A Solutıon Algorıthm for Interval Transportatıon Problems vıa Tıme-Cost Tradeoff

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Abstract

In this paper, an algorithm for solving interval time-cost tradeoff transportation problemsis presented. In this problem, all the demands are defined as intervalto determine more realistic duration and cost. Mathematical methods can be used to convert the time-cost tradeoff problems to linear programming, integer programming, dynamic programming, goal programming or multi-objective linear programming problems for determining the optimum duration and cost. Using this approach, the algorithm is developed converting interval time-cost tradeoff transportation problem to the linear programming problem by taking into consideration of decision maker (DM).

Indexing terms/Keywords: time-cost tradeoff, transportation problem, interval linear programming, decision making.

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Introduction

Optimization plays a major role for decision making. There are many scientific methods that can be selected for finding the best value of objective function and satisfy the conditions. Two major divisions, linear and nonlinear programming problems, optimize an objective function subject to a linear or/and nonlinear constraints. Interval arithmetic was introduced as an approach to bound rounding errors in the most of scientific and numerical computations that are based on conventional operations underlying the real arithmetic. The interval analysis method was developed for uncertain optimization problems when the bounds of the uncertain coefficients and constant terms are known. If they are taken as close interval, it will be considered as interval-valued optimization problems that are matched with real problems. Tanaka et.all [9] first proposed the concept of fuzzy optimization on general interval. Zimmerman [11] proposed the first approach of the fuzzy linear programming.

In traditional mathematical programming problems, parameters and variables are considered as crisp numbers. However, in the real world, regarding these parameters as precise is an unrealistic assumption. Therefore, to obtain more realistic results, fuzzy and stochastic approaches are used to determine imprecise and uncertain parameters in a real life problem. Because of having no information to construct membership function or probability distribution in reality, decision makers prefer using interval parameters. The interval linear programming is a method for decision making under uncertainty. Many researchers worked on several cases of linear programming problem with interval parameters. Senguptaet all [8] defined an interval linear programming problem as an extension of classical linear programming problem. Hladík [4] dealt with a linear programming problem in which input data varying in some given real compact intervals, and aimed to compute the exact range of the optimal value function. Luo and Li [6] introduced new concepts of optimal solutions of interval linear programming problems. Allahdadi and Nehi [1] determined optimal solution set of the interval linear programming problem as the intersection of some regions, by the best and the worst case methods, when the feasible solution components of the best problem are positive, and aimed to determine this optimal solution set by the best and the worst problem constraints.

Generally, time-cost tradeoff problems are known as one of the main aspects of project scheduling. This is because schedule planners want to find the most cost-effective way to complete a project within a desirable completion time. Therefore, for completing the project with the least possible time and cost, obtaining a tradeoff between cost and time of project is worthy. Several mathematical models have been developed to solve these time-cost tradeoff problems. In recent years, schedule planners have paid attention to uncertain scheduling and some have claimed that fuzzy set theory is moreappropriate for modelling such problems. Leuet all [5] proposed a new method using fuzzy set theory modelling the uncertainties of activity durations. Using genetic algorithms, a searching technique is adopted to obtain the optimal project time-cost tradeoff profiles. Chao-Guanget all [2] proposed a new solution approach for fuzzy time-cost tradeoff problems based on genetic algorithms. Ghazanfariet all [3] presented a new model for solving time-cost tradeoff problem in fuzzy environment, and they developed a new solution method for possibility goal programming problems.

In this paper, we present the tradeoffs between the total cost and the completion time to transport all the demands defined as interval, and an algorithm for solving interval time-cost tradeoff transportation problems. The main objective of such problems is to determine the optimum duration and the cost using the mathematical methods converting the time-cost tradeoff problems to linear programming, integer programming, dynamic programming, goal programming or multi-objective linear programming problems. Many models have focused on deterministic situations. However, during the transportation implementation, many uncertain variables affect the costs and the duration. Therefore, considering the demands as interval would be beneficial to schedule more realistic duration and cost. From this point of view, the developed algorithm solves interval time-cost tradeoff transportation problems converting the linear programming problem by taking into consideration of decision maker.

This paper is organized as follows: preliminaries of interval arithmetic and mathematical formulation of the interval transportation problem is given in Section 2 and Section 3, respectively. In Section 4, the proposed algorithm is handled. Section 5 and Section 6 consist of numerical example and conclusion, respectively.

Preliminaries

In this section, some brief information about interval arithmetic is given.

Definition 1[10]: Let I be a class of all closed intervals in . Closed interval C is demonstrated as $C=[c_L,c_U]$ where c_L and c_U mean the lower and upper bounds of C, respectively.

Definition 2[10]: Let $C=[c_L,c_U]$ and $D=[d_L,d_U]$ be closed intervals in I. Then,

- $C+D=[c_L+d_L,c_U+d_U]$
- $-C=[-c_U,-c_L]$
- $C-D=[c_L-d_U,c_U-d_L]$
- $kC=\begin{cases}[kc_L,kc_U], & if\ k\geq 0\\ [kc_U,kc_L], & if\ k<0\end{cases}$

where k is a real number.

Definition 3[8]: Let $C=[c_L,c_U]$ and $D=[d_L,d_U]$ be closed intervals in I. $C=D$ if $c_L=d_L$ and $c_U=d_U$.

Mathematical Formulation of IntervalTransportation Problem

Sets:

I : The set of sources, $i=1,\ldots,m$

J : The set of destinations, $j=1,\ldots n$

Parameters:

c_{ij} : Unit cost of transported product from the source i to the destination j

s_l^i : The lower bound of the units of the product available at source i

s_u^i : The upper bound of the units of the product available at source i

d_l^j : The lower bound of demand at the destination j

d_u^j : The upper bound of demand at the destination j

Decision Variables:

x_{ij} : The number of units of the product transported from the source i to the destination j

t_{ij} : Unit time of transported products from the source i to the destination j

y_i : Available capacity level of source i

z_j : Quantity level of desirable demand of destination j

Destinations / Sources	$j=1$	$\cdots$	$j=n$	
$i=1$	t_{11} x_{11} c_{11}	$\cdots$	t_{1n} x_{1n} c_{1n}	$\left[s_l^1, s_u^1\right]$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$i=m$	t_{m1} x_{m1} c_{m1}	$\cdots$	t_{mn} x_{mn} c_{mn}	$\left[s_l^m, s_u^m\right]$
	$\left[d_l^1, d_u^1\right]$	$\cdots$	$\left[d_l^n, d_u^n\right]$	$\left[\sum_{j=1}^{n} d_l^j, \sum_{j=1}^{n} d_u^j\right] = \left[\sum_{i=1}^{m} s_l^i, \sum_{i=1}^{m} s_u^i\right]$

Figure 1. Table of interval transportation problem

The mathematical model of time-cost tradeoff interval transportation problem is given below:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1.a}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = s_l^i + \left(s_u^i - s_l^i\right) y_i \quad , \forall i \tag{1.b}$$

$$\sum_{i=1}^{m} x_{ij} = d_l^j + \left(d_u^j - d_l^j\right) z_j \quad , \forall j \tag{1.c}$$

$$y_i \leq 1 \quad , \forall i \tag{1.d}$$

$$z_j \leq 1 \quad , \forall j \tag{1.e}$$

$$x_{ij} \geq 0 \quad , \forall i, \forall j \tag{1.f}$$

$$\left[\sum_{j=1}^{n} d_l^j, \sum_{j=1}^{n} d_u^j\right] = \left[\sum_{i=1}^{m} s_l^i, \sum_{i=1}^{m} s_u^i\right] \tag{1.g}$$

Here, constraint (1.b) states that the sum of products from specified source i to all destinations must be equal to the available capacity. Constraint (1.c) defines that the sum of products transported from all sources to particular destination j must be equal to desirable demand. Constraint (1.d) and (1.e) determine that available capacity level of source i and quantity level of desirable demand of destination j. Constraint (1.f) guarantees a reliable solution space. Constraint (1.g) applies when the total availability equals to the total demand.

Proposed Algorithm of Interval Transportation Problem

Step 1. Solve transportation LP problem (1) and find $x_{ij}\ (i=1,\ldots,m),(j=1,\ldots,n)$.

Step 2. Determine

$$T_k = \max\left\{t_{ij} : x_{ij} \neq 0, i=1,..,m, j=1,\ldots,n\right\}$$

$$C_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

$$P_k = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}$$

where T_k, C_k, P_k are transportation time, total transportation cost and quantity of transported products, respectively and k is the iteration number, $k=1,\ldots,l$.

Step 3. The k[th] proposal is offered as (T_k, C_k, P_k).

Step 4. Decision maker wants to transport more products in a short time determined T_k days. Therefore, assign big M number to the cost c_{ij} satisfying $t_{ij} \geq T_k$, i.e., $c_{ij} = M >> 0$.

Step 5. Solve the obtained LP problem.

Step 6. The (k+1)[th] proposal is offered as $(T_{k+1}, C_{k+1}, P_{k+1})$.

Step 7. Compare successive transportation times: If $T_{k+1} < T_k$, assign the value of T_{k+1} to the T_k and go to Step 4. Else, continue.

Step 8. All proposals are offered to decision maker.

Step 9. If decision maker prefers one of the proposals, it is accepted the best decision and STOP. Else, continue.

Step 10. Determine points

$$(x_k, y_k) = \left(\frac{P_k}{T_k}, \frac{C_k}{T_k}\right) \quad , k=1,\ldots,l-1$$

Step 11. Find a linear equation using Least Square Method.

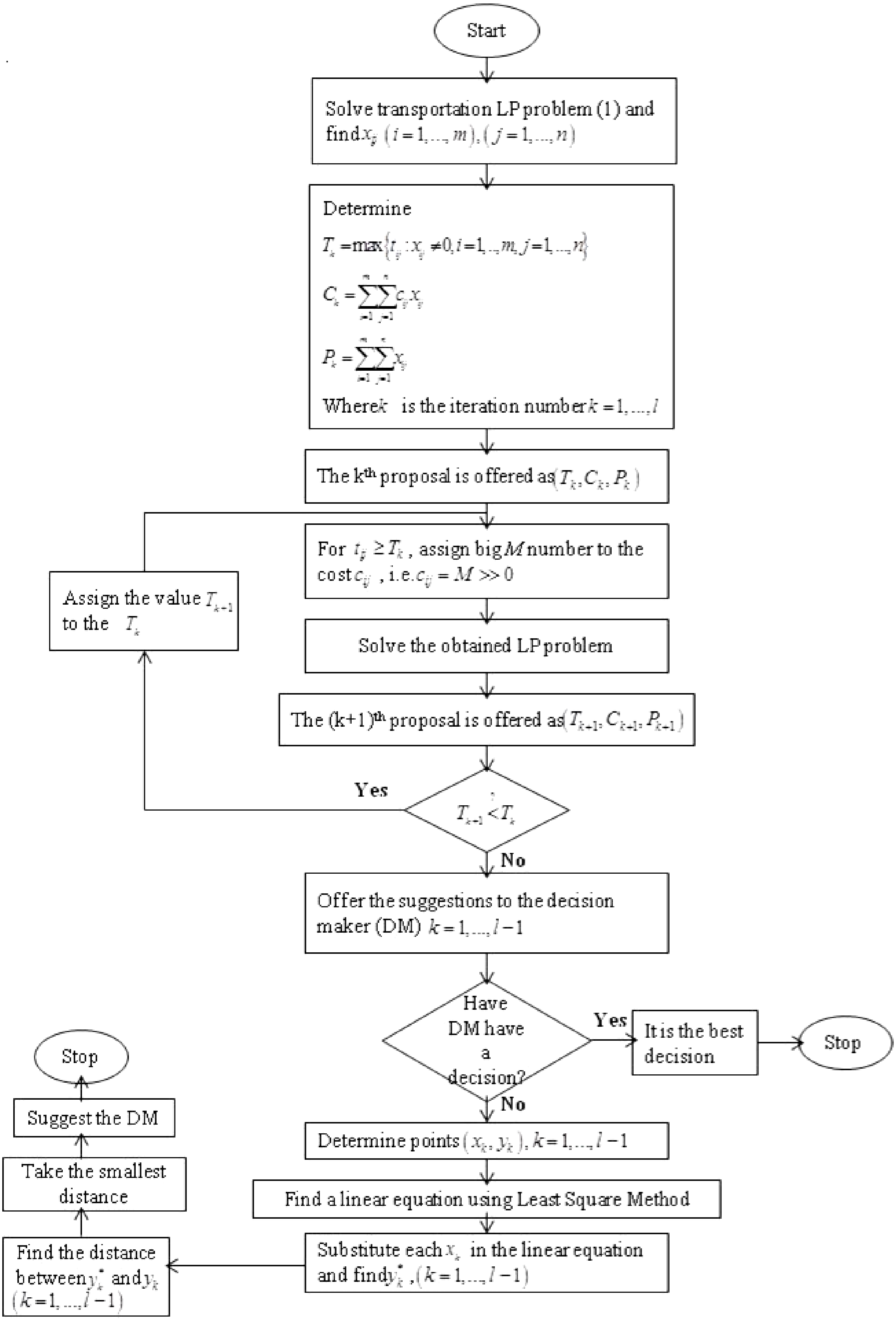
Step 12. Substitute each $x_k\ (k=1,\ldots,l-1)$ in this linear equation and find points $y_k^*\ (k=1,\ldots,l-1)$.

Step 13. Find distances between y_k^* and $y_k\ (k=1,\ldots,l-1)$.

Step 14. Suggest the related proposal corresponding to the smallest distance offering to the decision maker and STOP.

Flow chart of the proposed algorithm is given in Figure 2.

Figure 2. The flow chart of the algorithm



Numerical Experiment

Transportation times (day) and unit costs (dollar) of each cell is given in Figure 3.

Destinations / Sources	$j=1$	$j=2$	$j=3$	$j=4$	
$i=1$	13 5	9 7	17 8	21 11	$[16,20]$
$i=2$	11 13	16 10	13 7	24 6	$[12,15]$
$i=3$	32 13	28 12	17 3	9 5	$[19,29]$
	$[7,11]$	$[15,20]$	$[9,13]$	$[16,20]$	

Figure 3. Transportation table of numerical experiment

Using the proposed algorithm, the mathematical model of the numerical experiment is given below:

$$\min \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} \tag{2.a}$$

subject to

$$\sum_{j=1}^{4} x_{ij} = s_l^i + \left(s_u^i - s_l^i\right) y_i \quad i=1,2,3 \tag{2.b}$$

$$\sum_{i=1}^{3} x_{ij} = d_l^j + \left(d_u^j - d_l^j\right) z_j \quad j=1,2,3,4 \tag{2.c}$$

$$y_i \le 1 \quad i=1,2,3 \tag{2.d}$$

$$z_j \le 1 \quad j=1,2,3,4 \tag{2.e}$$

1st iteration:

Step 1-2-3: The mathematical model of the numerical experiment is

$$\min \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij}$$

$$x_{11} + x_{12} + x_{13} + x_{14} - 4y_1 = 16$$

$$x_{21} + x_{22} + x_{23} + x_{24} - 3y_2 = 12$$

$$x_{31}+x_{32}+x_{33}+x_{34}-10y_3=19$$

$$x_{11}+x_{21}+x_{31}-4z_1=7$$

$$x_{12}+x_{22}+x_{32}-5z_2=15$$

$$x_{13}+x_{23}+x_{33}-4z_3=9$$

$$x_{14}+x_{24}+x_{34}-4z_4=16$$

$$y_1\le 1,\quad y_2\le 1,\quad y_3\le 1$$

$$z_1\le 1,\quad z_2\le 1,\quad z_3\le 1,\quad z_4\le 1$$

and the solution is obtained as

$$x_{11}=7,\ x_{12}=9,\ x_{22}=6,\ x_{24}=6,\ x_{33}=9$$

$$y_1=y_2=y_3=0$$

$$z_1=z_2=z_3=z_4=0$$

It is found that $T_1=\max\{t_{11},t_{12},t_{22},t_{24},t_{33}\}=24$ days, $C_1=271$ dollars, $P_1=47$ units. As a result, 1st proposal is offered as $(T_1,C_1,P_1)=(24,271,47)$.

2nd iteration:

Step 4:Because products are wanted to transport less than 24 days, assign a big number M to the cells c_{31} and c_{32}.

Step 5: The constraints will remain same because the decision variables $y_i\,(i=1,2,3)$ and $z_j\,(j=1,2,3,4)$ are found zero at the end of the 1st iteration. However, the objective function will change regarding as the alterations explained above. Obtained mathematical model is solved and the solution is obtained as

$$x_{11}=7,\ x_{12}=9,\ x_{22}=6,\ x_{23}=6,\ x_{33}=3,\ x_{34}=16$$

$$y_1=y_2=y_3=0$$

$$z_1=z_2=z_3=z_4=0$$

Step 6: $T_2=17$ days, $C_2=289$ dollars, $P_2=47$ units are found. As a result, 2st proposal is offered as $(T_2,C_2,P_2)=(17,289,47)$.

Step 7: Successive transportation time values should be compared for making a decision whether to continue iterations or not. Because the transportation time is found shorter than previous iteration regarding as identical transporting products, 3rd iteration can be applied.

3rd iteration:

Step 4: Because products are requested to transport less than 17 days, assign a big number M to the cells c_{14}, c_{24}, c_{31} and c_{32}.

Step 5: The constraints will remain same because the decision variables $y_i\left(i=1,2,3\right)$ and $z_j\left(j=1,2,3,4\right)$ are found zero at the end of 2nd iteration. However, the objective function willvariate because of the arised changes. Obtained mathematical model is solved and the solution is

$$x_{11}=7,\ x_{12}=12,\ x_{22}=3,\ x_{23}=9,\ x_{34}=19$$

$$y_1=0.75,\quad y_2=y_3=0$$

$$z_1=z_2=z_3=0\quad z_4=0.75$$

Step 6: It is found that $T_3=16$ days, $C_3=307$ dollars, $P_3=50$ units. As a result, 3rd proposal is offered as $\left(T_3, C_3, P_3\right)=\left(16, 307, 50\right)$.

Step 7: By comparing successive transportation time values, 4th iteration is applied.

4th iteration:

Step 4: Because products are wanted to transport less than 16 days, assign a big number M to the cells $c_{13}, c_{14}, c_{24}, c_{31}, c_{32}$ and c_{33}.

Step 5: The constraints are changed because the decision variables are found as $y_1=0.75$ and $z_4=0.75$ at the end of 3rd iteration. Thus, obtained mathematical model is solved and the solution is

$$x_{11}=5,\ x_{12}=15,\ x_{21}=2,\ x_{23}=10,\ x_{34}=19$$

$$y_1=y_2=y_3=0$$

$$z_1=z_2=z_3=z_4=0$$

Step 6: It is found that $T_4=13$ days, $C_4=321$ dollars, $P_4=51$ units and the 4th proposal is offered as $\left(T_4, C_4, P_4\right)=\left(13, 321, 51\right)$.

Step 7: By comparing successive transportation time values, 5th iteration is applied.

5th iteration:

Step 4: Because products are requested to transport less than 13 days, assign a big number M to the cells $c_{13}, c_{14}, c_{22}, c_{24}, c_{31}, c_{32}$ and c_{33}.

Step 5: The constraints will remain same because the decision variables $y_i\left(i=1,2,3\right)$ and $z_j\left(j=1,2,3,4\right)$ are found zero at the end of 4th iteration. Obtained mathematical model is solved and the solution is

$$x_{11}=15,\ x_{13}=1,\ x_{21}=7,\ x_{23}=5,\ x_{33}=3,\ x_{34}=16$$

$$y_1=y_2=y_3=0$$

$$z_1 = z_2 = z_3 = z_4 = 0$$

Step 6: $T_5 = 17$ days, $C_5 = 9276$ dollars, $P_5 = 51$ units, and the 5[th] proposal is offered as $(T_5, C_5, P_5) = (17, 9276, 51)$.

Step 7: Because it is seen that both the transportation time and cost increased in the 5[th] iteration, iterations are finalized.

Step 8: All proposals are offered to decision maker. List of proposals are given in Table 1.

Table 1. Proposals offered to decision maker

	Number of Product	Transportation Time	TransportationCost
1[st]proposal	47	24	271
2[nd]proposal	47	17	289
3[rd]proposal	50	16	307
4[th]proposal	51	13	321

Step 9-10. Because decision maker has no decision, the points (x_k, y_k), $k = 1,2,3,4$ are determined which are given in Table 2.

Table 2. Determined points

	(x_k, y_k)
$k=1$	$(11.29, 1.95)$
$k=2$	$(17, 2.76)$
$k=3$	$(19.18, 3.13)$
$k=4$	$(24.64, 3.92)$

Step 11. Using these points, a linear equation is formed as $y = 7.11x - 2.89$ by applying least squares method.

Step 12.Each $x_k, (k = 1,2,3,4)$ is substituted in the equation and new $y_k^*, (k = 1,2,3,4)$ points are obtained.

Step 13-14. Distance between y_k and $y_k^* \; (k = 1,2,3,4)$ is determined and the related proposal corresponding to the smallest distance is suggested to the decision maker, which is 3[rd] one.

Conclusion

In this paper, to transport all the demands the tradeoff between the total cost and the completion time is considered. For solving interval time-cost tradeoff problems, the main objective is to determine the optimum duration and cost by using the mathematical methods. The proposed algorithm solves the time-cost tradeoff transportation problem taking into consideration of decision maker by converting it to linear programming problem and considering the demands as interval.

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