connection where ∇ is Riemannian connection. A quarter symmetric connection $\tilde{\nabla}$ satisfying following condition

$$(\tilde{\nabla}_X g)(Y,Z) = 0 \quad (21)$$

for all $X, Y \& Z \in TM$, where TM is Lie algebra of vector fields of the manifold M. The quarter symmetric connection satisfying (21) for the metric g is called quarter symmetric metric connection. If $\tilde{R} and\ R$ are curvature tensors with respect to connections $\tilde{\nabla}$ and ∇ respectively then we have

$$\tilde{R}(X,Y,U) = R(X,Y,U) + 3g(X,U)Y - 3g(Y,U)X - 2g(X,U)\eta(Y)\xi$$
$$+ 2g(Y,U)\eta(X)\xi - 2\eta(X)\eta(U)Y + 2\eta(Y)\eta(U)X \quad (22)$$

$$\forall X, Y \in TM.$$

Definition 2.1 Let M be an odd dimensional P-Sasakian manifold of dimension n then $W_2 -$ curvature tensor of M with respect to Levi-Civita connection ∇ is defined by [8]

$$W_2(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[g(Y,Z)QX - g(X,Z)QY] \tag{23}$$

$$\forall X, Y \in TM.$$

Definition 2.2 Let M be an odd dimensional P-Sasakian manifold of dimension n then the pseudo-projective curvature tensor $\overline{P}$ is defined by

$$\overline{P}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] \tag{24}$$

$$-\frac{r}{n}\left\{\frac{a}{n-1} + b\right\}[g(Y,Z)X - g(X,Z)Y], \forall X, Y \in TM$$

where 'a' and 'b' are constants such that both $a, b \neq 0$ *and* R, S and r are the curvature tensor, Ricci tensor and scalar curvature respectively [8].

Definition 2.3 Let M be an n-dimensional P-Sasakian manifold where n is odd then the Pseudo projective curvature tensor $\overline{P}$ is defined as

$$\overline{P}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y]$$

$$-\frac{r}{n}\left\{\frac{a}{n-1} + b\right\}[g(Y,Z)X - g(X,Z)Y] \tag{25}$$

$$\forall X, Y \in TM,$$

where R is curvature tensor, S is Ricci tensor & r is scalar curvature & a,b are parameters such that $a, b \neq 0$.

Definition 2.4 The con circular curvature tensor [2] is

$$\tilde{C}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] \tag{26}$$

$$\forall X, Y \in TM.$$

Definition 2.5 The Weyl curvature tensor is defined by

$$\tilde{W}(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)LX - g(X,Z)LY]$$

$$+\frac{r}{(n-1)(n-2)}\{g(Y,Z)X - g(X,Z)Y\} \tag{27}$$

for all $X, Y, Z \in TM$, where r is scalar curvature & is symmetric endomorphism of the tangent space corresponding to Q the Ricci Tensor [2].

Definition 2.6 A para sasakian manifold is called special para sasakian manifold if M admits a 1-form η satisfying

$$(\nabla_X \eta)(Y) = -g(X,Y) + \eta(X)\eta(Y) \tag{28}$$

Definition 2.7 An n-dimensional (where n is odd) P-Sasakian manifold is η -Einstein if the Ricci tensor S satisfies

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) \tag{29}$$

$$\forall X,Y \in TM$$

where ‘a’ and ‘b’ are smooth functions on the manifold. The η -Einstein manifold reduces to an Einstern manifold if $b \neq 0$

3 Curvature properties in quarter symmetric connection

The curvature tensor with respect to quarter symmetric metric connection $\tilde{\nabla}$ is quarter defined by

$$\tilde{R}(X,Y)Z = \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]} Z$$

$$= \tilde{\nabla}_X (\nabla_Y Z + \eta(Z)FY) - \tilde{\nabla}_Y [\nabla_Z X + \eta(Z)FX]$$

$$-[\nabla_{[X,Y]} Z + \eta(Z)F(X,Y)]$$ where F= Φ

$$= \tilde{\nabla}_X (\nabla_Y Z) + \eta(Z)\tilde{\nabla}_X (FY) - \tilde{\nabla}_Y (\nabla_X Z) - \eta(Z)\tilde{\nabla}_Y (FX) - \nabla_{[X,Y]} Z - \eta(Z)F[X,Y]$$

... ... by help of equation (20)

$$= \nabla_X \nabla_Y Z + \eta(\nabla_Y Z)FX + \eta(Z)\{\nabla_X FY + \eta(FY)FX\} - \{\nabla_Y \nabla_X Z + \eta(\nabla_X Z)FY\}$$

$$-\eta(Z)\{\nabla_Y FX + \eta(FX)FY\} - \nabla_{[X,Y]} Z - \eta(Z)F[X,Y]$$

$$= R(X,Y,Z) + \eta(\nabla_Y Z)FX + \eta(Z)\nabla_X FY$$

$$-\eta(\nabla_X Z)FY - \eta(Z)\nabla_Y FX - \eta(Z)F[\nabla_X Y - \nabla_Y X].$$

$$= R(X,Y,Z) + [\nabla_Y \eta(Z) - \eta(Z)\eta(FY)]FX - [\nabla_X \eta(Z) - \eta(Z)\eta(FX)]FY$$

$$= [R(X,Y,Z) + \alpha(Y,Z)FX - \alpha(X,Z)FY]$$

where

$$\alpha(Y,Z) = \nabla_Y \eta(Z) - \eta(Z)\eta(FY)$$

$$\tilde{R}(X,Y,Z) + \tilde{R}(Y,Z,X) + \tilde{R}(Z,X,Y) = R(X,Y,Z) + R(Y,Z,X) + R(Z,X,Y)$$

$$\therefore \tilde{R}(X,Y,Z) + \tilde{R}(Y,Z,X) + \tilde{R}(Z,X,Y) = 0 \tag{30}$$

which is Bianchi’s first identity for quarter symmetric metric connection. Thus we state the following theorem.

Theorem 3.1 The Riemannian curvature tensor with respect to quarter symmetric metric connection satisfies the following condition for P- Sasakian Manifold

$$\tilde{R}(X,Y,Z)+\tilde{R}(Y,Z,X)+\tilde{R}(Z,X,Y)=0 \quad (31)$$

$$\forall X,Y \,\&\, Z \in TM.$$

Since

$$\tilde{R}(X,Y,Z)=R(X,Y,Z)-\alpha(Y,Z)FX-\alpha(X,Z)FY$$

$$-[(\nabla_X\eta)Z-(\nabla_Z\eta)X]FY$$

For P-sasakian manifold

$$(\nabla_X\phi)(Y)=-g(X,Y)\xi-\eta(Y)X+2\eta(X)\eta(Y)\xi \quad (32)$$

$$\forall X,Y\in T(M)$$

$$\therefore \tilde{R}(X,Y,Z)=R(X,Y,Z)-[g(Y,\phi Z)-g(Z,\phi Y)]FX$$

$$-[g(X,\phi Z)-g(Z,\phi X)]FY$$

Taking covariant derivative with respect to X both sides

$$\nabla_X\tilde{R}(X,Y,Z)=\nabla_X R(X,Y,Z) \quad (33)$$

because for quarter symmetric metric connection $\nabla g=0$. Thus we state the following theorem.

Theorem 3.2 Covariant derivative of Riemannian curvature with respect to is Riemannian connection is equal to covariant derivative of Riemannian curvature with respect to quarter symmetric metric connection for P- Sasakian Manifold.

For P-Sasakian manifold

$$\alpha(X,\xi)=(\nabla_X\eta)\xi-\eta(X)\eta(\xi) \quad (34)$$

$$=(\nabla_X\eta)\xi-\eta(X). \quad (35)$$

$$=-\eta(X) \quad (36)$$

$$\alpha(X,\xi)=-g(X,\xi) \quad (37)$$

The curvature tensor $K \; and \; \overline{K}$ and is given by

$$K(X,Y,Z,U)=g(R(X,Y)Z,U) \quad (38)$$

and

$$\tilde{K}(X,Y,Z,U)=g(\tilde{R}(X,Y)Z,U) \quad (39)$$

For quarter symmetric metric connection $\nabla_X g=0$.

$$\Rightarrow \nabla K(X,Y,Z,U) = 0 = \nabla \tilde{K}(X,Y,Z,U)$$

Hence we state the following theorem.

Theorem 3.3 The covariant derivative of curvature tensor with respect to Riemannian connection and quarter symmetric metric connection both are equal to zero for P- Sasakian Manifold.

Since for P-Sasakian manifold

$$\alpha(X,\xi) = -g(X,\xi)$$

Taking covariant derivative both sides we get $\nabla_X \alpha(X,\xi) = -\nabla g(X,\xi) = 0$, for quarter symmetric metric connection

Corollary 1 *The tensor field* α *of type (0,2) satisfy*

$$\alpha(X,\xi) = -\eta(X), \alpha(X,\xi) = -g(X,\xi)$$

For P-Sasakian manifold

$$(\nabla_X \phi)\phi Y = -g(X,\phi Y)\xi - \eta(\phi Y)X - 2\eta(X)\eta(\phi Y)\xi$$
$$(\nabla_X \phi)\phi Y = g(\phi X, Y)$$

Thus we state the following theorem.

Theorem 3.4 The covariant derivative of ϕ for quarter symmetric metric connection satisfies

$$(\nabla_X \phi)\phi Y = g(X,\phi Y)\xi$$

.

Since

$$\alpha(Y,Z) = \nabla_Y \eta(Z) - \eta(Z)\eta(FY),$$
$$\alpha(Y,Z) = \nabla_Y \eta(Z)$$

$$\tilde{R}(X,Y,Z) = R(X,Y,Z) + \alpha(Y,Z)FX - \alpha(X,Z)FY$$

$$= R(X,Y,Z) + \nabla_Y \eta(Z)FX - \nabla_X \eta(Z)FY \quad (40)$$

Taking inner product of (40) with Z we get

$$\tilde{S}(X,Y) = S(X,Y) + \nabla_Y \eta(Z) g(FX,Z) - \nabla_X \eta(Z) g(FY,Z)$$

$$= S(X,Y) + g(Y,\phi Z)g(FX,Z) - g(X,\phi Z)g(FY,Z)$$
$$= S(X,Y) + \sum_{n=1}^{n} g(Y,\phi Z)g(FX,e_i) - g(X,\phi e_i)g(FY,e_i)$$

$$= S(X,Y) + (n-1)[g(Y,FX) - g(X,FY)]$$
$$= S(X,Y) + (n-1)[-g(FY,X) - g(X,FY)]$$

$$\tilde{S}(X,Y) = S(X,Y) - 2(n-1)g(X,FY) \quad (41)$$

The W_2 − curvature tensor is given by

$$\tilde{W}_2(X,Y)Z = \tilde{R}(X,Y)Z - \frac{1}{n-1}\{g(Y,Z)\tilde{Q}X - g(X,Z)\tilde{Q}Y\}$$

where $\tilde{Q}$ is Ricci Operator. Taking inner product

$$\tilde{W}_2(X,Y,Z,U) = \tilde{K}(X,Y,Z,U) - \frac{1}{n-1}\{g(Y,Z)g(\tilde{Q}X,U) - g(X,Z)g(\tilde{Q}Y,U)\}$$

$$K(X,Y,Z,U) - \frac{1}{n-1}\{g(Y,Z)\tilde{S}(X,U) - g(X,Z)\tilde{S}(Y,U)\}$$

Taking covariant derivative & using $\nabla g = 0$ we get

$$\nabla\tilde{W}_2 = \nabla\tilde{K} \tag{42}$$

Thus we state the following theorem.

Theorem 3.5 The covariant derivative of W_2 -curvature with respect to quarter symmetric metric connection is equal to covariant derivative of Riemannian Curvature for P- Sasakian Manifold.

4 Pseudo projective & con-circular curvature tensor

The Pseudo-projective curvature tensor $\tilde{P}$ with respect to the quarter symmetric metric connection $\tilde{\nabla}$ is defined as

$$\bar{P}(X,Y)Z = a\tilde{R}(X,Y)Z + b[\tilde{S}(Y,Z)X - \tilde{S}(X,Z)Y]$$

$$-\frac{\tilde{r}}{n}\left\{\frac{a}{n-1} + b\right\}[g(Y,Z)X - g(X,Z)Y]$$

$$= a[R(X,Y,Z) + \alpha(Y,Z)FX - \alpha(X,Z)FY]$$

$$+ b[\{S(Y,Z) - 2(n-1)g(Y,FZ)\}X$$

$$-\{S(X,Z) - 2(n-1)g(X,FZ)\}Y]$$

$$-\frac{r(n-1)\,trace\,(\alpha)}{n}\left\{\frac{a}{n-1} + b\right\}[g(Y,Z)X - g(X,Z)Y]$$

By (26) and taking covariant derivative both sides we get

$$\tilde{\nabla}\bar{P}(X,Y)Z = \nabla\bar{P}(X,Y)Z \tag{43}$$

Thus we state the following theorem.

Theorem 4.1 In an n dimensional(where n is odd) Sasakian manifold the pseudo projective curvature tensor is same with respect to Riemannian connection and quarter symmetric metric connection for P- Sasakian Manifold.

Now taking inner product of (26) by Z we get

$$\tilde{S}(X,Y) = S(X,Y) - \frac{r}{n(n-1)}[\sum g(Y,e_i)g(X,e_i) - \sum g(X,e_i)g(Y,e_i)]$$

$$= S(X,Y) - \frac{r}{n(n-1)}[S(Y,X) - S(X,Y)] \tag{44}$$

$$\therefore \tilde{S}(X,Y) = S(X,Y) \quad (45)$$

$$\forall X,Y \in TM.$$

Thus we state the following theorem.

Theorem 4.2 The Ricci tensor of concircular curvature tensor with respect to quarter symmetric metric connection and Riemannian connection are same for P- Sasakian Manifold.

Taking inner product of (27) with Z we have

$$\tilde{S}(X,Y) = S(X,Y) - \frac{1}{n-2}[S(Y,e_i)g(X,e_i)$$

$$-S(X,e_i)g(e_i,Y) + g(Y,e_i)g(e_i,LX) - g(X,e_i)g(LY,e_i)]$$

$$+\frac{r}{(n-1)(n-2)}[g(Y,e_i)g(X,e_i) - S(Y,e_i)g(e_i,Y)]$$

$$= S(X,Y) - \frac{1}{n-2}[S(Y,X) - S(X,Y)$$

$$+S(Y,LX) - S(X,LY)] + \frac{r}{(n-1)(n-2)}[S(Y,X) - S(X,Y)]$$

$$\Rightarrow \tilde{S}(X,Y) = S(X,Y) - \frac{1}{n-2}[S(Y,LX) - S(X,LY)]$$

$$= S(X,Y) - \frac{1}{n-2}[g(QY,LX) - g(QX,LY)] \quad (46)$$

Taking covariant derivative of (46) both sides we get $\nabla_Z \tilde{S}(X,Y) = \nabla_Z S(X,Y)$, because $\nabla_Z g(X,Y) = 0$, for quarter symmetric metric connection. Hence we conclude the following theorem.

Theorem 4.3 The covariant derivative of Ricci tensor with respect to quater symmetric metric connection for Weyl curvature tensor is same as covariant derivative of Ricci tensor with respect to Riemannian connection for P- Sasakian Manifold.

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