DOI: https://doi.org/10.24297/jam.v19i.8828

Estimation of Parameters For Exponentiated Burr Type XII Distribution Based on Ranked Set Sampling

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Abstract

The aim of this paper is to estimate the parameters of exponentiated Burr type XII distribution (EBXII) based on ranked set sampling (RSS) technique, and also simple random sampling(SRS) is provided by the method of maximum likelihood. Fisher information matrix for both (SRS) and (RSS) for the unknown parameters are derived. Simulation study compared between the estimators of both methods in terms of their biases, mean square errors, and efficiencies. It is shown that the estimators based on RSS are more efficient than those of SRS.

Keywords: Exponentiated Burr typeXII ,Fisher information matrix , Maximum likelihood estimation , Ranked set sampling.

Introduction

The Exponentiated Burr XII (EBXII) is an extended distribution to Burr type XII distribution . this model is an important model for several areas such as actuarial sciences , economic ,survival analysis. Al-Hussaini and Hussain(2011) paid a great concern to Exponentiated models in general and Exponentiated Burr type XII in particular. They estimated the parameters of EBXII distribution using type II Censoring, Bayes estimators are also provided under complete samples and censored type II samples. furthermore ranked set sampling (RSS) is a statistical technique for data collection that generally leads to more efficient estimators than those based on simple random sampling (SRS). The RSS method was first proposed by McIntyre (1952) as a cost-effective and more structural alternative approach to SRS. In RSS procedure , without any certain measurement ,sampling units can be ranked easily and cheaply with respect to characteristic of interest. The efficiency of RSS according to SRS has been investigated by several researchers, Takahashi and Wakimato (1968) showed that RSS mean is an unbiased estimator for the population mean with smaller variance compared to SRS mean. Dell and clutter(1972) showed that RSS is more efficient than SRS even with an error in ranking. RSS method has been modified to yield new sampling methods. Several modifications for RSS were introduced by several authors Samawi et al(1996) suggested using extreme ranked set samples to estimate the population mean. Muttlak(1997) introduced median ranked set sampling to estimate the population mean. AL-Saleh and AL-Kadiri (2000) considered double ranked set sample (DRSS) as a two-stage sampling technique that increase the efficiency of RSS estimator without increasing set size it is shown that double ranked set sampling is more efficient than RSS. Beside these studies ,Several authors have considered the estimation of the parameters of well-known distribution using RSS or modification of it for example , the estimation of unknown parameter of exponential and logistic distribution was estimated by lam et al (1994). Abu-Dayyeh and Assrhani (2011) estimate the shape and location parameters of the Pareto distribution based on RSS and SRS and compare between the two methods .the estimation of unknown parameters of normal ,the exponential and gamma distributions using median and extreme ranked set sample was studied by Shaibu and Muttlak (2004). Al-Omari and Al-Hadhrami (2011) estimate the parameters of the modified Weibull distribution using extreme ranked set sample (ERSS). Hassan (2013) derived the maximum likelihood and Bayes estimators of the shape and scale parameters of exponentiated exponential based on SRS and RSS. Khamnei and Mayan (2016) estimated the parameters of exponentiated gumbel based on simple random sample and ranked set sample and compare between two methods. Gurler and Esemen (2017)estimated the parameters of generalized Rayleigh distribution based on simple random sampling and ranked set sampling.

In this paper, the parameters of exponentiated BurrXII (EBXII) distribution will be estimated under both SRS and RSS methods, also fisher information matrix will be provided for both methods. The probability density function (pdf) and cumulative density function (cdf) of exponentiated BurrXII (EBXII) are given respectively

$$f(x) = \beta\theta\alpha\, x^{\beta-1}\,(1+x^{\beta})^{-\theta-1}\left[1-(1+x^{\beta})^{-\theta}\right]^{\alpha-1} \quad \alpha,\beta,\theta,x>0 \quad (1)$$

$$F(x) = [\ 1-(1+x^{\beta})^{-\theta}]^{\alpha} \quad \alpha, \beta, \theta, x > 0 \qquad (2)$$

Materials and Methods

1. Parameter Estimation and Fisher information matrix for EBXII Based on(SRS) Samples:

In this section we will deal with the point estimation of the unknown parameters of the exponentiated Burr XII distribution from simple random sample by the methods of maximum likelihood which is one of the most commonly used for the most theoretical distributions .also the ML estimators has desirable properties of being consistent and asymptotically normal for large sample under appropriate conditions. Let $x_1, x_2, \dots, x_n$ be a random sample of size n from EBXII ,then the likelihood function can be written as:

$$L_{SRS} = (\beta\theta\alpha)^n \prod_{i=1}^{n} x_i^{\beta-1} \prod_{i=1}^{n} (1+x_i^{\beta})^{-\theta-1} \prod_{i=1}^{n} \left[1-\left(1+x_i^{\beta}\right)^{-\theta}\right]^{\alpha-1} \qquad (3)$$

the log – likelihood function , logL_{SRS} ,denoted by $\ln L_{SRS}$ determined by taking the log of both sides.

$$\ln L_{SRS} = n\ln\beta + n\ln\theta + n\ln\alpha + (\beta-1)\sum_{i=1}^{n}\ln x_i - (\theta+1)\sum_{i=1}^{n}\ln\left(1+x_i^{\beta}\right) + (\alpha-1)\sum_{i=1}^{n} ln\left[1-\left(1+x_i^{\beta}\right)^{-\theta}\right]$$

For all unknown parameters, the maximum likelihood estimates of β,θ,α denoted by α_{SRS} , β_{SRS} , θ_{SRS} are obtained by setting the first partial derivatives of log L to be zero with respect to β,θ,α respectively.

$$\frac{\partial\, logL_{SRS}}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\ln\left[1-\left(1+x_i{}^{\beta}\right)^{-\theta}\right] = 0 \qquad (4)$$

$$\frac{\partial\, logL_{SRS}}{\partial\beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\ln x_i - (\theta+1)\sum_{i=1}^{n}\frac{x_i^{\beta}\ln x_i}{1+x_i{}^{\beta}} + (\alpha-1)\sum_{i=1}^{n}\frac{\theta x_i^{\beta} ln x_i\left(1+x_i{}^{\beta}\right)^{-\theta-1}}{[1-(1+x_i{}^{\beta})^{-\theta}]} = 0 \qquad (5)$$

$$\frac{\partial\, logL_{SRS}}{\partial\theta} = \frac{n}{\theta} + \sum_{i=1}^{n}\ln\left(1+x_i{}^{\beta}\right) + (\alpha-1)\sum_{i=1}^{n}\frac{\left(1+x_i{}^{\beta}\right)^{-\theta} ln\left(1+x_i{}^{\beta}\right)}{[1-(1+x_i{}^{\beta})^{-\theta}]} = 0 \qquad (6)$$

It is not easy to obtain a closed form solution to this system of equations (4),(5) and (6) therefore, an iterative method must be applied to solve these equations numerically to obtain α_{SRS} , β_{SRS} , θ_{SRS}

The fisher information matrix of the EBXII distribution of the simple random sample can be obtained by differentiating $\frac{\partial \ln l_{SRS}}{\partial\beta}$, $\frac{\partial \ln l_{SRS}}{\partial\theta}$, $\frac{\partial \ln l_{SRS}}{\partial\alpha}$ with respect to β ,θ and α to get the second partial derivative. And equate the result to zero. Then the fisher information matrix is a matrix its elements are the second partial derivatives as follows

$$\frac{\partial^2 \ln L_{SRS}}{\partial\beta^2} = var\,(\beta_{SRS}) = \frac{-n}{\beta^2} - (\theta-1)\sum_{i=1}^{n}\frac{x_i^{\beta}(\ln x_i)^2}{\left(1+x_i^{\beta}\right)^2}$$

$$+(\alpha-1)\theta\sum_{i=1}^{n}\left[\frac{x_i^{\beta}(\ln x_i)^2\left(1+x_i^{\beta}\right)^{-\theta-2}\left[\left(1-\theta x_i^{\beta}\right)-\left(1+x_i^{\beta}\right)^{-\theta}\right]}{\left[1-\left(1+x_i^{\beta}\right)^{-\theta}\right]^2} - \frac{x_i^{2\beta}\,\theta^2(\ln x_i)^2\left(1+x_i^{\beta}\right)^{-2\theta-2}}{\left[1-\left(1+x_i^{\beta}\right)^{-\theta}\right]^2}\right] (7)$$

$$\frac{\ln L_{SRS}}{\partial\beta\partial\theta} = cov(\beta_{SRS},\theta_{SRS})$$

$$= -\sum_{i=1}^{n}\frac{x_i^{\beta}\ln x_i}{(1+x_i^{\beta})} + (\alpha-1)\sum_{i=1}^{n}\frac{x_i^{\beta}\ln x_i(1+x_i^{\beta})^{-\theta-1}\left[\left(1-(1+x_i^{\beta})^{-\theta}\right)-\left(\theta\ln(1+x_i^{\beta})\right)\right]}{\left[1-(1+x_i^{\beta})^{-\theta}\right]^2}$$

$$+(\alpha-1)\sum_{i=1}^{n}\frac{x_i^{\beta}\ln x_i\,(1+x_i^{\beta})^{-2\theta-1}\left[\theta\ln(1+x_i^{\beta})\right]}{\left[1-(1+x_i^{\beta})^{-\theta}\right]^2} \qquad (8)$$

$$\frac{\ln L_{SRS}}{\partial\beta\partial\alpha} = cov(\beta_{SRS},\alpha_{SRS}) = \sum_{i=1}^{n}\frac{\theta(1+x_i^{\beta})^{-\theta}x_i^{\beta}\ln x_i}{\left[1-(1+x_i^{\beta})^{-\theta}\right]} \qquad (9)$$

$$\frac{\partial^2\ln l_{SRS}}{\partial\,\alpha^2} = var(\alpha_{SRS}) = -\frac{n}{\alpha^2} \qquad (10)$$

$$\frac{\partial^2\ln l_{SRS}}{\partial\,\alpha\partial\theta} = cov(\alpha_{SRS},\theta_{SRS}) = \frac{(1+x_i^{\beta})^{-\theta}\ln(1+x_i^{\beta})}{\left[1-(1+x_i^{\beta})^{-\theta}\right]} \qquad (11)$$

$$\frac{\partial^2\ln l_{SRS}}{\partial\,\theta^2} = var(\theta_{SRS}) = -\frac{n}{\theta^2}+(\alpha-1)\sum_{i=1}^{n}\frac{(1+x_i^{\beta})^{-\theta}\ln(1+x_i^{\beta})^2}{\left[1-(1+x_i^{\beta})^{-\theta}\right]^2} \qquad (12)$$

2. Parameter Estimation and Fisher information matrix for EBXII Based on(RSS) Samples:

In this section we will deal with the point estimation of the unkown parameters of the exponentiated Burr XII distribution from ranked set sample by the methods of maximum likelihood. The technique of ranked set sampling can be described in the following steps:

Step 1: selecting an independent *m* simple random samples from the population of interest.

Step 2: Each sample is of size *m* units and drawn without replacement, where the total initial sample size is m^2 and we call each simple random sample a set.

Step 3: Within each of *m* set, the sampled item are ranked (without yet knowing any values for the variable of interest, for example, visually)

Step4: After ranking the *m* items in each of the *m* sets, a subsample is drawn for measurement. This subsamples consist of the smallest ranked unit is chosen from the first set, the second smallest ranked units is chosen from the second set, continuing this process until the largest unit is chosen from the last set .so that the subsamples contains *m* units, each representing a different rank from the m sets . This entire process is referred to as a cycle and the number of observations in each random sample, m is called the set size.

Step 5: Repeating steps 1 through 4 for *r* cycles (times) until a total of rm^2units have been drawn from the population and actually the desired sample size(*rm*) have been measured. The*(rm)* measured observations from the ranked set sampling.

Suppose that X_{ij} , i = 1, ... , m , j = 1, ... r is is a ranked set sample from (EBXII) With a sample size $= rm$, m is the set size and r is the number of cycle Where , X_{ij} represents the ranked unit in the i[th] set and j[th] cycle for simplicity we denote $Y_{ij} = X_{ij}$ thenY_{ij} are independent with PDF given by

$$g(y_{ij}) = \frac{m!}{(i-1)!\,(m-i)!}\left[F(y_{ij})\right]^{i-1}f(y_{ij})\left[1-F(y_{ij})\right]^{m-i} \quad y_{ij}>0 \qquad (13)$$

The likelihood function of the ranked set sample $y_{1r}, y_{2r}, \dots, y_{mr}$

$$L_{RSS}=\prod_{j=1}^{r}\prod_{i=1}^{m}\left(\frac{m!}{(i-1)!\,(m-i)!}\right)(\beta\theta\alpha){y_{ij}}^{\beta-1}\left(1+y_{ij}^{\beta}\right)^{-\theta-1}\left[\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha i-1}\right]$$

$$\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^{m-i}\quad(14)$$

The log-likelihood function , denoted by $\ln L_{RSS}$ is given by:

$$logL_{RSS}=C+rm\ ln(\beta\theta\alpha)+(\beta-1)\sum_{j=1}^{r}\sum_{i=1}^{m}lny_{ij}-(\theta+1)\sum_{j=1}^{r}\sum_{i=1}^{m}ln\left(1+y_{ij}^{\beta}\right)^{-\theta}$$

$$+(\alpha i-1)\sum_{j=1}^{r}\sum_{i=1}^{m}\ln\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)+(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\ln\left(1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right)\quad(15)$$

Differentiate the ln Likelihood with respect to α ,β ,θ and equating to zero.

$$\frac{\partial Ln\ L_{RSS}}{\partial\alpha}=\frac{rm}{\alpha}+\sum_{j=1}^{r}\sum_{i=1}^{m}i\ \ln\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)+$$

$$(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\ln\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]}\quad(16)$$

$$\frac{\partial Ln\ L_{RSS}}{\partial\theta}=\frac{rm}{\theta}-\sum_{j=1}^{r}\sum_{i=1}^{m}\ln\left(1+y_{ij}^{\beta}\right)+(\alpha i-1)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\left(1+y_{ij}^{\beta}\right)^{-\theta}\ln\left(1+y_{ij}^{\beta}\right)}{\left[1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right]}$$

$$+(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\alpha\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha-1}\left(1+y_{ij}^{\beta}\right)^{-\theta}\ln\left(1+y_{ij}^{\beta}\right)}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]}\quad(17)$$

$$\frac{\partial Ln\ L_{RSS}}{\partial\beta}=\frac{rm}{\beta}+\sum_{j=1}^{r}\sum_{i=1}^{m}\ln y_{ij}-(\theta+1)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{y_{ij}^{\beta}\ln y_{ij}}{\left(1+y_{ij}^{\beta}\right)}$$

$$-(\alpha i-1)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta\left(1+y_{ij}^{\beta}\right)^{-\theta-1}y_{ij}^{\beta}\ln y_{ij}}{\left[1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right]}$$

$$-(m-i)\sum_{j=1}^{r}\sum_{i-1}^{m}\frac{\alpha\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha-1}\theta\left(1+y_{ij}^{\beta}\right)^{-\theta-1}y_{ij}^{\beta}\ln y_{ij}}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]}\quad(18)$$

The estimators of ML of α , θ , β say α_{RSS} ,β_{RSS} ,θ_{RSS} are the solution of the above non-linear equations as it seems it is difficult to find a closed form solution of the parameters so numerical technique is needed to solve them.

The fisher information matrix of the EBXII distribution of the ranked set sample can be obtained by differentiating $\frac{\partial\ln l_{RSS}}{\partial\beta}$,$\frac{\partial\ln l_{RSS}}{\partial\theta}$,$\frac{\partial\ln l_{RSS}}{\partial\alpha}$ with respect to β ,θ and α to get the second partial derivative. And equate the result to zero.

$$\frac{\partial^2 \ln l_{RSS}}{\partial \alpha^2} = var(\alpha_{RSS}) = -\frac{rm}{\alpha^2} + (m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\ln\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^2} \quad (19)$$

$$\frac{\partial^2 \ln l_{RSS}}{\partial \theta^2} = var(\theta_{RSS}) = \frac{-rm}{\theta^2} + (\alpha i-1)\frac{\left(1+y_{ij}^{\beta}\right)^{-\theta}\left(\ln(1+y_{ij}^{\beta})\right)^2}{\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^2} + (m-i)$$

$$\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\alpha\left(1+y_{ij}^{\beta}\right)^{-\theta}\left(\ln(1+y_{ij}^{\beta})\right)^2\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)\left[1-\alpha\left(1+y_{ij}^{\beta}\right)^{-\theta}-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^2}$$

$$-(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\alpha^2\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{2\alpha-2}\left(1+y_{ij}^{\beta}\right)^{-2\theta}\ln\left(1+y_{ij}^{\beta}\right)}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^2} \quad (20)$$

$$\frac{\partial^2 \ln l_{RSS}}{\partial \beta^2} = var(\beta_{RSS}) = \frac{-rm}{\beta^2} - (\theta+1)\sum_{c=1}^{m}\sum_{i=1}^{n}\frac{y_{ij}^{\beta}\left(\ln y_{ij}\right)^2}{\left(1+y_{ij}^{\beta}\right)^2}$$

$$-(\alpha i-1)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta\left(1+y_{ij}^{\beta}\right)^{-2\theta}y_{ij}^{\beta}\left(\ln y_{ij}\right)^2\left[1-\theta y_{ij}^{\beta}-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right]}{\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^2}$$

$$-(\alpha i-1)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta^2\left(1+y_{ij}^{\beta}\right)^{-2\theta-2}y_{ij}^{2\beta}\left(\ln y_{ij}\right)^2}{\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^2}$$

$$-(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta\alpha(\alpha-1)\left(1+y_{ij}^{\beta}\right)^{-\theta}y_{ij}^{\beta}\ln y_{ij}\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha-2}}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^2}$$

$$-(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta\left(1+y_{ij}^{\beta}\right)^{-\theta-2}y_{ij}^{\beta}\left(\ln y_{ij}\right)^2\left[1-\theta y_{ij}^{\beta}-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^2}$$

$$-(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta^2\alpha\left(1+y_{ij}^{\beta}\right)^{-2\theta-2}y_{ij}^{2\beta}\left(\ln y_{ij}\right)^2\left[\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha-1}\right]}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^2} \quad (21)$$

$$\frac{\partial^2 \ln l_{RSS}}{\partial \beta\partial\theta} = cov(\beta_{RSS},\theta_{RSS})$$

$$= -\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{y_{ij}^{\beta}\ln y_{ij}}{\left(1+y_{ij}^{\beta}\right)} - (\alpha i-1)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta\left(1+y_{ij}^{\beta}\right)^{-\theta-1}\ln(1+y_{ij}^{\beta})y_{ij}^{\beta}\ln y_{ij}}{\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^2}$$

$$-(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta\left(1+y_{ij}^{\beta}\right)^{-\theta-1} y_{ij}^{\beta}\ln y_{ij}\left(\ln\left(1+y_{ij}^{\beta}\right)+1\right)\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^{2}}$$

$$+(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta\alpha\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha-1}\left(1+y_{ij}^{\beta}\right)^{-2\theta-1}\ln\left(1+y_{ij}^{\beta}\right) y_{ij}^{\beta}\ln y_{ij}}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^{2}}$$

$$+(m-i)\sum_{c=1}^{r}\sum_{i=1}^{m}\frac{\alpha(\alpha-1)\left(1+y_{ij}^{\beta}\right)^{-\theta}\ln\left(1+y_{ij}^{\beta}\right)\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha-2}}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^{2}} \quad (22)$$

$$\frac{\partial^2 \ln l_{RSS}}{\partial\,\beta\partial\alpha}=cov(\beta_{RSS},\alpha_{RSS})=i\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\theta\left(1+y_{ij}^{\beta}\right)^{-\theta-1} y_{ij}^{\beta}\ln y_{ij}}{\left[1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right]}-(m-i)$$

$$\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha-1}\left[1+\alpha\ln\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]\theta\left(1+y_{ij}^{\beta}\right)^{-\theta-1} y_{ij}^{\beta}\ln y_{ij}}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^{2}} \quad (23)$$

$$\frac{\partial^2 \ln l_{RSS}}{\partial\,\theta\partial\alpha}=cov(\theta_{RSS},\alpha_{RSS})=-i\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\left(1+y_{ij}^{\beta}\right)^{-\theta}\ln\left(1+y_{ij}^{\beta}\right)}{\left[1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right]}+(m-i)$$

$$\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha-1}\left[1+\alpha\ln\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]\theta\left(1+y_{ij}^{\beta}\right)^{-\theta}\ln\left(1+y_{ij}^{\beta}\right)}{\left[1-\left(1-\left(1+y_{ij}^{\beta}\right)^{-\theta}\right)^{\alpha}\right]^{2}} \quad (24)$$

Simulation study:

To investigate the properties of the maximum likelihood estimators of the parameters of EBXII , a simulation study is conducted .in some situations , the whole procedure to generate an RSS of size n is repeated r times throughout in this paper we consider the case r=1. A Monto – carlo simulation is applied for different sample sizes n=(6,10,14,20,30) with four sets values of parameters were selected set 1 ($\alpha = 0.75$, $\beta = 0{,}5$, $\theta = 1.5$) and set 2 ($\alpha = ,75$, $\beta = 2$, $\theta = 1.5$) set 3 ($\alpha = 3$, $\beta = 2$, $\theta = 1.5$) , set 4 ($\alpha = 3$, $\beta = 2$, $\theta = 3$). The estimators α, β, θ based on SRS and RSS are obtained by solving equations (4,5,6) and (16,17,18) by the R package. This will be repeated 1000 times for each sample size and for selected sets of parameters. Then, the biases and MSEs of estimators of the unknown parameters are computed. also the relative efficiency is calculated

Numerical results are reported in Tables (4.1) to (4.4). from these tables, the following Results can be observed on the properties of estimated parameters from the EBXII distribution .

Results and conclusion:

Based on numerical study the following results can be observed

1) As the sample size increases, the bias and the mean square error decrease for all estimates based on SRS and RSS.
2) The mean square error for ML estimates based on RSS are usually better than that of SRS

3) The RSS estimates are usually more efficient than those estimates of SRS.(see tables from (1.1) to table (1.4).
4) As β increases the MSE of both Ɵ and α decreases in SRS but in RSS the MSE of both Ɵ and α increases
5) As α increases MSE of both Ɵ and β increases in SRS. But in RSS the MSE of β increases while the MSE of Ɵ is usually decreases.
6) As Ɵ increases the MSE of α increases but the MSE of β decreases in SRS. But in RSS the MSE of both α and β increase as Ɵ increases.

Tables

Table (4.1): Biases, MSEs and Efficiencies for estimators Based on SRS and RSS when $(\alpha = 0.75\,, \beta = 0{,}5\,, \theta = 1.5)$

		SRS		RSS		
Sample size m	Parameter	Bias	MSE	Bias	MSE	RE
	α	1.2970	4.3207	0.1563	0.2918	14.8071
6	β	0.1006	0.2700	0.1983	0.2627	1.02779
	θ	0.8745	4.6928	0.3039	0.7590	6.18277
	α	0.4914	1.0362	0.0932	0.1452	7.13636
10	β	0.0985	0.1482	0.0734	0.0691	2.14472
	θ	0.2036	1.0168	0.1077	0.3250	3.12862
	α	0.1923	0.4195	0.0212	0.0444	9.4482
14	β	0.0363	0.0618	0.0403	0.0301	2.05316
	θ	0.1368	0.4571	0.0307	0.0919	4.97388
	α	0.0519	0.0929	0.0149	0.0611	1.52046
20	β	0.0264	0.0295	0.0455	0.0276	1.06884
	θ	0.0301	0.1914	-0.0012	0.1421	1.34694
	α	0.0301	0.0383	-0.0130	0.0116	3.30172
30	β	0.0062	0.0073	0.0172	0.0059	1.23729
	θ	0.0255	0.0851	-0.0227	0.0281	3.02847

Table 4.2: Biases, MSEs and Efficiencies for estimators Based on SRS and RSS when $(\alpha = {,}75\,, \beta = 2\,, \theta = 1.5\,)$

		SRS		RSS		
Sample size m	Parameter	Bias	MSE	Bias	MSE	RE
	α	0.0930	3.9786	0.8863	2.2244	1.78862
6	β	0.4702	4.9399	0.0776	1.4202	3.47831
	θ	0.9588	4.3807	0.9972	2.3452	1.86794
	α	0.4954	0.9612	0.3548	0.7021	1.36904
10	β	0.3036	0.9938	0.0638	0.5862	1.69533
	θ	0.2352	0.9916	0.3539	0.7405	1.33910
	α	0.1844	0.5350	0.2664	0.3466	1.54357
14	β	0.1366	0.7712	-0.0376	0.3684	2.09338

	θ	0.1253	0.4529	0.1129	0.4077	1.11087
	α	0.1488	0.1530	0.0786	0.1247	1.22694
20	β	-0.0618	0.1390	0.0719	0.1163	1.19518
	θ	0.1077	0.2158	0.0536	0.2011	1.0731
	α	0.1149	0.0825	0.0327	0.0346	2.38439
30	β	-0.0567	0.1389	0.0184	0.1089	1.27548
	θ	0.1356	0.446	0.0297	0.0810	1.78519

Table 4.3 : Biases, MSEs and Efficiencies for estimators Based on SRS and RSS when ($\alpha = 3, \beta = 2, \theta = 1.5$)

		SRS		RSS		
Sample size m	Parameter	Bias	MSE	Bias	MSE	RE
	α	1.3872	4.3504	0.4720	1.1008	2.7630
6	β	0.3329	4.6847	0.3244	0.9630	5.3323
	θ	1.6058	3.9517	0.0738	0.1990	7.0946
	α	1.0253	2.2148	0.1719	0.2693	4.8530
10	β	0.0741	1.7379	0.1288	0.2312	8.0720
	θ	1.0503	1.6492	0.0311	0.0628	8.8371
	α	0.8916	1.6209	0.1980	0.5933	1.4908
14	β	0.0292	1.0908	0.0705	0.2007	5.5675
	θ	0.3726	0.6270	0.0331	0.0800	6.1835
	α	0.6948	0.9170	0.0825	0.2901	1.5330
20	β	0.0644	0.7079	0.0481	0.0794	9.1317
	θ	0.2407	0.4723	0.0059	0.0444	9.3488
	α	0.1050	0.3144	0.0139	0.0325	9.3861
30	β	-0.0246	0.3970	0.0277	0.0243	16.8780
	θ	0.0630	0.1883	-0.0014	0.0067	27.3434

Table 4.4: Biases, MSEs and Efficiencies for estimators Based on SRS and RSS when($\alpha = 3, \beta = 2, \theta = 3$)

		SRS		RSS		
Sample size m	Parameter	Bias	MSE	Bias	MSE	RE
	α	0.6293	4.6658	0.5806	4.2503	1.0911
6	β	0.6040	2.3119	0.3148	1.0813	1.9825
	θ	0.1706	1.3156	0.0494	0.9866	1.3073
	α	0.5254	4.6718	0.5323	4.1929	1.1244
10	β	0.2305	2.2099	0.3647	1.0123	2.4529
	θ	0.3182	1.0514	0.0770	0.8069	1.0546
	α	0.4743	1.9326	0.1038	1.5132	1.1367
14	β	0.0885	0.6324	0.2085	0.4309	1.6118
	θ	0.2249	0.9249	-0.0381	0.4152	2.1134
	α	0.2886	0.9252	0.0374	0.2504	3.3807
20	β	0.0183	0.2131	0.0477	0.0672	3.2780

	θ	0.1368	0.5940	0.0117	0.0757	7.6134
	α	0.1315	0.5461	-0.0039	0.2922	1.8096
30	β	0.0007	0.0875	0.0567	0.0620	1.4876
	θ	0.0656	0.2234	-0.0107	0.0814	2.6947

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