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Comparison of Halley-Chebyshev Method with Several Nonlinear equation Solving Methods Methods

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Abstract

In this paper, we present one of the most important numerical analysis problems that we find in the roots of the nonlinear equation. In numerical analysis and numerical computing, there are many methods that we can approximate the roots of this equation. We present here several different methods, such as Halley's method, Chebyshev's method, Newton's method, and other new methods presented in papers and journals, and compare them. In the end, we get a good and attractive result.

Keywords: Converge, Newton Method, Halley Method, Chebyshev Method, Nonlinear Equation, Hybrid Method, Steffenson Method.

1. Introduction

One of the most important problems in Numerical Analysis is finding different values of the x variable in the $f(x)=0$ equation that there exist variate methods for solution and it exist in [1], [2], [6], [8], [9] and [10]. Mostly this equation is solved by the Newton iteration method, that is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1}$$

It is known that Newton method has the convergence of the second order term. However, in the method which is mentioned here, the convergence is of higher order term. In fact, this new method is more convergent than the other methods, that is, the Newton method and the hybrid method [7] and Fang [5] and new method [3] and et cetera.

2. Several methods for equation solution

A new iteration method based on Taylor theorem was proposed to solve nonlinear algebraic equations by Nasr [7]. It is claimed that this new method has better convergent characteristics than the well-known Newton method. This method has iteration formula like below formula

$$x_{k+1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{2}$$

Where $A = f''(x_k)$, $B = 6f'(x_k) - 2f''(x_k)x_k$ and $C = 6f(x_k) - 6f'(x_k)x_k + f''(x_k)x_k^2$.

The other method that here oppugn is Halley method [11]. This method has iteration formula too

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{(f'(x_n))^2 - \frac{1}{2}f(x_n)f''(x_n)} \tag{3}$$

The next method is Chebyshev method [12] and we have

$$x_{n+1} = x_n - \left(1 + \frac{1}{2}\frac{f''(x_n)f(x_n)}{(f'(x_n))^2}\right)\frac{f(x_n)}{f'(x_n)} \tag{4}$$

Another method that it is new method and we named that Halley-Chebyshev method [4] is

$$x_{n+1} = x_n - \left(\omega \frac{f(x_n)f'(x_n)}{(f'(x_n))^2 - \frac{1}{2}f(x_n)f''(x_n)} + (1-\omega)\left(1 + \frac{1}{2}\frac{f''(x_n)f(x_n)}{(f'(x_n))^2}\right)\frac{f(x_n)}{f'(x_n)}\right) \tag{5}$$

Where ω is an arbitrary parameter and $0 \le \omega \le 1$.

And the last method that we have too is Steffenson method [1] and we have iteratin formula

$$\mathrm{x}_{n+1} = \mathrm{x}_n - \frac{(\mathrm{f}(\mathrm{x}_n))^2}{\mathrm{f}(\mathrm{x}_n + \mathrm{f}(\mathrm{x}_n)) - \mathrm{f}(\mathrm{x}_n)} \tag{6}$$

3. Comparison of several methods with example

In this section we perform some numerical examples using our new methods and we will compare these results to famous methods. All computations were done MAPLE. We know that an approximate solution rather than the accurate root, depending on the precision (ε) of computer. We use the following stopping test for computer programs: *(i)* $|\mathrm{x}_{n+1} - \mathrm{x}_n| < \varepsilon$, *(ii)* $|\mathrm{f}(\mathrm{x}_{n+1})| < \varepsilon$.

In here, we have real several methods of non-linear equation that we compare them with together in tables.

Example1

Consider the equation $f(x) = (x^2 - 5x + 7)e^x + x - 3$. This function has a single root with accurate amount -1.248014042. In order to find a root close to $x = -1.248014042$, we let $x_0 = 1$. The obtained results by Newton iteration, new hybrid iteration [3], and Halley method [11] are presented in the under table. (Table 1 with $\varepsilon = 10^{-100}$)

Table 1

Methods	The numbers of iterations	approximate solution
Newton method [1], [2], [6], [8], [9] and [10]	9	-1.248014041
Hybrid method [7]	5	- 1.248014038
New Hybrid method [3] with $\omega = 0$	4	- 1.248014040

New Hybrid method [3] with $\omega = 0.35$	5	- 1.248014040
Chebyshev method [12]	9	- 1.248014042
Halley method [11]	5	- 1.248014041
Steffenson method [1]	233	- 1.248014042
Halley-Chebyshev method [4] with $\omega = 0.35$	6	- 1.248014041

Halley-Chebyshev method has better performance than other methods

Example2

Consider the equation $f(x) = 2x^3 - 4x + 5\cos x$. This function has a single root with accurate amount -1.472154984. In order to find a root close to $x = -1.472154984$, we let $x_0 = 0$. The obtained results by Newton iteration, new hybrid iteration [3], and Halley method [11] are presented in the under table. (Table 2 with $\varepsilon = 10^{-1000}$)

Table 2

Methods	**The numbers of iterations**	**approximate solution**
Newton method [1], [2], [6], [8], [9] and [10]	65	-1.472154984
Hybrid method [7]	9	-1.472154984
New Hybrid method [3] with $\omega = 0$	6	-1.472154984
New Hybrid method [3] with $\omega = 0.35$	9	-1.472154984
Chebyshev method [12]	30	-1.472154984
Halley method [11]	18	-1.472154984
Steffenson method [1]	36	-1.472154984
Halley-Chebyshev method [4] with $\omega = 0.35$	14	-1.472154985

Halley-Chebyshev method has better performance than other methods

4. Conclusions

In this paper, we compare several iteration formulas to solve the roots of the nonlinear equations. Tables and results show that Halley-Chebyshev method has better performance than other methods. It can be seen from the examples that the current method is enough a faster method that takes lesser number of iterations, needs lesser number of functional evaluations in final as well as in individual step as compared to the other methods.

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