



PSEUDO-SLANT SUBMANIFOLD IN KENMOTSU SPACE FORMS

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ABSTRACT

In this paper, we study geometry of the pseudo-slant submanifold of a Kenmotsu space form. Necessary and sufficient conditions are given for a submanifold to be a pseudo-slant submanifold in Kenmotsu manifolds. Finally, we give some results for totally umbilical pseudo-slant submanifold in a Kenmotsu manifold and Kenmotsu space form.

Keywords:

Kenmotsu manifold, Kenmotsu space form, Slant submanifold, Pseudo-slant submanifold.

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1. INTRODUCTION

The differential geometry of slant submanifolds has shown an increasing development since B-Y. Chen [9, 10] defined slant submanifolds in complex manifolds as a natural generalization of both the holomorphic and totally real submanifolds. Many research articles have been appeared on the existence of these submanifolds in different known spaces. The slant submanifolds of an almost contact metric manifolds were defined and studied by A. Lotta [14]. After, these submanifolds were studied by J. L. Cabrerizo et. al [6] of Sasakian manifolds. Recently, in [3]. M. Atçeken studied slant and pseudo-slant submanifold in $(LCS)_n$ -manifolds.

The notion of semi-slant submanifolds of an almost Hermitian manifold was introduced by N. Papaghiuc [15]. Cabrerizo et al. studied and characterized slant submanifolds of K- contact and Sasakian manifolds and have given several examples of such submanifolds. Recently, Carizzo [7, 6] defined and studied bi-slant immersions in almost Hermitian manifolds and simultaneously gave the notion of pseudo-slant submanifold in almost Hermitian manifolds. The contact version of pseudo-slant submanifolds has been defined and studied by V. A. Khan and M. A Khan [12]. The present paper is organized as follows.

In this paper, we study pseudo-slant submanifolds of a Kenmotsu manifold. In section 2, we review basic formulas and definitions for a Kenmotsu manifold and their submanifolds. In section 3, we recall the definition and some basic results of a pseudo-slant submanifold of almost contact metric manifold. In section 4, we will give some results for totally umbilical pseudo-slant submanifold in a Kenmotsu manifold and Kenmotsu space form $\tilde{M}(c)$.

2. PRELIMINARIES

In this section, we give some notations used throughout this paper. We recall some necessary facts and formulas from the theory of Kenmotsu manifolds and their submanifolds.

Let \tilde{M} be a $(2m+1)$ -dimensional almost contact metric manifold together with a metric tensor g a tensor field φ of type $(1,1)$, a vector field ξ and a 1-form η on \tilde{M} which satisfy

$$\varphi^2(X) = -X + \eta(X)\xi \quad (1)$$

$$\varphi\xi = 0, \quad \eta(\varphi X) = 0, \quad \eta(\xi) = 1, \quad \eta(X) = g(X, \xi) \quad (2)$$

and

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \quad g(\varphi X, Y) = -g(X, \varphi Y) \quad (3)$$

for any vector fields X, Y on \tilde{M} . If in addition to above relations

$$(\tilde{\nabla}_X \varphi)Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X \quad (4)$$

then, \tilde{M} is called Kenmotsu manifold, where $\tilde{\nabla}$ is Levi-Civita connection of g . We have also on a Kenmotsu manifold \tilde{M}

$$\tilde{\nabla}_X \xi = X - \eta(X)\xi = -\varphi^2 X \quad (5)$$

for any $X, Y \in \Gamma(T\tilde{M})$.

Let \tilde{R} be the curvature tensor of the connection $\tilde{\nabla}$. The sectional curvature of a φ -section is called a φ -holomorphic sectional curvature. A Kenmotsu manifold with constant φ -holomorphic sectional curvature c is said to be a Kenmotsu space form and it is denoted by $\tilde{M}(c)$. The curvature tensor \tilde{R} of a Kenmotsu space form $\tilde{M}(c)$ is given by

$$\begin{aligned} \tilde{R}(X, Y)Z = & \frac{(c-3)}{4} \{g(Y, Z)X - g(X, Z)Y\} + \frac{(c+1)}{4} \{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ & + \eta(Y)g(X, Z)\xi - \eta(X)g(Y, Z)\xi + g(X, \varphi Z)\varphi Y - g(Y, \varphi Z)\varphi X + 2g(X, \varphi Y)\varphi Z\} \end{aligned} \quad (6)$$

for any $X, Y \in \Gamma(T\tilde{M})$.



Now, let M be a submanifold of a contact metric manifold \tilde{M} with the induced metric g . Also, let ∇ and ∇^\perp be the induced connections on the tangent bundle TM and the normal bundle $T^\perp M$ of M , respectively. Then the Gauss and Weingarten formulas are, respectively, given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y) \tag{7}$$

and

$$\tilde{\nabla}_X Y = -A_V X + \nabla^\perp_X Y, \tag{8}$$

where σ and A_V are the second fundamental form and the shape operator (corresponding to the normal vector field V), respectively, for the submanifold of M into \tilde{M} . The second fundamental form σ and shape operator A_V are related by

$$g(A_V X, Y) = g(\sigma(X, Y), V) \tag{9}$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$.

Let $p \in M$ and $\{e_1, e_2, \dots, e_{2m+1}\}$ be an orthonormal basis of tangent space $T_M(p)$ such that $\{e_1, e_2, \dots, e_l\}$ are tangent to M at p . We H denote the mean curvature vector, that is

$$H(p) = \frac{1}{l} \sum_{i=1}^l \sigma(e_i, e_i). \tag{10}$$

A submanifold M of an contact metric manifold \tilde{M} is said to be totally umbilical if

$$\sigma(X, Y) = g(X, Y)H. \tag{11}$$

A submanifold M is said to be totally geodesic submanifold if $\sigma(X, Y) = 0$, for each $X, Y \in \Gamma(TM)$ and M is said to be minimal submanifold if $H = 0$.

Also, we get

$$\sigma_{ij}^r = g(\sigma(e_i, e_j), e_r), \quad 1 \leq i, j \leq l, \quad l+1 \leq r \leq 2m+1,$$

where, the coefficients σ_{ij}^r denote of the second fundamental form σ with respect to $\{e_1, e_2, \dots, e_{2m+1}\}$, and

$$\|\sigma\|^2 = \sum_{i,j=1}^l g(\sigma(e_i, e_j), \sigma(e_i, e_j)).$$

For any submanifold M of a Riemannian manifold \tilde{M} , the equation of Gauss is given by

$$\tilde{R}(X, Y)Z = R(X, Y)Z + A_{\sigma(X,Z)}Y - A_{\sigma(Y,Z)}X + (\tilde{\nabla}_X \sigma)(Y, Z) - (\tilde{\nabla}_Y \sigma)(X, Z), \tag{12}$$

for any $X, Y \in \Gamma(TM)$, where \tilde{R} and R denote the Riemannian curvature tensor of \tilde{M} and M , respectively. The covariant derivative $\tilde{\nabla} \sigma$ of σ is defined by

$$(\tilde{\nabla}_X \sigma)(Y, Z) = \nabla_X^\perp \sigma(Y, Z) - \sigma(\nabla_X Y, Z) - \sigma(\nabla_X Z, Y) \tag{13}$$

and the covariant derivative ∇A is also defined by

$$(\nabla_X A)_V Y = \nabla_X (A_V Y) - A_{\nabla_X V} Y - A_V \nabla_X Y, \tag{14}$$



for any $X, Y, Z \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$. The normal and tangent components of (12) are, respectively, given by

$$(\tilde{R}(X, Y)Z)^\perp = (\tilde{\nabla}_X \sigma)(Y, Z) - (\tilde{\nabla}_Y \sigma)(X, Z), \quad (15)$$

and

$$(\tilde{R}(X, Y)Z)^T = R(X, Y)Z + A_{\sigma(X, Z)}Y - A_{\sigma(Y, Z)}X. \quad (16)$$

If $(\tilde{R}(X, Y)Z)^T$, then M is said to be curvature – invariant submanifold of \tilde{M} . The Ricci equation is also given by

$$g(\tilde{R}(X, Y)V, U) = g(\tilde{R}^\perp(X, Y)V, U) + g([A_U, A_V]X, Y), \quad (17)$$

for any $X, Y \in \Gamma(TM)$ and $V, U \in \Gamma(T^\perp M)$, where \tilde{R}^\perp denotes the Riemannian curvature tensor of the normal $T^\perp M$. Here, if $\tilde{R}^\perp(X, Y)V = 0$, then the normal connection of M is called flat.

A Kenmotsu manifold \tilde{M} is said to be η – Einstein if its Ricci tensor S of type $(0, 2)$ is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y) \quad (18)$$

where a, b are smooth functions on \tilde{M} . From the definition it is clear that if $b = 0$, then the manifold is called Einstein.

3. PSEUDO-SLANT SUBMANIFOLD OF KENMOTSU MANIFOLD

In this section, we will study pseudo-slant submanifolds in a Kenmotsu manifold, give some characterization and submanifold is characterized.

Now, let M be a submanifold of an almost contact metric manifold \tilde{M} , then for any $X \in \Gamma(TM)$ we can write

$$\varphi X = TX + NX, \quad (19)$$

where TX is the tangential component and NX is the normal component of φX . Similarly for $V \in \Gamma(T^\perp M)$, we can write

$$\varphi V = tV + nV, \quad (20)$$

where tV is the tangential component and nV is also the normal component of φV . Thus by using (1), (19) and (20), we obtain

$$T^2 = -I + \eta \otimes \xi - tN, \quad Nt + nN = 0 \quad (21)$$

and

$$Tt + tn = 0, \quad NT + n^2 = -I. \quad (22)$$

Furthermore, the covariant derivatives of the tensor field T, N, t and n are, respectively, defined by

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y, \quad (23)$$

$$(\nabla_X N)Y = \nabla_X^\perp NY - N\nabla_X Y, \quad (24)$$

$$(\nabla_X t)V = \nabla_X tV - t\nabla_X^\perp V, \quad (25)$$

and

$$(\nabla_X n)V = \nabla_X^\perp nV - n\nabla_X^\perp V. \quad (26)$$



Furthermore, for any $X, Y \in \Gamma(TM)$, we have $g(TX, Y) = -g(X, TY)$ and $V, U \in \Gamma(T^\perp M)$, we get $g(U, nV) = -g(nU, V)$. These Show that T and n are also skew-symmetric tensor fields. Moreover, for any $X \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$, we have

$$g(NX, V) = -g(X, tV), \tag{27}$$

which gives the relation between N and t .

Taking into account (6) and (17), we have

$$g(\tilde{R}^\perp(X, Y)V, U) = \frac{(c+1)}{4} \{g(X, \phi V)g(U, \phi Y) - g(Y, \phi V)g(\phi X, U) + 2g(X, \phi Y)g(\phi V, U)\} - g([A_\nu, A_\nu]X, Y) \tag{28}$$

for any $X, Y \in \Gamma(TM)$ and $V, U \in \Gamma(T^\perp M)$. By using (6) and (12), the Riemannian curvature tensor R of an immersed submanifold M of a Kenmotsu space form $\tilde{M}(c)$ is given by

$$R(X, Y)Z = \frac{(c-3)}{4} \{g(Y, Z)X - g(X, Z)Y\} + \frac{(c+1)}{4} \{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + \eta(Y)g(X, Z)\xi - \eta(X)g(Y, Z)\xi + g(X, \phi Z)TY - g(Y, \phi Z)TX + 2g(X, \phi Y)TZ\} + A_{\sigma(Y, Z)}X - A_{\sigma(X, Z)}Y. \tag{29}$$

From (6) and (12), for a submanifold, Codazzi equation is given by

$$(\tilde{\nabla}_X \sigma)(Y, Z) - (\tilde{\nabla}_Y \sigma)(X, Z) = \frac{(c+1)}{4} \{g(X, \phi Z)NY - g(Y, \phi Z)NX + 2g(X, \phi Y)NZ\}. \tag{30}$$

A submanifold M is said to be invariant if N is identically zero, that is, $\phi X \in \Gamma(TM)$ for all $X \in \Gamma(TM)$. On the other hand, M is said to be anti-invariant if T is identically zero, that is, $\phi X \in \Gamma(T^\perp M)$ for all $X \in \Gamma(TM)$. By an easy computation, we obtain the following formulas

$$(\nabla_X T)Y = A_{NY}X + t\sigma(X, Y) + g(TX, Y)\xi - \eta(Y)TX \tag{31}$$

and

$$(\nabla_X N)Y = n\sigma(X, Y) - \sigma(X, TY) - \eta(Y)X. \tag{32}$$

Similarly, for any $V \in \Gamma(T^\perp M)$ and $X \in \Gamma(TM)$, we obtain

$$(\nabla_X t)V = g(NX, V)\xi + A_{nV}X - TA_\nu X \tag{33}$$

and

$$(\nabla_X n)V = -\sigma(tV, X) - NA_\nu X. \tag{34}$$

Since M is tangent to ξ , making use of (5) and (9) we obtain

$$A_\nu \xi = \sigma(X, \xi) = 0 \tag{35}$$

for all $V \in \Gamma(T^\perp M)$ and $X \in \Gamma(TM)$.

In contact geometry, A. Lotta introduced slant immersions as follows [14].



Definition 3. 1. Let M be a submanifold of a Kenmotsu manifold \tilde{M} . A distribution D on M is said to be a slant if for each non-zero vector $X \in D_p$, the angle θ between φX and D_p is constant, that is, independent of $p \in \tilde{M}$ and $X \in D_p$. The constant angle θ is called the slant angle of the slant distribution. So a submanifold M of \tilde{M} is said to be a slant submanifold if the tangent bundle TM of M is slant [14].

So invariant and anti-invariant submanifolds are slant submanifolds with slant angle $\theta=0$ and $\theta = \frac{\pi}{2}$ respectively. A proper slant submanifolds is neither invariant nor anti-invariant. Thus invariant and anti invariant submanifolds are special cases of slant submanifolds.

If M is a slant submanifold of an almost contact metric manifold, then the tangent bundle TM of M can be decomposed as

$$TM = D \oplus \xi,$$

where ξ denotes the distribution spanned by the structure vector field ξ and D is complementary of distribution of ξ in TM , known as the slant distribution on M . Recently, Cabrerizo et al. [6] extended the above result in to a characterization for a slant submanifold in a contact metric manifold. In fact, they obtained the following crucial theorem.

Theorem 3. 1. [6]. Let M be a slant submanifold of an almost contact metric manifold \tilde{M} such that $\xi \in \Gamma(TM)$. Then, M is slant submanifold if and only if there exist a constant $\lambda \in [0,1]$ such that

$$T^2 = -\lambda(I - \eta \otimes \xi) \quad (36)$$

furthermore , in this case, if θ is the slant angle of M , then $\lambda = \cos^2 \theta$.

Corollary 3. 1. [6]. Let M be a slant submanifold of an almost contact metric manifold \tilde{M} with slant angle θ . Then for any $X, Y \in \Gamma(TM)$, we have

$$g(TX, TY) = \cos^2 \theta \{g(X, Y) - \eta(X)\eta(Y)\} \quad (37)$$

and

$$g(NX, NY) = \sin^2 \theta \{g(X, Y) - \eta(X)\eta(Y)\}. \quad (38)$$

Definition 3. 2. [12]. Let M be a slant submanifold of an almost contact metric manifold \tilde{M} . M is said to be pseudo-slant of \tilde{M} if there exit two orthogonal distributions D^\perp and D_θ on M such that:

- i) TM has the orthogonal direct decomposition $TM = D^\perp \oplus D_\theta$, $\xi \in \Gamma(D_\theta)$.
- ii) The distribution D^\perp is anti-invariant, that is, $\varphi(D^\perp) \subset T^\perp M$.
- iii) The distribution D_θ is a slant, that is, the slant angle between of D_θ and $\varphi(D_\theta)$ is a constant.

Let d_1 and d_2 be dimensional of distributions D^\perp and D_θ , respectively. Then

- i) If $d_2 = 0$ then, M is an anti-invariant submanifold.
- ii) If $d_1 = 0$ and $\theta = 0$ then, M is an invariant submanifold.
- iii) If $d_1 = 0$ and $\theta \in (0, \frac{\pi}{2})$ then, M proper slant submanifold.



- iv) If $\theta = \frac{\pi}{2}$ then, M is an anti-invariant submanifold.
- v) If $d_1, d_2 \neq 0$ and $\theta \in (0, \frac{\pi}{2})$ then, M is a proper pseudo-slant submanifold.
- vi) If $d_2, d_1 \neq 0$ and $\theta = 0$ then, M is a semi-invariant submanifold.

If we denote the projections on D^\perp and D_θ by P_1 and P_2 , respectively, then for any vector field $X \in \Gamma(TM)$, we can write

$$X = P_1X + P_2X + \eta(X)\xi. \tag{39}$$

On the other hand, applying ϕ on both sides of equation (39), we have

$$\phi X = \phi P_1X + \phi P_2X$$

and

$$TX + NX = NP_1X + TP_2X + NP_2X, \quad TP_1X = 0. \tag{40}$$

From which, we can easily to see

$$TX = TP_2X, \quad NX = NP_1X + NP_2X$$

and

$$\phi P_1X = NP_1X, \quad TP_1X = 0, \quad \phi P_2X = TP_2X + NP_2X, \quad TP_2X \in \Gamma(D_\theta). \tag{41}$$

If we denote the orthogonal complementary of ϕTM in $T^\perp M$ by μ , then the normal bundle $T^\perp M$ can be decomposed as follows

$$T^\perp M = N(D^\perp) \oplus N(D_\theta) \oplus \mu. \tag{42}$$

We can easily see that the bundle μ is an invariant subbundle with respect to ϕ . Since D^\perp and D_θ are orthogonal distribution on M , $g(Z, X) = 0$ for each $Z \in \Gamma(D^\perp)$ and $X \in \Gamma(D^\theta)$. Thus, by equation (3) and (19), we can write

$$g(NZ, NX) = g(\phi Z, \phi X) = g(Z, X) = 0$$

that is, the distributions $N(D^\perp)$ and $N(D_\theta)$ are also mutually perpendicular. In fact, the decomposition (42) is an orthogonal direct decomposition.

Theorem 3. 2. Let D be a distribution on M , orthogonal to ξ . Then D is a slant if and only if there is a constant $\lambda \in [0, 1]$ such that

$$(TP)^2 X = -\lambda X \tag{43}$$

for all $X \in \Gamma(D)$, where P denotes the orthogonal projection on D . Furthermore, in this case, the slant angle θ of M satisfies $\lambda = \cos^2 \theta$ [6].

It is well known that $t\sigma = 0$ plays an important role in the geometry of submanifolds. This means that the induced structure T is a Kenmotsu structure on M . Then (31) reduces to

$$(\nabla_X T)Y = g(TX, Y)\xi - \eta(Y)TX, \tag{44}$$



for $X, Y \in \Gamma(D_\theta)$. For example, this means that the induced structure T is a Kenmotsu structure on M if the ambient manifold \tilde{M} is a Kenmotsu manifold.

Moreover, for any $Z, W \in \Gamma(D^\perp)$ and $U \in \Gamma(TM)$, also by using (4) and (9), we have

$$\begin{aligned}
 g(A_{NZ}W - A_{NW}Z, U) &= g(\sigma(W, U), NZ) - g(\sigma(Z, U), NW) \\
 &= g(\tilde{\nabla}_U W, \varphi Z) - g(\tilde{\nabla}_U Z, \varphi W) \\
 &= g(\varphi \tilde{\nabla}_U Z, W) - g(\varphi \tilde{\nabla}_U W, Z) \\
 &= g(\tilde{\nabla}_U \varphi Z, W) - g((\tilde{\nabla}_U \varphi)Z, W) \\
 &\quad - g(\tilde{\nabla}_U \varphi W, Z) + g((\tilde{\nabla}_U \varphi)W, Z) \\
 &= g(\tilde{\nabla}_U \varphi Z, W) - g(g(\varphi U, Z)\xi - \eta(Z)\varphi U, W) \\
 &\quad - g(\tilde{\nabla}_U \varphi W, Z) + g(g(\varphi Z, U)\xi - \eta(U)\varphi Z, W) \\
 &= -g(A_{NZ}U, W) + g(A_{NW}U, Z) \\
 &= g(A_{NW}Z - A_{NZ}W, U).
 \end{aligned}$$

It follows that

$$A_{NW}Z = A_{NZ}W, \tag{45}$$

for any $Z, W \in \Gamma(D^\perp)$.

Theorem 3.3. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . The tensor field N is parallel if and only if shape operator A_V satisfies

$$A_V Y = \sec^2 \theta A_{n_V} T Y$$

for any $Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$.

Proof. If N is parallel, from (32), we have

$$n\sigma(X, Y) - \sigma(X, TY) - \eta(Y)NX = 0$$

for any $X, Y \in \Gamma(TM)$. This implies

$$\begin{aligned}
 0 &= n\sigma(X, TY) - \sigma(X, T^2 Y) \\
 &= n\sigma(X, TY) + \cos^2 \theta \sigma(X, Y - \eta(Y)\xi).
 \end{aligned}$$

Thus we have

$$\begin{aligned}
 0 &= g(n\sigma(X, TY), V) + \cos^2 \theta g(\sigma(X, Y - \eta(Y)\xi), V) \\
 &= -g(\sigma(X, TY), nV) + \cos^2 \theta g(\sigma(X, Y - \eta(Y)\xi), V) \\
 &= -g(A_{n_V} T Y, X) + \cos^2 \theta g(A_V X, Y - \eta(Y)\xi) \\
 &= -g(A_{n_V} T Y, X) + \cos^2 \theta g(A_V X, Y) \\
 &= -g(A_{n_V} T Y, X) + \cos^2 \theta g(A_V X, Y)
 \end{aligned}$$

for any $V \in \Gamma(T^\perp M)$. This equivalent to



$$A_V Y = \sec^2 \theta A_{nV} T Y.$$

Theorem 3. 4. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . If the shape operator of the pseudo-slant submanifold M is parallel, then M is a totally geodesic submanifold.

Proof. If the shape operator A_V of M is parallel, then by using (5) and (14), we have

$$\nabla_X (A_V Y) - A_{\nabla_X V} Y - A_V \nabla_X Y = 0,$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$. Here, choosing $Y = \xi$ and taking into account (14) and (35), we obtain

$$-A_{\nabla_X V} \xi - A_V \nabla_X \xi = -A_V (X - \eta(X)\xi) = 0.$$

This yields to $A_V X = 0$. This proves our assertion.

Theorem 3. 5. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . If the endomorphism T is parallel if and only if M is anti-invariant submanifold \tilde{M} .

Proof. If T is parallel, then from (31) and (35), we have

$$\begin{aligned} 0 &= g(A_{NV} X, \xi) + g(TX, Y) - \eta(Y)g(TX, \xi) \\ &= g(\sigma(X, \xi), NY) + g(TX, Y) \\ &= g(TX, Y). \end{aligned} \tag{46}$$

Here, taking $Y = TX$ equation (46), we have $g(TX, TX) = 0$ for any $X, Y \in \Gamma(TM)$. This implies that $\cos^2 \theta \{g(X, X) - \eta^2(X)\} = 0$, that is, M is anti-invariant submanifold.

Theorem 3. 6. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . If the endomorphism N is parallel on M , then M is an invariant submanifold \tilde{M} .

Proof. If N is parallel, then from (32), we have

$$n\sigma(X, Y) - \sigma(X, TY) - \eta(Y)NX = 0,$$

for any $X, Y \in \Gamma(TM)$. Here, choosing $Y = \xi$ and taking into account that $\sigma(X, \xi) = 0$, (2) and (38), we conclude that $\sin^2 \theta \{g(X, X) - \eta^2(X)\} = 0$. This proves our assertion.

Theorem 3. 7. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . Then the tensor N is parallel if and only if the tensor t is parallel.

Proof. By using (32) and (33), we have

$$\begin{aligned} g((\nabla_X N)Y, V) &= g(n\sigma(X, Y), V) - g(\sigma(X, TY), V) - \eta(Y)g(NX, V) \\ &= -g(h(X, Y), nV) - g(A_V X, TY) - g(NX, V)\eta(Y) \\ &= -g(A_{nV} X, Y) + g(TA_V X, Y) - g(NX, V)\eta(Y) \\ &= -g(-A_{nV} X + TA_V X - g(NX, V)\xi, Y) \\ &= -g((\nabla_X t)V, Y), \end{aligned}$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$. This proves our assertion.

Theorem 3. 8. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . The derivation of T is skew-symmetric, that is



$$g((\nabla_X T)Y, Z) = -g((\nabla_X T)Z, Y)$$

for any $X, Y, Z \in \Gamma(TM)$.

Proof. For any $X, Y, Z \in \Gamma(TM)$, by using (9), (27), (31), we obtain

$$\begin{aligned} g((\nabla_X T)Y, Z) &= g(A_{NY}X + t\sigma(X, Y) + g(TX, Y)\xi - \eta(Y)TX, Z) \\ &= g(\sigma(X, Z), NY) - g(\sigma(X, Y), NZ) + \eta(Z)g(TX, Y) - \eta(Y)g(TX, Z) \\ &= -g(t\sigma(X, Z), Y) - g(A_{NZ}X, Y) + \eta(Z)g(TX, Y) - \eta(Y)g(TX, Z) \\ &= -g(A_{NZ}X + t\sigma(X, Z) + g(TX, Z)\xi - \eta(Z)TX, Y) \\ &= -g((\nabla_X T)Z, Y). \end{aligned}$$

This complete of the proof.

Theorem 3. 9. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . Then we have the following assertion

$$g((\nabla_X n)V, U) = -g((\nabla_X n)U, V),$$

for any $X \in \Gamma(TM)$ and $V, U \in \Gamma(T^\perp M)$.

Proof. For any $X \in \Gamma(TM)$ and $V, U \in \Gamma(T^\perp M)$, from (9), (27) and (34), we reach

$$\begin{aligned} g((\nabla_X n)V, U) &= -g(-\sigma(tV, X) - NA_V X, U) \\ &= -g(-A_U X, tV) + g(A_V X, tU) \\ &= g(NA_U X, V) + g(\sigma(X, tU), V) \\ &= -g(-NA_U X - \sigma(X, tU), V) \\ &= -g((\nabla_X n)U, V). \end{aligned}$$

This proves our assertion.

Theorem 3. 10. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . Then the tensor n is parallel if and only if the shape operator A_V of M satisfies the condition

$$A_V tU = A_U tV, \tag{47}$$

for all $V, U \in \Gamma(T^\perp M)$.

Proof. For all $V, U \in \Gamma(T^\perp M)$, from (27) and (34), we have

$$\begin{aligned} 0 &= -g(\sigma(tV, X), U) - g(NA_V X, U) \\ &= -g(A_U tV, X) + g(A_V X, tU) \\ &= g(A_V tU - A_U tV, X), \end{aligned}$$

for all $X \in \Gamma(TM)$. The proof is complete.

Theorem 3. 11. Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold \tilde{M} . If n is parallel then, M is totally geodesic submanifold of \tilde{M} .

Proof. If n is parallel, from (34) and (19), we have

$$\sigma(tV, X) + \varphi A_V X = 0 \tag{48}$$

for all $X \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$. Applying φ to (48) and taking into account (1) and (35), we obtain



$$\begin{aligned} 0 &= \varphi^2 A_V X + \varphi \sigma(tV, X) \\ &= -A_V X + \eta(A_V X) \xi + t\sigma(tV, X) + n\sigma(TV, X). \end{aligned}$$

This yields to

$$-A_V X + t\sigma(tV, X) = 0. \quad (49)$$

On the other hand, also by using (9), (22), (27) and (47), we conclude that

$$\begin{aligned} g(A_V X, Z) &= g(t\sigma(tV, X), Z) = -g(\sigma(tV, X), NZ) \\ &= -g(A_{NZ} tV, X) = -g(A_V tNZ, X) \end{aligned}$$

for $Z \in \Gamma(TM)$. Taking into account of $tNZ = -Z + \eta(Z)\xi - T^2Z$, we obtain

$$\begin{aligned} g(A_V Z, X) &= -g(-A_V Z + \eta(Z)A_V \xi - A_V T^2Z, X) \\ &= g(A_V Z, X) + g(A_V X, T^2Z) \\ &= g(A_V X + T^2A_V X, Z). \end{aligned}$$

Here, by using (21), we conclude

$$\begin{aligned} 0 &= g(A_V X, T^2Z) = g(A_V X, -Z + \eta(Z)\xi - tNZ) \\ &= -g(A_V X, Z) + g(NA_V X, NZ) \\ &= -\cos^2 \theta g(A_V X, Z) \end{aligned}$$

for all $Z \in \Gamma(TM)$. Since M is a proper pseudo-slant submanifold, we arrive at $A_V = 0$, that is, M is totally geodesic in \tilde{M} .

4. PSEUDO-SLANT SUBMANIFOLD IN KENMOTSU SPACE FORMS

In this section, we will study pseudo-slant submanifolds in a Kenmotsu space form, give some characterization and submanifold is characterized.

Let $\left\{ e_1, e_2, \dots, e_p, e_{p+1} = \sec \theta T e_1, e_{p+2} = \sec \theta T e_2, \dots, \right.$ be an orthonormal basis of $\Gamma(TM)$ such that

$\left. \left\{ e_{2p} = \sec \theta T e_p, e_{2p+1} = \xi, e_{2p+2}, e_{2p+3}, \dots, e_{2p+q+1} \right\} \right\}$ are tangent to $\Gamma(D_\theta)$ and $\left\{ e_{2p+2}, e_{2p+3}, \dots, e_{2p+q+1} \right\}$ are tangent $\Gamma(D^\perp)$.

Theorem 4. 1. Let M be a pseudo-slant submanifold of a Kenmotsu manifold space form $\tilde{M}(c)$ such that $c \neq -1$. If M is a pseudo-slant curvature-invariant submanifold, then M is either semi-invariant or anti-invariant submanifold.

Proof. We suppose that M is a pseudo-slant curvature-invariant submanifold of a Kenmotsu space form $\tilde{M}(c)$ such that $c \neq -1$. Then from (30), we have

$$g(X, TZ)NY - g(Y, TZ)NX + 2g(X, TY)NZ = 0, \quad (50)$$

for any $X, Y, Z \in \Gamma(TM)$. Taking $X = Z$ in equation (50), we have

$$0 = -g(Y, TZ)NZ + 2g(Z, TY)NZ = -3g(TZ, Y)NZ. \quad (51)$$

Here taking $Y = TZ$ in equation (51), we have



$$g(TZ, TZ)NZ = 0.$$

From (37) and (38), we obtain

$$\cos^2 \theta \sin^2 \theta \{g(Z, Z) - \eta^2(Z)\}^2 = 0.$$

This implies that $\sin 2\theta \{g(Z, Z) - \eta^2(Z)\} = 0$, that is, M is either a semi-invariant or an anti-invariant submanifold. Thus the proof is complete.

Theorem 4. 2. Let M be a pseudo-slant submanifold of a Kenmotsu manifold of a Kenmotsu space form $\tilde{M}(c)$ with flat normal connection such that $c \neq -1$. If $TA_V = A_V T$ for any vector V normal to M , then M is either an anti-invariant or generic submanifold of $\tilde{M}(c)$.

Proof. If the normal connection of M is flat, then from (28), we have

$$g([A_U, A_V]X, Y) = \frac{(c+1)}{4} \{g(X, \phi V)g(U, \phi Y) - g(Y, \phi V)g(\phi X, U) + 2g(X, \phi Y)g(\phi V, U)\}$$

for all $X, Y \in \Gamma(TM)$ and $V, U \in \Gamma(T^\perp M)$. Here, choosing $U = nV$ and $Y = TX$, by direct calculations, we can state

$$g([A_U, A_{nV}]X, TX) = -\frac{(c+1)}{2} \{g(TX, TX)g(nV, nV)\},$$

that is,

$$g(A_{nV}A_V TX - A_V A_{nV} TX, X) = -\frac{(c+1)}{2} \{g(TX, TX)g(nV, nV)\},$$

from which

$$tr(A_{nV}A_V T) - tr(A_V A_{nV} T) = \frac{(c+1)}{2} tr(T^2)g(nV, nV).$$

If $TA_V = A_V T$ and because of T is skew-symmetric, then we conclude that $tr(A_{nV}A_V T) = tr(A_V A_{nV} T)$ and thus

$$\frac{(c+1)}{2} tr(T^2)g(nV, nV),$$

from here $\dim(TM) = 2p + q + 1$, the we can easily to see that

$$(2p + q + 1) \cos^2 \theta \frac{(c+1)}{2} g(nV, nV) = 0.$$

Thus θ is either vanishes or $n = 0$. This implies that M is either an anti-invariant or it is a generic submanifold.

Theorem 4. 3. Let M be a pseudo-slant submanifold of a Kenmotsu manifold of a Kenmotsu space form $\tilde{M}(c)$. There exist no totally umbilical proper pseudo-slant submanifolds in a Kenmotsu space form $\tilde{M}(c)$ such that $c \neq -1$.

Proof. We suppose that M is totally umbilical pseudo-slant submanifold in Kenmotsu space form $\tilde{M}(c)$. Since every totally umbilical submanifold in Kenmotsu manifold is totally geodesic, we have



$$g(\tilde{R}(X, Y)Z, \varphi Z) = g((\tilde{\nabla}_X \sigma)(Y, Z) - (\tilde{\nabla}_Y \sigma)(X, Z), \varphi Z) = 0 \tag{52}$$

for any $X, Y \in \Gamma(D_\theta)$ and $Z \in \Gamma(D^\perp)$. Since the ambient space M is a Kenmotsu space form, from (6) we infer

$$g(\tilde{R}(X, Y)Z, \varphi Z) = \frac{(c+1)}{2} g(X, \varphi Y)g(NZ, NZ). \tag{53}$$

Taking $Y = TX$ in equation (53), we have

$$\frac{(c+1)}{2} g(X, \varphi TX)g(NZ, NZ) = 0. \tag{54}$$

Here, taking $X = Z$ in equation (54). From, (37) and (38), we obtain

$$\cos^2 \theta \sin^2 \theta \{g(Z, Z) - \eta^2(Z)\}^2 = 0.$$

This implies that $\sin 2\theta \{g(Z, Z) - \eta^2(Z)\} = 0$, that is, M is either a semi-invariant or an anti-invariant submanifold. This proves our assertion.

Theorem 4. 4. Let M be a pseudo-slant submanifold of a Kenmotsu manifold of a Kenmotsu space form $\tilde{M}(c)$. Then the Ricci tensor S of M is given by

$$\begin{aligned} S(X, W) = & \left\{ \frac{(c-3)}{4} (2p+q) + \frac{(c+1)}{4} (3\cos^2 \theta - 1) \right\} g(X, W) \\ & + \frac{(c+1)}{4} (1-q-2p-3\cos^2 \theta) \eta(X)\eta(W) \\ & + (2p+q+1)g(\sigma(X, W), H) - \sum_{m=1}^{2p+q} g(\sigma(e_m, W), \sigma(X, e_m)) \end{aligned} \tag{55}$$

for any $X, W \in \Gamma(TM)$.

Proof. For any $X, Y, Z \in \Gamma(TM)$, by using (6) and (12), we have

$$\begin{aligned} g(R(X, Y)Z, W) = & \frac{(c-3)}{4} \{g(Y, Z)g(X, W) - g(X, Z)g(Y, W)\} \\ & + \frac{(c+1)}{4} \{\eta(X)\eta(Y)g(Y, W) - \eta(Y)\eta(Z)g(X, W) \\ & + \eta(Y)\eta(W)g(X, Z) - \eta(X)\eta(W)g(Y, Z) \\ & + g(X, \varphi Y)g(\varphi Y, W) - g(Y, \varphi Z)g(\varphi X, W) \\ & + 2g(X, \varphi Y)g(\varphi Z, W)\} + g(\sigma(X, W), \sigma(Y, Z)) \\ & - g(\sigma(Y, W), \sigma(X, Z)). \end{aligned}$$

Let $\left\{ \begin{aligned} e_1, e_2, \dots, e_p, e_{p+1} = \sec \theta T e_1, e_{p+2} = \sec \theta T e_2, \dots, \\ e_{2p} = \sec \theta T e_p, e_{2p+1} = \xi, e_{2p+2}, e_{2p+3}, \dots, e_{2p+q+1} \end{aligned} \right\}$ be an orthonormal basis of $\Gamma(TM)$ such that



$\left\{ e_1, e_2, \dots, e_p, e_{p+1} = \sec \theta T e_1, e_{p+2} = \sec \theta T e_2, \dots, e_{2p} = \sec \theta T e_p, e_{2p+1} = \xi \right\}$ are tangent to $\Gamma(D_\theta)$ and $\{e_{2p+2}, e_{2p+3}, \dots, e_{2p+q+1}\}$ are tangent $\Gamma(D^\perp)$. Hence, taking $Y = Z = e_i, e_j, e_k, \xi$ and $1 \leq i \leq p, p+1 \leq j \leq 2p, \xi, 2p+2 \leq k \leq 2p+q+1$ then, we obtain

$$S(X, W) = \sum_{i=1}^p g(R(X, e_i)e_i, W) + \sum_{j=p+1}^{2p} g(R(X, \sec \theta T e_j) \sec \theta T e_j, W) + g(R(X, \xi)\xi, W) + \sum_{k=2p+2}^{2p+q+1} g(R(X, e_k)e_k, W).$$

Here

$$g(R(X, e_i)e_i, W) = \frac{(c-3)}{4} \{pg(X, W) - g(X, e_i)g(e_i, W)\} + \frac{(c+1)}{4} \{-p\eta(X)\eta(W) + 3g(TX, e_i)g(e_i, TW)\} + g(\sigma(X, W), \sigma(e_i, e_i)) - g(\sigma(e_i, W), \sigma(X, e_i)), \tag{56}$$

$$g(R(X, \sec \theta T e_j) \sec \theta T e_j, W) = \frac{(c-3)}{4} \{pg(X, W) - g(X, \sec \theta T e_j)g(W, \sec \theta T e_j)\} + \frac{(c+1)}{4} \{-p\eta(X)\eta(W) + 3g(TX, \sec \theta T e_j)g(\sec \theta T e_j, TW)\} + g(\sigma(X, W), \sigma(\sec \theta T e_j, \sec \theta T e_j)) - g(\sigma(\sec \theta T e_j, W), \sigma(X, \sec \theta T e_j)), \tag{57}$$

$$g(R(X, \xi)\xi, W) = \frac{(c-3)}{4} \{g(X, W) - g(X, \xi)g(\xi, W)\} + \frac{(c+1)}{4} \{\eta(X)\eta(W) - g(X, W)\} \tag{58}$$

and

$$g(R(X, e_k)e_k, W) = \frac{(c-3)}{4} \{qg(X, W) - g(X, e_k)g(e_k, W)\} + \frac{(c+1)}{4} \{-\eta(X)\eta(W)q + 3g(TX, e_k)g(e_k, TW)\} + \{g(\sigma(X, W), \sigma(e_k, e_k)) - g(\sigma(e_k, W), \sigma(X, e_k))\}. \tag{59}$$

From here



$$\begin{aligned} \sum_{m=1}^{2p+q} g(\sigma(e_m, W), \sigma(X, e_m)) &= \sum_{i=1}^p g(\sigma(e_i, W), \sigma(X, e_i)) \\ &+ \sum_{j=p+1}^{2p} g(\sigma(\sec \theta T e_j, W), \sigma(X, \sec \theta T e_j)) \\ &+ \sum_{k=2p+2}^{2p+q+1} g(\sigma(e_k, W), \sigma(X, e_k)). \end{aligned}$$

Hence, equation (56), (57), (58) and (59), we obtain

$$\begin{aligned} S(X, W) &= \left\{ \frac{(c-3)}{4} (2p+q) + \frac{(c+1)}{4} (3\cos^2 \theta - 1) \right\} g(X, W) \\ &+ \frac{(c+1)}{4} (1-q-2p-3\cos^2 \theta) \eta(X) \eta(W) \\ &+ (2p+q+1) g(\sigma(X, W), H) - \sum_{m=1}^{2p+q} g(\sigma(e_m, W), \sigma(X, e_m)). \end{aligned}$$

This complete the proof.

Thus we have the following corollary.

Corollary 4. 1. Every totally umbilical pseudo-slant submanifold M of a Kenmotsu space form $\tilde{M}(c)$ is an η -Einstein submanifold. Since totally umbilical pseudo-slant submanifold of a Kenmotsu minimal.

Theorem 4. 5. Let M be a pseudo-slant submanifold of a Kenmotsu manifold of a Kenmotsu space form $\tilde{M}(c)$. Then the scalar curvature ρ of M is given by

$$\begin{aligned} \rho &= \left\{ \frac{(c-3)}{4} (2p+q) + \frac{(c+1)}{4} (3\cos^2 \theta - 1) \right\} (2p+q+1) \\ &- \frac{(c+1)}{4} (3\cos^2 \theta + 2p+q-1) + (2p+q+1)^2 \|H\|^2 - \|\rho\|^2. \end{aligned} \tag{60}$$

Proof. From (55) by using $X = W = e_m$, we have

$$\rho = \sum_{m=1}^{2p+q+1} S(e_m, e_m)$$

which gives (60). Thus the proof is complete.

Thus we have the following corollary.

Corollary 4. 2. Let M be a pseudo-slant submanifold of a Kenmotsu manifold of a Kenmotsu space form $\tilde{M}(c)$. Then scalar curvature ρ of M is given by

$$\begin{aligned} \rho &= \left\{ \frac{(c-3)}{4} (2p+q) + \frac{(c+1)}{4} (3\cos^2 \theta - 1) \right\} (2p+q+1) \\ &- \frac{(c+1)}{4} (3\cos^2 \theta + 2p+q-1). \end{aligned}$$

Example 4. 1. Let M be a submanifold of \square^7 defined by the equation

$$(\sqrt{3}u, v, v \sin \alpha, v \cos \alpha, w \cos \beta, -w \cos \beta, z).$$

We can easily to see that the tangent bundle of M is spanned by the tangent vectors



$$e_1 = \sqrt{3} \frac{\partial}{\partial x_1}, \quad e_2 = \frac{\partial}{\partial y_1} + \sin \alpha \frac{\partial}{\partial x_2} + \cos \alpha \frac{\partial}{\partial y_2}, \quad e_3 = \cos \beta \frac{\partial}{\partial x_3} - \cos \beta \frac{\partial}{\partial y_3},$$

$$e_4 = -w \sin \beta \frac{\partial}{\partial x_3} + w \sin \beta \frac{\partial}{\partial y_3}, \quad e_5 = \xi = \frac{\partial}{\partial z}.$$

For the almost contact structure φ of \square^7 whose coordinate systems $(x_1, y_1, x_2, y_2, x_3, y_3, z)$ choosing

$$\varphi\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial y_i}, \quad \varphi\left(\frac{\partial}{\partial y_j}\right) = -\frac{\partial}{\partial x_j}, \quad 1 \leq i, j \leq 3$$

then, we have

$$\varphi e_1 = \sqrt{3} \frac{\partial}{\partial y_1}, \quad \varphi e_2 = -\frac{\partial}{\partial x_1} + \sin \alpha \frac{\partial}{\partial y_2} - \cos \alpha \frac{\partial}{\partial x_2},$$

$$\varphi e_3 = \cos \beta \frac{\partial}{\partial y_3} + \cos \beta \frac{\partial}{\partial x_3}, \quad \varphi e_4 = -w \sin \beta \frac{\partial}{\partial y_3} - w \sin \beta \frac{\partial}{\partial x_3}.$$

By direct calculations, we can infer $D_\theta = \text{span}\{e_1, e_2\}$ is a slant distribution with slant angle

$$\cos \theta = \frac{g(e_2, \varphi e_1)}{\|e_2\| \|\varphi e_1\|} = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ.$$

Since

$$g(\varphi e_3, e_1) = g(\varphi e_3, e_2) = g(\varphi e_3, e_4) = g(\varphi e_3, e_5) = 0,$$

$$g(\varphi e_4, e_1) = g(\varphi e_4, e_2) = g(\varphi e_4, e_3) = g(\varphi e_4, e_5) = 0$$

orthogonal to M , $D^\perp = \text{span}\{e_3, e_4\}$ is an anti-invariant distribution. Thus M is a 5-dimensional proper pseudo-slant submanifold of \square^7 with its usual almost contact metric structure.

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