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Decrypting the Central Mystery of Quantum Mathematics: Part 3. A Non-Einstein, non-QM View of Bell Test Experiments

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Abstract

The fact that loophole-free Bell test experiments have proved Einstein's local realism wrong, does not prove that the quantum mechanical (QM) model is correct, because the Theory of Elementary Waves (TEW) Axioms can also explain the Bell test experiments. Bi-Rays are a pair of coaxial elementary rays traveling at the speed of light in countervailing directions. In a Bell test experiment a Bi-Ray stretches from Alice's equipment, through the fiberoptic cable, across the 2-photon source, through more fiberoptic cable, to Bob's equipment. A pair of entangled photons is born into that Bi-Ray. Each photon follows the same Bi-Ray in opposite directions. This model produces the same Bell test results found by QM. According to QM this would be classified as a "non-local" model, so it is no surprise that it can explain the Bell test results. But it is a different model than QM. TEW supports a more realistic view of Nature, based on better Axioms. Although QM can explain quantum experiments, it requires that you believe the quantum world is weird. TEW Axioms explain the quantum world in a way that is more intuitively similar to the world of everyday experience.

Keywords Theory of Elementary Waves

Mathematics Subject Classification (MSC2010): 81Q65 Alternative Quantum Mechanics

1 Introduction

This is the third in a series of four articles focused on decrypting the central mystery of quantum mathematics.[1-4] The first article introduced three new Axioms and new mathematics. The second showed that this new approach can explain six quantum experiments published in mainstream scientific journals. This article will show that this approach can explain the Bell test experiments, and sheds new light on quantum information systems.

The "second quantum revolution" is sometimes defined as the defeat of Einstein's local realism by quantum mechanics in loophole-free Bell test experiments. Quantum information systems are a prominent part of this revolution. The speed of light has allegedly been replaced by instantaneous entanglement, and it has allegedly been "proved" that reality comes into existence only when we measure it.

There is a third competitor on the field: the Theory of Elementary Waves (TEW). To say that Einstein has been defeated and therefore QM is correct, is a denial of this reality. TEW Axioms can explain the results of the Bell test experiments, and can explain how quantum information systems work, yet TEW is different than QM.[5-22]

There are advantages to the TEW picture instead of the QM picture of the quantum world. Many people are uncomfortable with the QM picture of Nature (witness Schrődinger's cat). It doesn't square with our common sense. It is difficult describe what is wrong. It is subtle. The term, "quantum weirdness" is often used. No one wants to go back to Einstein's local realism. But if QM is the only alternative, then there is something "unnatural" about Nature. Our plan is to use equations to un-discombobulate Nature.

As we showed in the first article, the Axioms of QM, which led to Schrődinger's cat, can be replaced by Axioms of TEW that lead to a more satisfying picture of the quantum world. The quantum world is surprisingly similar to our world of everyday experience. We showed that the Axioms of TEW provide a solid platform for quantum math, and for understanding six published experiments. Now we propose to extend that to include the Bell test experiments.

TEW Axioms are:

A. Wave function collapse occurs *before* we measure something,
B. There is *no* wave particle duality,

C. Waves travel in the *opposite* direction as particles.

We showed earlier that the corresponding three Axioms of QM are flawed, and need to be replaced by these three. But we are about to add a footnote to the third TEW Axiom. The primary wave travels in the opposite direction as a particle, but Elementary Waves travel in both directions.

To explain the Bell test experiments and quantum computers we need to introduce an advanced version of TEW, based on Bi-Rays.

2 The concept of Bi-Rays

When we think of elementary rays, we quickly discover why there must be a more advanced level of TEW that is based on Bi-Rays.

In a double slit experiment we find zero energy waves coming from the target screen, refracting through the two slits, and interfering as they impinge on the particle gun. Each particle can only follow an elementary ray with a frequency corresponding to the de Broglie frequency of the particle, which is based on the energy of the particle: $f = E/h$. Since many different particles can be used in a double slit experiment, from photons to Buckminsterfullerene to phenylalanine, we hypothesize that all particles follow Elementary Waves. This implies that there are waves of all frequencies always available.

How is it possible that just when we need a zero energy wave of a certain frequency, traveling in a specific direction, such a wave is readily available? Where do these waves come from? The implication is that everywhere in space there are an infinite number of such waves traveling in all directions at the speed of light, and at all frequencies. That suggests some kind of aether as the medium through which Elementary Waves travel. That will be the subject of the fourth article in this series.

If at all points of space there are elementary rays traveling in all directions, that means that every elementary ray has a mate: an identical elementary ray traveling in the opposite direction, coaxially. We refer to this as a **Bi-Ray,** meaning a pair of countervailing coaxial elementary rays, each traveling at the speed of light in opposite directions.

We previously defined the symbol Æ (pronounced "ash") as the name of an elementary ray. We will now define a new symbol:

Æ⇄Æ

to be the name of a Bi-Ray. We are using the color red to refer to the Ælementary ray moving to the right. We use the color blue to refer to the Ælementary ray moving to the left.

To say that a pair of particles, such as photons are "entangled" in TEW means that both particles are attached to the same Bi-Ray, and following that Bi-Ray in opposite directions. Think of a train track with two locomotives moving in opposite directions away from each other. The railroad tracks are Bi-Rays; the engines are particles. All the energy comes from the particles, none from the tracks.

The probability of a particle following a Bi-Ray is the amplitude of it following one ray, times the amplitude of it following the countervailing ray. A locomotive follows both rails of the track. The probability of a locomotive following the track requires that it remain attached to both rails. For each rail it has an amplitude for that attachment. So the most simple entangled situation involves two locomotives, each of which is following the same pair of "Bi-Rays" in opposite directions, away from each other.

Why would the two elementary rays, traveling coaxially in opposite directions, be coherent or cohesive with one another? We believe the particles impart the coherence, the way a bead on a necklace imparts unity to a pair of strings threaded through the center of the bead. This author learned this from Lewis E. Little, the founder of TEW.

Although what we just said may have the reader's head spinning, still it is not a lot of assumptions. The sense of vertigo in the reader's head is because these are new ideas, not because it is complicated. It is therefore **astonishing** when we now declare that the propositions in the last few paragraphs **are sufficient to explain the mathematics of the Bell test experiments!**

Notice that we developed these ideas without mentioning the EPR experiment or fifty years closing loopholes. We pulled a familiar rabbit out of an unfamiliar hat. Some readers wonder, "How could something so important be produced by something so simple?" The answer, we believe, is that when you are looking at how Nature actually works, things are simple, even though they are so unfamiliar as to induce vertigo.

How do we reconcile Bi-Rays with the third Axiom of TEW:

C. Waves travel in the *opposite* direction as particles?

An entangled particle is following two countervailing elementary rays. The primary ray is the one traveling in the opposite direction as the particle. The secondary ray is the one traveling in the same direction as the particle.

We are about to develop the mathematics of Bi-Rays. The question arises, what is the boundary between this new mathematics and the mathematics of mono-rays that occupied us in the previous two articles. The answer is: We don't know where the boundary is. We know that the mathematics of Bi-Rays works for the Bell test experiments, but does not work for a double slit experiment. We know that the mathematics of mono-rays (from the last two articles) works for the double slit experiment but not for the Bell test experiments. But that is all we are sure about. This author's experience is that the mono-ray model is applicable to most situations, whereas the Bi-Ray model is applicable to few situations.

3 Four EIGENSTATES

If each elementary ray is projected onto horizontal and vertical planes ($\overrightarrow{V}$, $\overrightarrow{H}$, $\overleftarrow{V}$ and $\overleftarrow{H}$), then each of them has an eigenstate as shown in the left side of Figure 1 (below). When we combine them into a Bi-Ray we produce the four EIGENSTATES shown on the right side of Figure 1. We will use capital letters "EIGENSTATE" and the labels "A, B, C and D" to refer to the four arrangements on the right side of that diagram ($\overrightarrow{V}\ \overleftarrow{V}$, $\overrightarrow{H}\ \overleftarrow{V}$, $\overrightarrow{V}\ \overleftarrow{H}$ and $\overrightarrow{H}\ \overleftarrow{H}$).

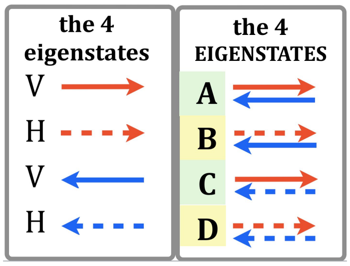

Figure 1: Left are the 4 eigenstates of the elementary rays individually ("V"=Vertical and"H"= Horizontal) ; Right are the 4 EIGENSTATES of the Bi-Rays: VV, HV, VH and HH.

Now imagine an experiment in which a Bi-Ray, Æ⇄Æ, stretches across an entire Bell test experiment, from Alice, through fiberoptic cable, through the two-photon source, through more fiberoptic cable all the way to Bob. In the center, when the Bi-Ray crosses the two-photon source, an entangled pair of photons is born into the context of the Bi-Ray (Figure 2).

Alice and Bob have polarizer equipment which they can randomly set to angle θ_1 and θ_2, as shown in Figure 2. Each of them records whether they see a photon or not, after it has passed through their polarizer. At the end of the experiment a computer correlates these data based on time, and calculates the probability that both Alice and Bob see a photon simultaneously, and records what polarizer angles were used at that instant.

3.1 Trigonometry

Figure 2 (next page) is the source of our trigonometry. If we direct our attention to the top line of Figure 2, that represents EIGENSTATE A: the two countervailing elementary rays are both projected onto a vertical plane ($\overrightarrow{V}\ \overleftarrow{V}$).

The probability of a photon moving towards Alice in EIGENSTATE A is defined as the amplitude of it following one mono-ray times the amplitude of it following the other mono-ray. This depends on the cosine of the angle θ_1 (the angle of Alice's polarizer) onto the vertical plane.

$$[cos(\theta_1 - V)cos(\theta_1 - V)] \tag{1}$$

Similarly the probability of a photon moving toward Bob in EIGENSTATE A is: $[cos(\theta_2 - V)cos(\theta_2 - V)]$ and the probability of both Alice and Bob seeing a photon is the product of those, which is

Probability of Alice —X— Probability of Bob

$$[cos(\theta_1 - V)cos(\theta_1 - V)] \text{ X } [cos(\theta_2 - V)cos(\theta_2 - V)] \tag{2}$$

Note that the angle θ_1 only appears on the left side of this equation, never on the right. Similarly the angle θ_2 only appears on the right side of this equation, never on the left.

The probability of both people seeing a photon simultaneously is the sum of the probabilities in each of the four EIGENSTATES ($\overrightarrow{V}\overleftarrow{V}$, $\overrightarrow{H}\overleftarrow{V}$, $\overrightarrow{V}\overleftarrow{H}$ and $\overrightarrow{H}\overleftarrow{H}$), which is shown in Figure 3, and also below.

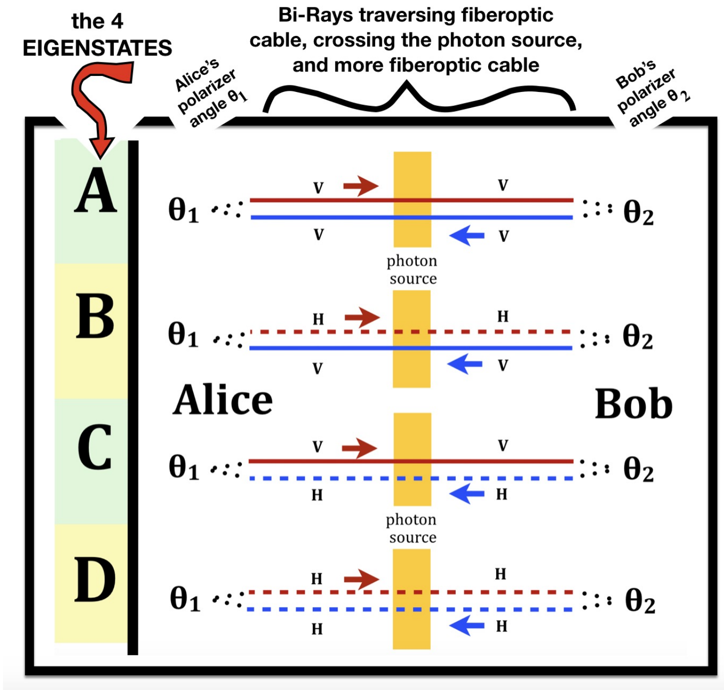

Figure 2: Alice and Bob look at incident photons through polarizers set at random angles θ_1 and θ_2 and record whether they do, or do not see a photon: YES / NO.

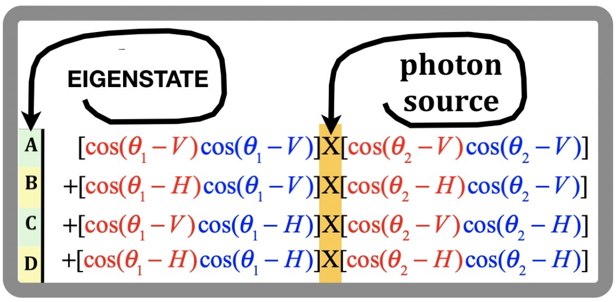

Figure 3: Trigonometry read directly off Figure 2.

When we turn the crank of the trigonometry machinery the trigonometry does the work for us. The probability of both Alice and Bob seeing a photon simultaneously is:

$$\begin{aligned}
&EigenstateA \quad [cos(\theta_1 - V)cos(\theta_1 - V)] \text{ X } [cos(\theta_2 - V)cos(\theta_2 - V)] \\
&EigenstateB + [cos(\theta_1 - H)cos(\theta_1 - V)] \text{ X } [cos(\theta_2 - H)cos(\theta_2 - V)] \\
&EigenstateC + [cos(\theta_1 - V)cos(\theta_1 - H)] \text{ X } [cos(\theta_2 - V)cos(\theta_2 - H)] \\
&EigenstateD + [cos(\theta_1 - H)cos(\theta_1 - H)] \text{ X } [cos(\theta_2 - H)cos(\theta_2 - H)]
\end{aligned}$$

If we use polar coordinates, so the angle V is zero, and H is $\pi/2$, then we get:

$$\begin{aligned}
&= [cos(\theta_1)cos(\theta_1)] \text{ X } [cos(\theta_2)cos(\theta_2)] \\
&+[sin(\theta_1)cos(\theta_1] \text{ X } [sin(\theta_2)cos(\theta_2)] \\
&+[cos(\theta_1)sin(\theta_1)] \text{ X } [cos(\theta_2)sin(\theta_2)] \\
&+[sin(\theta_1)sin(\theta_1)] \text{ X } [sin(\theta_2)sin(\theta_2)]
\end{aligned}$$

which can be factored:

$$\begin{aligned}
&= [cos(\theta_1)cos(\theta_2) + sin(\theta_1)sin(\theta_2)] \\
&\bullet[cos(\theta_1)cos(\theta_2) + sin(\theta_1)sin(\theta_2)]
\end{aligned}$$

for which there is a trigonometry equation, which gives us:

$$= [cos(\theta_2 - \theta_1) \bullet cos(\theta_2 - \theta_1)] \quad (3)$$

$$= cos^2(\theta_2 - \theta_1) \quad (4)$$

The result, $\cos^2(\theta_2 - \theta_1)$, is the probability of both Alice and Bob seeing a photon simultaneously. In the literature about Bell test experiments, this is called a "Coincidence Rate." It is exactly the answer found by QM if the two photons are emitted with the same orientation.

Figures 4 and 5 show three dimensional graphs of the equation $Z = \cos^2(\theta_2 - \theta_1)$. It looks like blue ocean waves. If Alice chooses angle θ_1, that can be graphed as the red line undulating across the waves. That leaves a full range of possible values of θ_2 for Bob to choose from. Alice's choice does not constrain or influence Bob's choice.

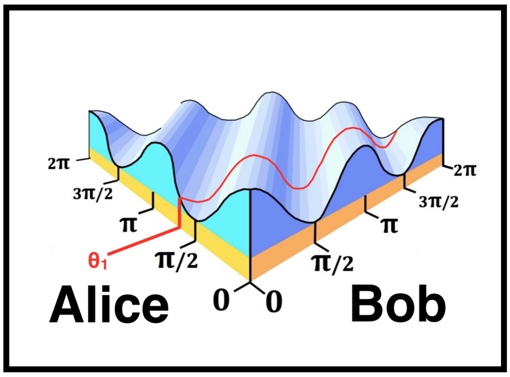

Figure 4: This graph of $Z = \cos^2(\theta_2 - \theta_1)$ has an undulating red line indicating that Alice chose angle $\theta_1 = 0.7\pi$. Bob can choose angle θ_2 at random and the height of the curve shows the probability that they each see a photon at that pair of polarizer angles.

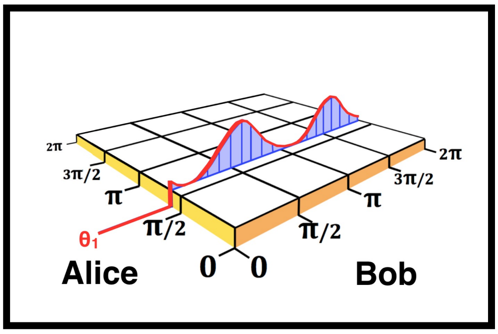

Figure 5: This is a vertical slice of exactly the same graph as above.

Figure 4 shows that for any value of θ_1 chosen by Alice, if Bob chooses a random angle θ_2, then the height of the graph will give the probability that they will both simultaneously see a photon at angles θ_1 and θ_2.

The results reported so far are based on a 2-photon source that emits photons with a correlated polarization. For example, the famous Aspect, Dalibard and Roger experiment of 1982 used a calcium-40 source that produced two photons with correlated polarization and obtained similar results as ours.[23-25]

There would be different results if the two photons were orthogonal to one another at birth. That would happen for example if the pair of photons was produced by a Wollaston prism. Then the final probability would be $Z = \sin^2(\theta_2 - \theta_1)$.

3.2 What does this tell us about Nature?

What we have been discussing so far is the mathematical machinery that arises from the definition of Bi-Rays (Æ⇄Æ). The only assumption we made is that the amplitude of either of the entangled photons following a Bi-Ray is the amplitude of it following one ray times the amplitude of it following the other ray. That is the definition of what a Bi-Ray is. No additional assumptions are needed in order to explain the Bell test experimental results.

These results agree with QM and disagree with Einstein's local realism.

However, our approach explains the results in a different way than does QM. With Bi-Rays there is no instantaneous communication across space. Furthermore when Alice looks at her photon, that measurement does not cause wave function collapse. Whether Alice looks or doesn't look, makes no difference. Alice's equipment sends no signal to Bob's equipment. Remember the first Axiom of TEW:

A. Wave function collapse occurs *before* we measure something.

Wave function collapse occurs at the 2-photon source, and consists of a pair of photons being born into the environment of one specific Bi-Ray and not some other Bi-Ray. There are no "hidden variables" internal to the particles, but there are "hidden variables" embedded in the Bi-Ray. The Bi-Ray traverses the fiberoptic cables, so the photons are always embedded in that same environment. The information embedded in the Bi-Ray is Figure 4 (a graph of $Z = \cos^2(\theta_2 - \theta_1)$).

This mathematical model gives interesting answers to the question "Local versus non-local?" Since John Bell and CHSH (Clauser, Horne, Shimony and Holt) define "non-local" to mean that the information available to Alice is also available to Bob, therefore the TEW model would be classified as "non-local." It has been known for decades that Bell test results can be explained by non-local arrangements.[26, 27]

However, there is a different way of defining "non-local" that is vital to science. If Alice does an experiment in her lab, can she ignore the influence of the Andromeda galaxy, and assume that the only relevant variables are those that are within a meter of her experiment? In TEW nothing moves faster than the speed of light. Therefore the TEW model is "local" in the sense that Alice can ignore the Andromeda galaxy.

There has been intense interest in the question, "How rapidly can Alice's equipment send a signal to Bob's equipment?" People ask whether the speed of transmission is light speed or something faster, such as instantaneous. TEW provides the following answer: the signal was sent yesterday, before the electricity was turned on. The "signal" consists of the Bi-Ray itself and the content of the signal is Figures 4 and 5. The Bi-Ray is not changed by Alice making an observation. Furthermore, it does not matter if, or when Bob looks at his data. The answer is not going to change. This was a theme throughout the first article in this series (about a double slit experiment): observation or measurement is not important. We demote measurement, compared with QM, which promotes it.

What is astonishing about this discussion of the Bell test experiments is how abbreviated it is. It is simple. We don't need the thousands of academic articles and scholarly symposia and the hundreds of millions of dollars of research that have been required to produce "loophole free" Bell tests that can decisively defeat Einstein.

In their heart of hearts many mathematicians are dissatisfied with the QM theory of the Bell test experiments because it leaves us with a view of Nature that doesn't add up, that is at odds with our common sense. Perhaps the TEW view of Nature will be easier to live with. Time will tell.

4 Two worldviews: QM versus TEW

When the "second quantum revolution" is discussed, quantum computers are prominent in the discussion in addition to the Bell test experiments. TEW provides an alternative view of both superposition. It would be reasonable to expect TEW to shed light on entanglement in quantum computers, but, as you will see, TEW is mute on that subject.

4.1 Superposition - a different approach

Superposition is startlingly different in TEW than in QM. In the first article in this series we focused on the double slit experiment. QM experts say that particles are in a superposition in that experiment. They allege (incorrectly) that a particle leaves the gun, passes through both slits as a wave, and interferes with itself as a wave. At the screen it becomes a particle with a discreet position and makes a single dot. The double slit experiment is irrefutable proof of wave particle duality, they say, incorrectly.

TEW replies that this is nonsense. The QM experts have failed to recognize a covert assumption they made, which is that waves and particles travel in the same direction. From the very beginning this assumption was made, without seeking evidence for whether it was a reasonable or unreasonable assumption. Max Planck, Niels Bohr, Albert Einstein, Werner Heisenberg, Wolfgang Pauli, Louis de Broglie, Erwin Schrődinger, John von Neumann, Paul Dirac and Enrico Fermi all made this assumption. None of them ever asked, "What is the evidence that this is a reasonable assumption?"

In the classical world it would be a reasonable assumption. In our everyday experience we don't notice particles following waves backwards. This is precisely how the quantum world differs from the classical world.

If you only ***ask that question*** suddenly the most solid doctrine of QM comes crashing down, because the evidence supporting that which is “obvious” is lacking. It is simple to explain the double slit experiment, the Stern Gerlach magnet experiments, and lots of other experiments without any need for wave particle duality, as we demonstrated in the previous two articles. Therefore the whole doctrine of superposition collapses.

The first article in this series of four articles showed that it is easy to see that waves come from the target screen in the double slit experiment, refract through the two slits, and particles follow the waves backwards. Particles are never in a superposition. Waves are sometime in a superposition, but not always. When the waves through slit A interfere with the waves through slit B, as they impinge on the particle gun, there is superposition additivity.

Since the idea of qubit superposition is central to any discussion of quantum computers, the question arises how the absence of particle superposition in TEW changes our perspective. The first thing to recognize is that qubits are not always identical to particles.

A Josephson junction, for example, supports two currents flowing forever clockwise and counterclockwise. If those currents are added or subtracted from one another $|R\rangle \pm |L\rangle$ we arrive at different energy states, which are interpreted symbolically as kets $|0\rangle$ and $|1\rangle$. So in this case the qubit looks more like a wave than a particle. In QM this is not a problem, because QM draws no distinction between waves and particles. But TEW makes a sharp distinction between them because waves and particles travel in opposite directions in a double slit experiment.

How should TEW interpret the doctrine of qubit superposition? Consider a Hadamard gate, which produces a superposition of the kets $|0\rangle$ and $|1\rangle$. We propose this should be understood as a decision point inside the quantum circuit. What comes out of the downstream side of the gate is not a superposition $((|0\rangle \pm |1\rangle)/\sqrt{2})$ but either ket $|0\rangle$ or $|1\rangle$, but not both.

Since TEW rejects the doctrine that wave function collapse occurs at the detector in an experiment, decisions are always made earlier in TEW than in QM. **It is the decision point that is important.** In a double slit experiment that is at the particle gun. At a Hadamard gate, the decision point is at the exit to the Hadamard gate, which the qubit must leave as $|0\rangle$ or $|1\rangle$ but not both.

A superposition that is created inside a Hadamard gate is instantly resolved, so that only one of the two kets exits the downstream side of the gate. The word “superposition” is a code word for “decision point.” In other words, a “qubit superposition” can only exist inside a gate, not outside.

Entanglement in quantum computers will be discussed later.

5 When and Where are decisions made?

Rather than speaking of “Wave Function Collapse” it is less confusing if we speak of “Decision Points,” as we said before. The advantage of the term "Decision Point" is that it is easier to define and measure, whereas "Wave Function Collapse" is very abstract and impossible to measure when exactly it happens.

Sometimes decisions are made when particles are emitted (as in the double slit experiment), but in other experiments decisions are made *after* that.

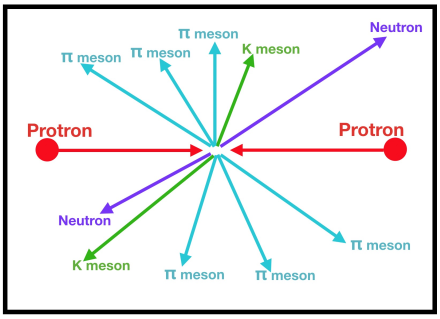

Figure 6: The decision point in a scattering experiment

Take for example a high energy collider that accelerates each of two protons going in opposite directions to almost the speed of light, then collides them into one another, as shown in Figure 8. Clearly the decision point is when there is an impact.

In a digital computer decisions are made inside electrical circuits. The three circuits in Figure 9 constitute a “universal set,” meaning that any logical computer circuit can be simulated using just these three. Obviously decisions are made inside the circuits.

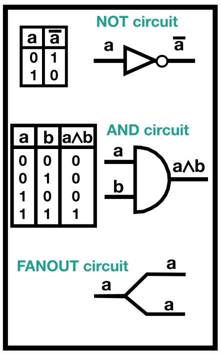

Figure 7: Three circuits in a digital computer.

We claim that decisions are made inside the gates in a quantum computer, in a similar way. No one ever says of a digital computer, the "AND circuit produces a superposition that will collapse at a later time."

The QM idea of decisions being held in suspension (in a superposition) until the qubit is measured at the end of the circuit, is a contorted and unnatural way of speaking, a vocabulary that does not reflect how programmers actually think.

Our discussion will assume an IBM online quantum computer, based on superconducting Josephson circuits. We do not mean that the IBM approach is better than the Google, D-Wave, Intel, Microsoft, Strangeworks, Zapata, Coldquanta or other platforms. We needed to choose one system for simplicity, and so we happened to select the IBM system. This author has learned only the basics of how a quantum computer works by learning some rudimentary skills with an IBM online quantum computer.[1]

The TEW vocabulary differs from the QM vocabulary, and in many ways is more natural and simple to learn. This author is an expert in TEW, not quantum computers.

6 Quantum computers

Here is a two sentence summary of what we are about to say. If you want to know how a computer circuit works, look inside the skull of a programmer and learn how the compiler actually works at a nuts and bolts level. Probably there is nothing in there that corresponds to a "qubit superposition."

Here is a more detailed account of IBM style quantum computers. A superconducting Josephson junction has current flowing in both directions: one clockwise, the other counter-clockwise. The ground state $|R\rangle + |L\rangle$ has the least amount of energy and is interpreted to represent a $|0\rangle$. The first excited state $|R\rangle - |L\rangle$ is interpreted to represent $|1\rangle$. The advantage of the Josephson junction is that the energy levels are unevenly spaced, and therefore can be distinguished from one another. Furthermore, at 0.015 Kelvin, the currents sustain themselves forever.

6.1 A Hadamard gate

How do programmers think when they are writing code for a Hadamard gate? They think that the gate in Figure 8 (next page) accomplishes something.

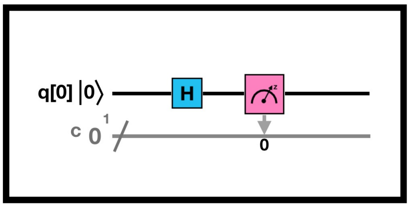

Figure 8: A Hadamard circuit.

[1]https://quantum-computing.ibm.com (access date Sept 2, 2019)

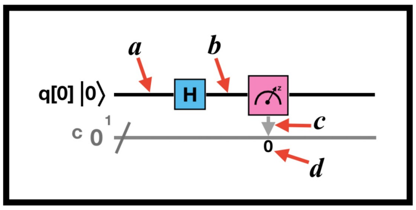

Figure 9: Possible decision points inside a Hadamard circuit.

$$\begin{bmatrix}1\\0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} = \frac{\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}}{\sqrt{2}} \tag{5}$$

If there is a superposition like $\Psi = \alpha\,|0\rangle + \beta\,|1\rangle$ and $\alpha = \beta$ then in a computer circuit we would arrange the Hadamard gate as shown in Figure 9 with letters "a" through "d" inserted into the diagram as benchmarks for when and where a decision might be made.

According to QM the Hadamard gate produces a superposition, and that superposition makes a decision to be $|0\rangle$ or $|1\rangle$ somewhere between letters "c" and "d" in Figure 9. The official QM view is that the information inside the computer comes out of a superposition and becomes either $|0\rangle$ or $|1\rangle$ when a measurement is made. A measurement is made in the pink box in Figure 9.

According to TEW there is a decision point is much earlier. A decision is made in the middle of the Hadamard gate:

$$\left\{\frac{\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}}{\sqrt{2}}\right\} = \text{Decision Point A} \tag{6}$$

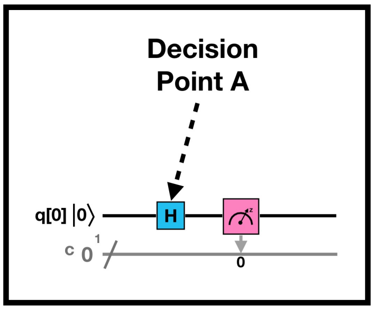

Figure 10: Decision point located inside a Hadamard gate.

In Figure 9, TEW would say that a decision has already been made (i.e. the qubit takes a value of $|0\rangle$ or $|1\rangle$) by the point marked "b." What happens when a measurement is made (the pink box in Figure 9) is that we measure the state that has existed from earlier in the process. This illustrates the difference between the first Axiom of TEW versus the first Axiom of QM. We say that wave function collapse occurs PRIOR TO measurement, whereas QM says wave function collapse occurs WHEN something is measured. QM still carries a "positivist" metaphysics even while they denounce metaphysics. We claim that our way of thinking is closer to what happens inside the brain of a quantum computer programer, and we claim that is the place to look if you want to see how a compiler actually works for such a computer.

If we test this circuit hundreds of times, we would find that approximately 50% of the time the answer would be 0, and the other 50% would be 1. Yes, the measuring equipment tells us "0 or 1" but that is just informing us about conditions inside the circuit, prior to measurement. The decision is made in the obvious location, which is inside the Hadamard gate. If you doubt what we are saying, look inside the brain of a programmer, and see how the quantum compiler actually works!

Although the "official party line" of QM about when a decision is made inside a quantum computer is convoluted, the history of QM is even more convoluted. According to historically pure QM, as evident in John von Neumann's book *Mathematical Foundations of Quantum Mechanics*, wave function collapse occurs somewhere *after* the letter "d" in

Figure 9. It happens when a human sees and understands the data. It might be a long time after "d", perhaps weeks. Wave function collapse occurs when an observation enters human consciousness.[25] Why would consciousness be so decisive? Today such issues are considered boring, and no one ever thinks about such issues.

Furthermore, the question is stupid because we make thousands of "shots" at the circuitry and the final data tell us the distribution of probabilistic results. In a circuit the output data might read, "P(0) = 0.496, P(1) = 0.504." A quantum computer does not produce one result that subsequently enters human consciousness; it produces probabilistic results. If we take a thousand shots and get one set of results, then another thousand shots will undoubtedly produce different results.

6.2 First Bell state

The four Bell states provide a basis for maximally entangled qubits. In quantum computers the word "entangled" means that it takes at least two qubits to represent a state. An entangled state cannot be reproduced by multiplying two independent qubits.

We will start by discussing the first Bell state.

$$\left|\Psi^+\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{7}$$

This can be simulated with a Hadamard and CNOT gate.

$$H \equiv \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \tag{8}$$

QM says Hadamard gate takes the upper qubit $|0\rangle$ and maps it into the superposition

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \tag{9}$$

.

After the H gate, if the first qubit is in state $|0\rangle$ (i.e. the CNOT gate will do nothing), then the lower qubit remains in state $|0\rangle$ and the final result is $|00\rangle$. However, if the first qubit comes out of the Hadamard gate in state $|1\rangle$, then the lower qubit would be changed by CNOT from $|0\rangle$ to $|1\rangle$ and the final result is $|11\rangle$. Therefore this H-CNOT circuit

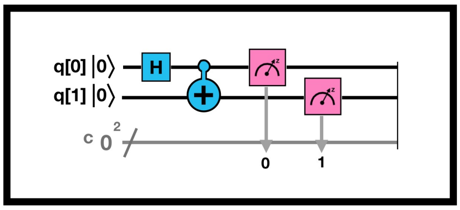

Figure 11: Hadamard and CNOT gates produce the first Bell state.

would yield results as $|00\rangle$ half the time, $|11\rangle$ the other half, but never $|01\rangle$ nor $|10\rangle$. Here is what happens on the upper wire when the qubit enters:

$$|0\rangle \equiv \begin{bmatrix}1\\0\end{bmatrix} \rightarrow \begin{bmatrix}1\\0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \frac{\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}}{\sqrt{2}} \tag{10}$$

According to TEW there is a decision point in the middle of the Hadamard gate:

$$\left\{\frac{\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}}{\sqrt{2}}\right\} = \textbf{Decision Point A} \tag{11}$$

If the value of the top vector is

$$|0\rangle \equiv \begin{bmatrix}1\\0\end{bmatrix} \tag{12}$$

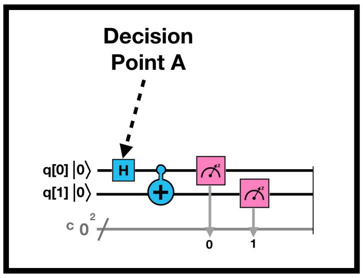

Figure 12: Decision point A, which produces a superposition that is instantly resolved so that either $|0\rangle/\sqrt{2}$ or $|1\rangle/\sqrt{2}$ leaves the gate; the superposition itself does not leave the gate.

then the CNOT gate is the identity matrix. When the upper qubit $|0\rangle$ joins the lower qubit $|0\rangle$ we have

$$\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}=\frac{1}{\sqrt{2}}|00\rangle \tag{13}$$

But if the value of the top vector in the superposition is

$$|1\rangle\equiv\begin{bmatrix}0\\1\end{bmatrix} \tag{14}$$

when the qubit exits the Hadamard gate, then the CNOT gate is arranged like this

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix} \tag{15}$$

and when the upper qubit $|1\rangle$ joins the lower qubit $|0\rangle$ we have

$$\begin{bmatrix}0\\1\\1\\0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\1\end{bmatrix}=\frac{1}{\sqrt{2}}|11\rangle \tag{16}$$

Thus we have the first Bell state:

$$|\Psi^+\rangle=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) \tag{17}$$

If we step back and look at the page of code above, what stands out is that in the center of it we find the words, **“Decision Point A.”** Those words mark the spot when a super-position $((|0\rangle+|1\rangle)/\sqrt{2})$ emerges and the qubit is immediately resolved in favor of $|0\rangle$ or $|1\rangle$ before it leaves the gate.

We see that the upper qubit (meaning the qubit on the upper wire) is the decisive one. It is in control. The lower qubit is the slave, doing exactly what the upper qubit says to do. The lower qubit enters the circuit in a position of $|0\rangle$ and then changes or does not change depending on how the upper qubit has effected the CNOT gate.

The circuit we considered in Figure 12 doesn’t have a lot of entanglement. The upper wire is actually in control of both wires.

6.3 Bell ZW state

To make the results more interesting we can change our attention from the first Bell state, to another Bell state called the “Bell state ZW.” We define “W” as follows:

$$W=\frac{1}{\sqrt{2}}(X+Z). \tag{18}$$

Figure 13 starts exactly the same as the previous circuit, with a $|0\rangle$ in the upper circuit controlling the CNOT gate.

The interesting part of this circuit is not the upper wire but the lower circuit which is manipulated to turn it into a "W" axis. The manipulation consists of extra gates shown in Figures 13 and 14. To define "ZW" we insert four other gates to roll the second Bloch sphere around to impart information into the "W" axis, then roll the Bloch sphere again so that we can read that information on the "Z" axis.

Following the gates as they appear from left to right on the bottom wire of Figure 13: The "S" gate causes a $\pi/2$ rotation around the Z axis. The Hadamard gate H interchanges the X and Z axes. The gate T causes a $\pi/4$ rotation around the Z axis. Then the final Hadamard gate H interchanges the Z and X axes again. This sequence of gates makes visible the "ZW Bell state." This paragraph comes close to articulating what happens inside the compiler inside the brain of a programmer. Most programmers are not thinking about the sterile abstract idea that "the superposition makes a decision at the end when everything is measured." TEW theory provides a vocabulary closer to the reality inside the programmer's brain, whereas QM theory speaks in obscure abstractions.

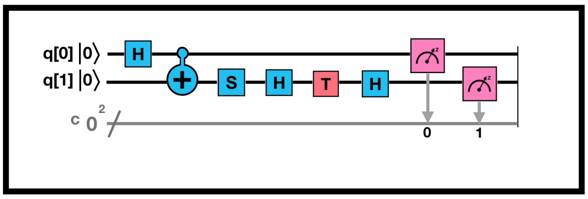

Figure 13: Gates to measure Bell state ZW (where $W = \frac{1}{\sqrt{2}}(X + Z)$).

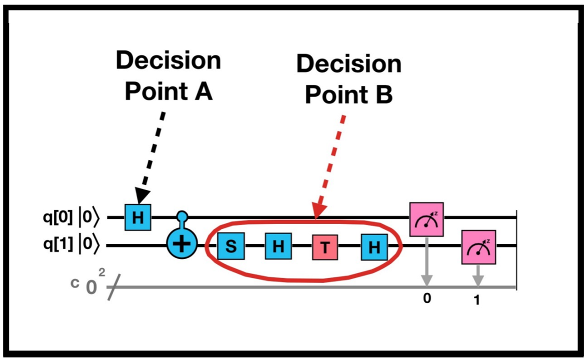

Figure 14: Decision points A and B

Table 1: Results after 2,048 shots at Bell state ZW

Ket	Shots	Percent
$\|00\rangle$	889	43.4 %
$\|11\rangle$	778	38.0 %
$\|01\rangle$	143	07.0 %
$\|10\rangle$	238	11.6 %

If we take 2,048 shots at this circuit, we might obtain the results shown in Table 1. Probably the table for the next 2,048 shots would be slightly different.

It is important to remember that these data represent the ZW Bell state, not the pure "first Bell state" $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

Our purpose is to discuss TEW as a new set of assumptions for looking at quantum circuits. To reiterate our basic idea: if you want to know how a Bell ZW circuit works, look inside the skull of a programmer and learn how the compiler actually works at a nuts and bolts level. Probably there is nothing in there that corresponds to a "qubit superposition."

7 Entanglement in Q-computers

Although TEW has explained the Bell test experiments, we do not yet have a coherent TEW perspective on entanglement of qubits in a quantum computer. We have some sketchy ideas, but the overall picture is not yet clear. Here are our

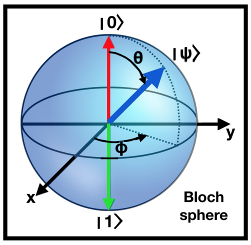

Figure 15: A Bloch sphere.

sketchy ideas. There are two ways to attack the problem: with equations or diagrams. Starting with equations, we propose that for a qubit in a Bloch sphere, $|\alpha|^2 + |\beta|^2 = 1$.

The generic state of a qubit may be written as $|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\theta}\sin\frac{\phi}{2}|1\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\theta}\sin\frac{\phi}{2} \end{bmatrix}$.

We can apply the projector $P = |\Psi\rangle\langle\Psi|$ to get the matrix representation of the operator P on the basis $(|0\rangle, |1\rangle)$, which is

$$P = \begin{bmatrix} \cos^2\frac{\theta}{2} & e^{-i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{bmatrix} \tag{19}$$

If the internal workings of a quantum computer were described along these lines, we would have a trigonometric approach that would be closer to the trigonometry that was used to describe the Bell test experiments in the first half of this article.

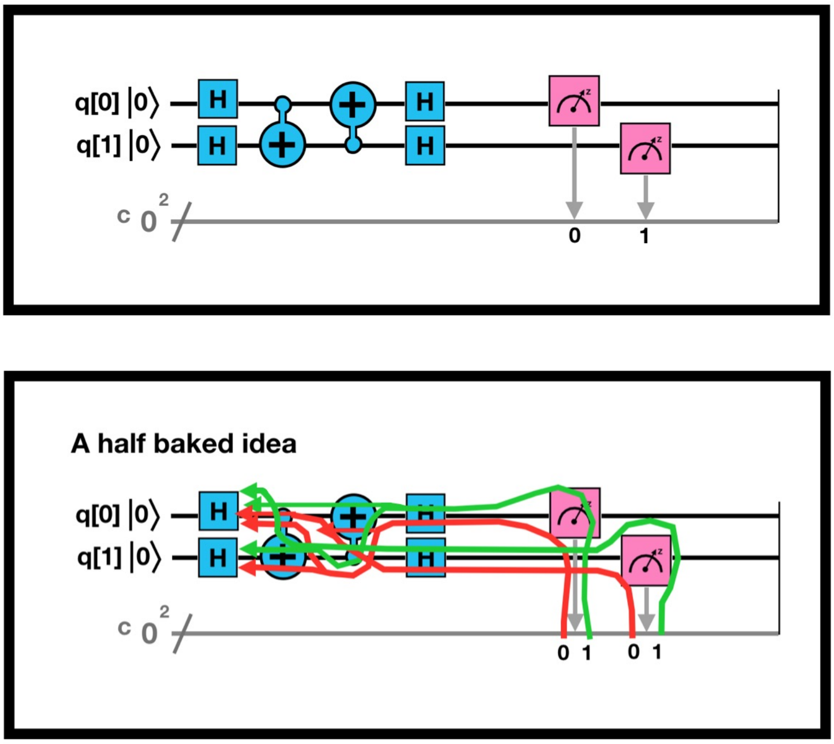

Figure 16: The same circuit (top and bottom) with Elementary Rays (red and green lines) starting at the detectors and threading backwards, so that each Hadamard decision point has both a green and a red thread available at the exit of the decision point.

Another way to attack the problem is by diagrams of computer circuits (see Figure 15 above). We are sketching possible quantum circuits for the purpose of illustrating ideas that we have not yet presented in this article, namely to describe where Bi-Rays might live inside a quantum computer. This circuit is not supposed to accomplish anything other than illustrating a new way of thinking.

If we think of each qubit following an Elementary Ray in the opposite direction, then the Elementary Rays would start at the detectors and snake backwards through the equipment. We will use a red line to symbolize the wave starting at the "zero" value of each detector, and green for the "one" value. The idea is that one or more pair of threads (a red and a green) are present at the back door of each Hadamard gate, so a qubit, as it exits the gate, could choose whether to follow a red or a green path backwards. If there are several decision points, as in Figure 15, the red/green threads would bifurcate so as to present a branch to each of the decision points. The details of this red/green tree have not been worked out.

In Figure 15 the pair of C-Not gates might have red and green arrows traveling in both directions (up and down). Why is that important? That is in such an area ($\rightleftarrows$) of a quantum circuit within which you might search for a Bi-Ray, consisting of a pair of Elementary Waves traveling in countervailing directions ($\rightleftarrows$). It is in the arena of Bi-Rays that we might develop the TEW idea of "entanglement" based on two qubits following a single Bi-Ray in opposite directions. The details have not yet been worked out.

These ideas are not satisfactorily clear.

8 Conclusions

It is difficult to talk about either the Bell test experiments or quantum computers using the vocabulary of QM. The problem is that QM starts with bad Axioms:

A. Wave function collapse occurs when we measure something,

B. There is wave particle duality,

C. Waves travel in the same direction as particles.

The first Axiom is the most problematic. When we try to understand the Bell test experiments, that Axiom forces us to say that something decisive happens when Alice observes a photon. That is patently absurd. There is nothing important that happens at that moment. If Alice leaves for a coffee break and doesn't make any observation, it is of no significance! Nature is set up in such a way that if humans die or disappear, Nature can endure just fine, and perhaps better. For billions of years and in billions of cubic lightyears of space human observation (or measurement) has not occurred, and the universe wasn't troubled by our absence.

The first Axiom also forces us to say that the focus in a quantum computer should be on the final detector, where wave function collapse allegedly occurs. Actually the final detector is the least interesting part of a computer. What is interesting are the gates upstream, where all the decisions are made. Let's take a second to think about those last two sentences. It indicates something is wrong with a theory if it focuses on the most boring, the least interesting part of a quantum computer, by saying that is where wave function collapse occurs (i.e. where THE decision is made). We propose that is now how programmers actually think, and we propose that the internal workings of the mental compiler in programmers' brains is what should be adopted as our definition of "reality."

Trying to use the Axioms of QM is like trying to dress a child in clothing that doesn't fit. No matter how carefully you do it, the kid still looks poorly dressed. Not sharp!

The way to make life easier is to switch to Axioms that actually fit:

A. Wave function collapse occurs *before* we measure something,

B. There is *no* wave particle duality,

C. Waves travel in the *opposite* direction as particles.

In the Bell test experiments the decisive moment is when the photon pair is born. Einstein was right about that. The problem with Einstein is that he thought "hidden variables" such as particle spin, must be intrinsic characteristics of particles. He was wrong. If he had known about TEW he would have done better saying that "hidden variables" are intrinsic to Bi-Rays.

Once a pair of photons is born into one specific Bi-Ray and not some other Bi-Ray, the final data are already determined. The answer will be $Z = \cos^2(\theta_2 - \theta_1)$ or $Z = \sin^2(\theta_2 - \theta_1)$, depending whether the two photons were born with the same polarization or orthogonal to one another.

If the photons have the same polarization at birth, and Alice observes a photon, what is the content of the so-called "signal" she sends to Bob? It is $Z = \cos^2(\theta_2 - \theta_1)$. So when Bob looks at his photon and sees that it conforms to Alice's message, in QM Bob says to himself, "This is an astonishing coincidence that cannot be caused by chance alone! Clearly Alice sent me a message faster than the speed of light!"

8.1 Three different views of Nature

Compare three different views of Nature. Einstein's local realism pictures us living in a world in which objects have intrinsic characteristics, no matter whether you look at them or not. That view of Nature has been dispelled by the Bell test experiments.

The QM view of Nature is that things don't exist until they are observed or measured. As Franco Selleri said, "A lot of miracles happen in QM!"

The TEW view emphasizes relationships. Alice and the photon source and Bob have relationships with one another. If Alice or Bob changes their angle of observation, things look different. But the core relationship is still the same.

Elementary Waves and Bi-Rays concern how things interact with one another. That interaction is not pre-determined by the Nature of the thing, because if you look at it from a different perspective, you see something different. Nor does the thing observed pop into reality because it is observed.

We are immersed in an ecological ocean of things and people affecting one another. Elementary Waves are relationships waiting to happen. They are possibilities.

8.2 How to think about a change of Axioms

In 1912 Alfred Wegener said that all the continents had been part of a super-continent, for which he coined the name "Pangea." The continents have been drifting apart ever since Pangea broke up. All scientists at that time declared that this was utterly insane. Even his father-in-law told Wegener this was a stupid idea. It was clear to all rational people that no force on earth was strong enough to move continents. Wegener's "stupid idea" was not mentioned in textbooks of science, and was treated as rubbish for decades.[30]

New Axioms that are tossed aside have a way of boomeranging and hitting you in the back of the head. Today the dominant paradigm in geology is an Axiom named "plate tectonics." It is assumed that convection currents in the magma carry heat from the core of the earth to the surface. This allows us to understand the ring of fire: volcanoes and earthquakes around the Pacific rim.

Usually an Axiomatic change signals a tectonic change in science. A change of Axioms is by nature almost impossible to understand or discuss, because it sounds like unintelligible gibberish to supporters of the old Axioms. Sometimes in mathematics, when someone solves an intractable problem, or proves something akin to the Riemann hypothesis, the article is rejected by math journals because no one can understand it.

8.3 Prestige in science and mathematics

In addition to that obstacle, this particular Axiomatic change has an additional problem. Almost no one is motivated to understand it. With zero energy comes zero prestige. Energy brings prestige. The ideas that are exciting in physics today are dark energy and dark matter (which is a condensed form of energy). There is also excitement about winning the race to build the quantum computer that will dominate computer science in the future. A lot of money is gambled on one research team versus another.

Just as energy brings prestige, the absence of energy brings contempt. When we say that Elementary Waves convey zero energy, scholars respond, "They cannot carry zero energy! If they are capable of doing anything then they must carry a tiny amount of energy!"

It is like politics. If you have power and prestige then everyone is interested in you. If you have zero power, then you are held in contempt. You are not even noticed, because no one is motivated to see you.

We are wrong, however, to pay so much attention to power, prestige, mass and energy. TEW has to do with relationships between things. If you look at this sentence, as we said before, then you give this sentence permission to respond to you. It is subtle, gentle, interactive, ecological, not coercive. It is like the long-distance relationship between Alice and Bob, a relationship that endures for a long time but never travels faster than light.

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