Multi-Source Backlogged Probabilistic Inventory Model for Crisp and Fuzzy Environment

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Abstract

This paper proposed a multi-item multi-source probabilistic periodic review inventory model under a varying holding cost constraint with zero lead time when: (1) the stock level decreases at a uniform rate over the cycle. (2) some costs are varying. (3) the demand is a random variable that follows some continuous distributions as (two-parameter exponential, Kumerswamy, Gamma, Beta, Rayleigh, Erlang distributions).

The objective function under a constraint is imposed here in crisp and fuzzy environment. The objective is to find the optimal maximum inventory level for a given review time that minimize the expected annual total cost. Furthermore, a comparison between given distributions is made to find the optimal distribution that achieves the model under considerations. Finally, a numerical example is applied.

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1. Introduction

In the practical situation, some costs are relating to some variables such as quantity (Q)or length of the cycle (N) … etc.,. Many researchers dealing with inventory models with varying costs for example, Chu et al. [6] and Fergany [8] illustrated probabilistic multi-item inventory model with varying mixture shortage cost under restrictions. Fergany and El-Wakeel [10] illustrated probabilistic single item inventory problem with varying order cost under two linear constraints. Abuo - El-Ata et al. [1] introduced probabilistic multi-item inventory model with varying order cost under two restrictions using a geometric programming approach. And other researchers concern with periodic review model for example, Silver and Robb [14] presented the model with some insights regarding the optimal reorder period. Fergany et al. [9] illustrated the model using Lagrange multiplier technique and fuzzy adaptive particle swarm optimization. Chiang [5] illustrated the model with stochastic supplier's visit intervals. Chiang [4] developed optimal replenishment for the model with two supply modes. Chiang [3] presented optimal ordering policies for the model with a refined intra-cycle time scale. Chiang [2] developed the model with a refined delivery scenario. Konstantaras and Papachristos [11] introduced manufacturing and logistics optimal policy and holding cost stability regions in the model with manufacturing and remanufacturing options. Yuyue and Hoong [15] developed the model with application to the continuous review obsolescence problem.

The cost parameters in real inventory systems and other parameters such as price, marketing and service elasticity to demand are imprecise and uncertain in nature. Since the proposed model is in a fuzzy environment, a fuzzy decision should be made to meet the decision criteria, and the results should be fuzzy as well. Fuzzy sets introduced by many researchers as a mathematical way of representing impreciseness or vagueness in everyday life. Rong et al. [12] presented a multi-objective wholesaler–retailers inventory-distribution model with controllable lead-time based on probabilistic fuzzy set and triangular fuzzy number. Sadjadi et al. [13] introduced fuzzy pricing and marketing planning model using a geometric programming approach.

This paper is formulated a multi-item multi-source periodic review inventory problem with a varying holding cost constraint when the holding and backlogged costs are varying. Also, shortages are permitted but fully backlogged and the demand considered to be a random variable that follows some continuous distributions as (two-parameter exponential, Kumerswamy, Gamma, Beta, Erlang, Raylieph distributions) without lead time. Also, the cost parameters under a constraint is considered here in crisp and fuzzy environment. The problem has been solved by Lagrange multiplier technique. The objective is to find the optimal maximum inventory level for a given review time which minimize the expected annual total cost under a restriction. And a comparison between given distributions is made to find the optimal distribution that achieves the model under considerations The results of the numerical example are got by Mathematica program.

2. Notations

Parameters for the r^{th} $(r = 1,2,\dots,n)$ item, s^{th} $(s = 1,2,\dots,m)$ source are:

Q_{mrs}	the maximum inventory level for r^{th} item, s^{th} source (decision variable).
N	the time of review (the cycle).
x_{rs}	the demand of the r^{th} item , s^{th} source during the cycle N (random variable).
C_{ors}	the order cost per unit item for r^{th} item, s^{th} source
C_{prs}	the purchase cost for r^{th} item for r^{th} item, s^{th} source
C_{hr}	the holding cost per unit item for r^{th} item.
C_{brs}	the backlogged cost per unit item for r^{th} item, s^{th} source
$C_{hr}(N)$	the varying holding cost for the r^{th}per cycle $= C_{hr}N^{-\beta_r}$
$C_{brs}(N)$	the varying backlogged cost for the r^{th}per cycle $= C_{br}\ N^{\beta_r}$
$\overline{D}_r$	the expected demand per unit item $\overline{D}_r \equiv (\overline{D}_1, \overline{D}_2, \dots, \overline{D}_n)$
k_{hrs}	the goal associated to expected holding cost
$\tilde{C}_{ors}$	the fuzzy set up cost per unit item per unit time.
$\tilde{C}_{hr}$	the fuzzy holding cost per unit item per unit time.
$\tilde{C}_{brs}$	the fuzzy backlogged cost per unit item per unit time.
$\tilde{C}_{hr}(N)$	the fuzzy varying holding cost for the r^{th}per cycle $= \tilde{C}_{hr}\ N^{-\tilde{\beta}_r}$
$\tilde{C}_{brs}(N)$	the fuzzy varying backlogged cost for the r^{th}per cycle $= \tilde{C}_{br}\ N^{\tilde{\beta}_r}$
$\tilde{k}_{hrs}$	the fuzzy goal associated to expected holding cost
$E(TC)$	the expected total average cost function $E(\ TC(Q_{mrs}, N))$

3. The mathematical model

This model developed the stock level decreases at a uniform rate over the cycle. Figure 2 exhibits the inventory flow process two final conditions may arise as indicated in Figure 1.

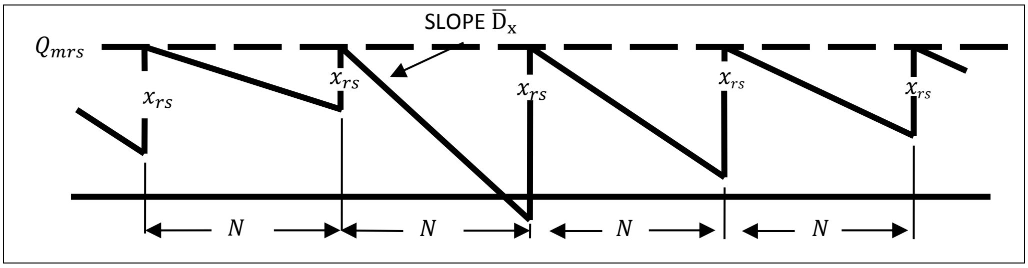

Figure (1): Inventory process with uniform demand

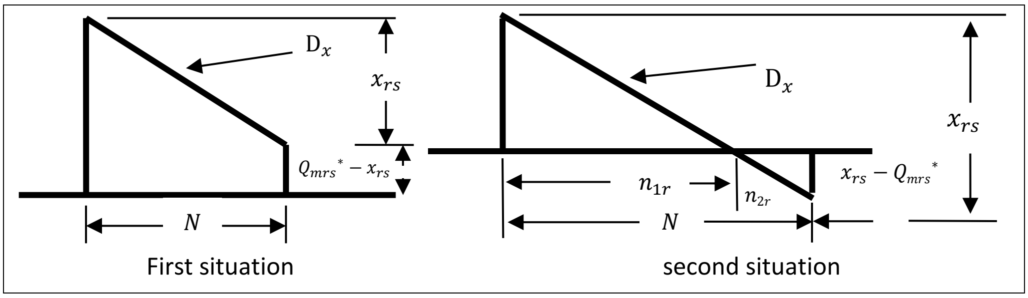

Figure (2): Two states of uniform model

The expected total cost of the cycle for multi-item multi source will be the sum of the expected purchase cost, the expected order cost, the expected varying holding cost, and the expected varying backlogged cost,

$$E(TC) = \sum_{r=1}^{n} [E(PC_{rs}) + E(OC_{rs}) + E(HC_{rs}(N)) + E(BC_{rs}(N))]$$

- The expected purchase cost for the cycle is given by

$$E(PC_{rs}) = C_{prs}\, \bar{x}_{rs} \tag{1}$$

- The expected order cost for the cycle is given by

$$E(OC_{rs}) = C_{ors} \tag{2}$$

- The expected varying holding cost for the cycle is given by

$$E(HC_{rs}) = C_{hr}(N)\, \bar{I} = C_{hr}\, N^{-\beta_r}\, \bar{I}$$

where $\bar{I}$ represents the expected average amount of inventory.

The first situation in Figure (2). If $\bar{x}_{rs} \leq Q_{mr}$. Then the average amount in inventory $\bar{I}$, is given by

$$\bar{I} = \frac{Q_{mrs} + (Q_{mrs} - x_{rs})}{2} = Q_{mrs} - \frac{x_{rs}}{2}$$

The second situation, the following relationships are evident:

$$n_{1r} = \frac{Q_{mrs}}{\overline{D}_r} \quad and \quad N = \frac{x_{rs}}{\overline{D}_r} \quad \text{then} \quad \frac{n_{1r}}{N} = \frac{Q_{mrs}}{x_{rs}}$$

hence, $$\bar{I} = \frac{Q_{mrs}}{2}\left(\frac{n_{1r}}{N}\right) \quad and \quad \bar{I} = \frac{Q_{mrs}{}^2}{2\, x_{rs}}$$

Thus, the expected average amount of inventory is given by

$$\bar{I} = \int_{x_{rs}=0}^{Q_{mrs}} \left(Q_{mrs} - \frac{x_{rs}}{2}\right) f(x_{rs})\, dx_{rs} + \int_{x=Q_{mrs}}^{\infty} \frac{Q_{mrs}{}^2}{2\, x_{rs}} f(x_{rs})\, dx_{rs}$$

- The varying expected varying holding cost for the cycle becomes

$$E(HC_{rs}) = C_{hr}\, N^{-\beta_r} \left(\int_{x_{rs}=0}^{Q_{mrs}} \left(Q_{mrs} - \frac{x_{rs}}{2}\right) f(x_{rs}) dx_{rs} + \int_{x=Q_{mrs}}^{\infty} \frac{Q_{mrs}{}^2}{2\, x_{rs}} f(x_{rs}) dx_{rs} \right) \tag{3}$$

- The expected varying backlogged cost for the cycle is given by $E(BC_{rs}) = C_{brs}\, N^{\beta_r}\, \bar{S}$

  where $\bar{S}$ represents the expected average backlogged.

In the first situation of Figure (2), the average backlogged is given by $\bar{S} = 0$

In the second situation, the following relationship is evident:

$$n_{2r} = \frac{x_{rs} - Q_{mrs}}{\overline{D}_r} \quad \text{Then,} \quad \frac{n_{2r}}{N} = \frac{x_{rs} - Q_{mrs}}{x_{rs}}$$

$$\bar{S} = \frac{x_{rs} - Q_{mrs}}{2}\left(\frac{n_{2r}}{N}\right) = \frac{(x_{rs} - Q_{mrs})^2}{2x_{rs}}$$

Thus, the expected average backlogged is

$$\bar{S} = \int_{x=Q_{mrs}}^{\infty} \frac{(x_{rs} - Q_{mrs})^2}{2x_{rs}} \, f(x_{rs})\, d(x_{rs})$$

- The expected varying backlogged cost for the cycle becomes

$$E(BC_{rs}) = C_{brs}\, N^{\beta_r} \int_{x=Q_{mrs}}^{\infty} \frac{(x_{rs} - Q_{mrs})^2}{2x_{rs}}\, f(x_{rs})\, d(x_{rs}) \tag{4}$$

Then, the expected total cost of the cycle for multi-item multi-source is the sum of Equations (1), (2), (3) and (4)

$$E(TC) = \sum_{r=1}^{n} \Bigg[C_{prs}\bar{x}_{rs} + C_{ors} + C_{hr}N^{-\beta_r} \left(\int_{x=0}^{Q_{mrs}} \left(Q_{mrs} - \frac{x_{rs}}{2}\right) f(x_{rs}) dx_{rs} \; + \int_{x=Q_{mrs}}^{\infty} \frac{{Q_{mrs}}^2}{2\, x_{rs}} f(x_{rs})\, dx_{rs} \right) + \; C_{brs}\, N^{\beta_r} \int_{x=Q_{mrs}}^{\infty} \frac{(x_{rs} - Q_{mrs})^2}{2x_{rs}}\, f(x_{rs})\, d(x_{rs}) \Bigg] \tag{5}$$

There is a limitation on the available expected varying holding cost;

$$E\big(HC(N)\big) = \sum_{r=1}^{n} C_{hr} N^{-\beta_r} \bar{I} \;\; \leq K_{hrs}$$

The problem is to find the optimal maximum inventory level for a given N which minimize the expected annual average total cost function (5) subject to the expected varying holding cost restriction. It may be written as

Min $E\,\big(TC(,Q_{mrs}, N)\big)$ for all $r = 1,2,\dots,n, \;\; s = 1,2,\dots,m$ (6)

subject to inequality constraint $E\big(HC(N)\big) \leq k_{hrs}$ (7)

To find the optimal values Q^{*}_{mrs} for a given N which minimize Equation (6) under the constraint (7), the Lagrange multipliers technique with the Kuhn-Tacker conditions is used, then the Lagrange function is given by:-

$$L(Q_{ms}, N)) = C_{prs}\bar{x}_{rs} + C_{ors} + C_{hr}N^{-\beta_r} \left(\int_{x=0}^{Q_{mrs}} \left(Q_{mrs} - \frac{x_{rs}}{2}\right) f(x_{rs})\, dx_{rs} + \int_{x=Q_{mrs}}^{\infty} \frac{{Q_{mrs}}^2}{2\, x_{rs}} f(x_{rs})\, dx_{rs} \right) + \; C_{brs}N^{\beta_r} \int_{x=Q_{mrs}}^{\infty} \frac{(x_{rs} - Q_{mrs})^2}{2x_{rs}}\, f(x_{rs})\, d(x_{rs}) + \lambda_{hrs} \left(C_{hr}\, N^{-\beta_r} \left\{ \int_{x=0}^{Q_{mrs}} \left(Q_{mrs} - \frac{x_{rs}}{2}\right) f(x_{rs})\, dx_{rs} \int_{x=Q_{mrs}}^{\infty} \frac{{Q_{mrs}}^2}{2\, x_{rs}} f(x_{rs})\, dx_{rs} \right\} - K_{hrs} \right) \tag{8}$$

where λ_{hr}, is the Lagrange multiplier.

The optimal values Q^{*}_{mrs} can be calculated by setting the corresponding first partial derivatives of Equation (8) equal to zero, and then the following equations are obtained.

$$\left. \frac{\partial E\big(TC(Q_{mrs}, N)\big)}{\partial Q_{mrs}} \right|_{Q_{mrs} \,=\, Q^{*}_{mrs}} = 0$$

$$\text{hence,} \int_{x_{rs}=0}^{Q^{*}_{mrs}} f(x_{rs})\, dx_{rs} + \int_{x_{rs}=Q^{*}_{mrs}}^{\infty} \frac{Q^{*}_{mrs}}{x_{rs}} f(x_{rs})\, dx_{rs} = \frac{C_{brs}\, N^{\beta_r}}{C_{brs}\, N^{\beta_r} + (1 + \lambda_{hrs})\, C_{hr}\, N^{-\beta_r}} \tag{9}$$

Special cases:

Unconstrained unconstraint single-item single source model when $\beta_r = 0, \lambda_{hrs} = 0$

$$\int_{x=0}^{Q^{*}_{m}} f(x)\, dx + \int_{x=Q_m}^{\infty} \frac{Q^{*}_{m}}{x} f(x)\, dx = \frac{C_s}{C_s + C_h} \tag{10}$$

Where C_s the shortage cost per unit item. (Fabrycky W. J. and Banks Jerry [7]).

4. The Model when all parameters are fuzzy numbers

The inventory cost coefficients, elasticity parameters and other coefficients in the model are fuzzy in nature. Therefore, the decision variable and the objective function should be fuzzy as well, it should find the right and the left shape functions of the objective function and decision variable, by find the upper bound and the lower bound of the objective function, i.e. $\tilde{L}^{L}(\propto)$ and$\tilde{L}^{R}(\propto)$. Recall that $\tilde{L}^{L}(\propto)$ and $\tilde{L}^{R}(\propto)$ represents the largest and the smallest values (The left and right $\propto$ cuts) of the optimal objective function $\tilde{L}(\propto)$. Using approximated value of TFN which observe in Figure 3

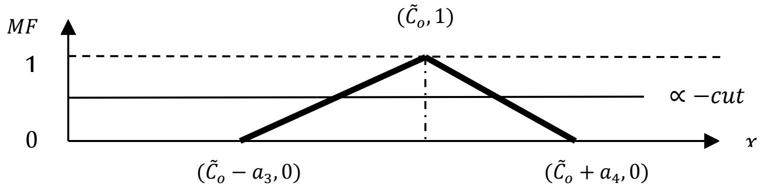

Figure 3: Order cost as triangular fuzzy number

Consider the model when all parameters are triangular fuzzy numbers (TFN) as given below

$$C_{prs} = (C_{pr} - a_{1r},\ C_{prs}\ , C_{prs} + a_{2r}), \qquad C_{ors} = (C_{ors} - a_{3r},\ C_{ors},\ C_{ors} + a_{4r}),$$
$$C_{hr} = (C_{hr} - a_{5r}\ , C_{hr},\ C_{hr} + a_{6r}), \quad \text{and} \quad C_{brs} = (C_{brs} - a_{7r},\ C_{brs},\ C_{brs} + a_{8r}),$$

where a_{ir} , $i = 1,2, \dots \dots ,8$ are arbitrary positive numbers under the following restrictions:

$0 \le a_{1r} \le C_{prs}, a_{2r} \ge 0$, $0 \le a_{3r} \le C_{ors}, a_{4r} \ge 0$, $0 \le a_{5r} \le C_{hr}, a_{6r} \ge 0$,and $0 \le a_{7r} \le C_{brs}, a_{8r} \ge 0$,

The left and right limits of $\propto cuts$ of $C_{prs}, C_{ors}, C_{hr}\ and\ C_{brs}$ are given by

$$\tilde{C}_{prs_L}(\propto) = C_{prs} - (1-\propto)a_{1r}, \qquad \tilde{C}_{prs_R}(\propto) = C_{prs} + (1-\propto)a_{2r},$$
$$\tilde{C}_{ors_L}(\propto) = C_{ors} - (1-\propto)a_{3r}, \qquad \tilde{C}_{ors_R}(\propto) = C_{ors} + (1-\propto)a_{4r},$$
$$\tilde{C}_{hr_L}(\propto) = C_{hr} - (1-\propto)a_{5r}, \qquad \tilde{C}_{hr_R}(\propto) = C_{hr} + (1-\propto)a_{6r},$$

and
$$\tilde{C}_{brs_L}(\propto) = C_{brs} - (1-\propto)a_{7r}, \qquad \tilde{C}_{brs_R}(\propto) = C_{brs} + (1-\propto)a_{8r}$$

where
$$\tilde{C}_{prs} = C_{prs} + \frac{1}{4}(a_{2r} - a_{1r}), \qquad \tilde{C}_{ors} = C_{ors} + \frac{1}{4}(a_{4r} - a_{3r}),$$
$$\tilde{C}_{hr} = C_{hr} + \frac{1}{4}(a_{6r} - a_{5r}) \quad \text{and} \quad \tilde{C}_{brs} = C_{brs} + \frac{1}{4}(a_{8r} - a_{7r})$$

Likewise, the same steps as in crisp case will be applied here with replacing $C_{prs}, C_{ors}, C_{hr}\ and\ C_{brs}$ by $\tilde{C}_{prs}, \tilde{C}_{ors}, \tilde{C}_{hr}\ and\ \tilde{C}_{brs}$. then the optimal value of Q^*_{mrs} for a given N which minimize expected annual total cost for fuzzy case can be calculated easily and the optimal value Q^*_{mrs} for fuzzy case can be calculated by the following equations:

$$\int_{x_{rs}=0}^{Q^*_{mrs}} f(x_{rs})\, dx_{rs} + \int_{x_{rs}=Q^*_{mrs}}^{\infty} \frac{Q_{mrs}}{x_{rs}} f(x_{rs})\, dx_{rs} = \frac{\tilde{C}_{brs}\, N^{\beta_r}}{\tilde{C}_{brs}\, N^{\beta_r} + (1+\lambda_{hrs})\, \tilde{C}_{hr}\, N^{-\beta_r}} \tag{14}$$

5. The model with some continuous distributions

Suppose that the demand for a particular item follows some continuous distribution such as:

5.1 The model with two-parameters exponential distribution: If the demand follows the two parameter exponential distribution then,

$f(x_{rs}) = \theta\, e^{-\theta\,(x_{rs}-\gamma)}, \quad \gamma < x_{rs} < \infty, \quad \theta > 0$

θ continuous inverse scale parameter $\theta > 0, \gamma$ continuous location parameter.

Hence, the optimal value Q^*_{mrs} can be calculated by the following equation

$$\theta\left(\int_{x_{rs}=\gamma}^{Q^*_{mrs}} e^{-\theta\,(x_{rs}-\gamma)}\, dx_{rs} + \int_{x_{rs}=Q^*_{mrs}}^{\infty} \frac{Q_{mrs}}{x_{rs}}\, e^{-\theta\,(x_{rs}-\gamma)}\, dx_{rs}\right) = \frac{C_{brs}\, N^{\beta_r}}{C_{brs}\, N^{\beta_r} + (1+\lambda_{hrs})\, C_{hr}\, N^{-\beta_r}}$$

5.2 The model with Kumaraswamy distribution: If the demand follows the Kumaraswamy distribution then,

$$f(x_{rs}) = \frac{\alpha_1\, \alpha_2 \left(\frac{x_{rs}-a}{b-a}\right)^{\alpha_1-1} \left(\left(\frac{x_{rs}-a}{b-a}\right)^{\alpha_1}\right)^{\alpha_2-1}}{b-a}, \quad a \le x_{rs} \le b,$$

α_1, α_2 continuous shape parameter $\alpha_1, \alpha_2 > 0, a, b$ continuous boundary parameters $a < b$.

Hence, the optimal value Q^*_{mrs} can be calculated by the following equation

$$\frac{\alpha_1\, \alpha_2}{b-a}\left[\int_{x_{rs}=a}^{Q^*_{mrs}} \left(\frac{x_{rs}-a}{b-a}\right)^{\alpha_1-1} \left(\left(\frac{x_{rs}-a}{b-a}\right)^{\alpha_1}\right)^{\alpha_2-1} dx_{rs} + \int_{x_{rs}=Q^*_{mrs}}^{b} \frac{Q_{mrs}}{x_{rs}} \left(\frac{x_{rs}-a}{b-a}\right)^{\alpha_1-1} \left(\left(\frac{x_{rs}-a}{b-a}\right)^{\alpha_1}\right)^{\alpha_2-1} dx_{rs}\right]$$
$$= \frac{C_{brs}\, N^{\beta_r}}{C_{brs}\, N^{\beta_r} + (1+\lambda_{hrs})\, C_{hr}\, N^{-\beta_r}}$$

5.3 The model with Gamma distribution: If the demand follows the Gamma distribution then,

$$f(x_{rs}) = \frac{(x_{rs}-\delta)^{\alpha-1}}{\sigma^{\alpha}\ \Gamma[\alpha]}\, e^{\frac{-(x_{rs}-\delta)}{\sigma}}, \delta \le x_{rs} < +\infty$$

α Continuous shape parameter $\alpha > 0, \sigma$ Continuous scale parameter $\sigma > 0$

δ Continuous location parameter.

Hence, the optimal value Q^*_{mrs} can be calculated by the following equation

$$\int_{x_{rs}=\gamma}^{Q^*_{mrs}} \frac{(x_{rs}-\delta)^{\alpha-1}}{\sigma^{\alpha}\Gamma[\alpha]}\, e^{\frac{-(x_{rs}-\delta)}{\sigma}}\, dx_{rs} + \int_{x_{rs}=Q^*_{mrs}}^{\infty} \frac{Q_{mrs}}{x_{rs}} \frac{(x_{rs}-\delta)^{\alpha-1}}{\sigma^{\alpha}\Gamma[\alpha]}\, e^{\frac{-(x_{rs}-\delta)}{\sigma}}\, dx_{rs} = \frac{C_{brs}\, N^{\beta_r}}{C_{brs}\, N^{\beta_r} + (1+\lambda_{hrs})\, C_{hr}\, N^{-\beta_r}}$$

5.4 The model with Beta distribution: If the demand follows the Beta distribution then,

$$f(x_{rs}) = \frac{1}{\mathrm{Beta}[\alpha_1,\alpha_2]} \frac{(x_{rs}-\mathrm{a})^{\alpha_1-1}\,(\mathrm{b}-x_{rs})^{\alpha_2-1}}{(\mathrm{b}-\mathrm{a})^{\alpha_1+\alpha_2-1}}, a \le x_{rs} \le b$$

α_1, α_2 Continuous shape parameter $\alpha_1, \alpha_2 > 0$, a and b Continuous boundary parameters $a < b$. Hence, the optimal value Q^*_{mrs} can be calculated by the following equation

$$\int_{x_{rs}=a}^{Q^*_{mrs}} \frac{1}{\mathrm{Beta}[\alpha_1,\alpha_2]} \frac{(x_{rs}-\mathrm{a})^{\alpha_1-1}\,(\mathrm{b}-x_{rs})^{\alpha_2-1}}{(\mathrm{b}-\mathrm{a})^{\alpha_1+\alpha_2-1}}\, dx_{rs} + \int_{x_{rs}=Q^*_{mrs}}^{b} \frac{Q_{mrs}}{x_{rs}} \frac{1}{\mathrm{Beta}[\alpha_1,\alpha_2]} \frac{(x_{rs}-\mathrm{a})^{\alpha_1-1}(\mathrm{b}-x_{rs})^{\alpha_2-1}}{(\mathrm{b}-\mathrm{a})^{\alpha_1+\alpha_2-1}}\, dx_{rs}$$
$$= \frac{C_{brs}\, N^{\beta_r}}{C_{brs}\, N^{\beta_r} + (1+\lambda_{hrs})\, C_{hr}\, N^{-\beta_r}}$$

5.5 The model with Raylieph distribution: If the demand follows the Raylieph distribution then,

$$f(x_{rs}) = \left(\frac{x_{rs}-\mathrm{a}}{{\alpha_2}^2}\right) e^{\frac{-1}{2}\left(\frac{x_{rs}-\mathrm{a}}{\alpha_2}\right)^2}, \quad \mathrm{a} \le x_{rs} < +\infty$$

α_2 continuous shape parameter $\alpha_2 > 0$, a continuous location parameter. Hence, the optimal value Q^*_{mrs} can be calculated by the following equation

$$\int_{x_{rs}=\gamma}^{Q^*_{mrs}} \left(\frac{x_{rs}-\mathrm{a}}{{\alpha_2}^2}\right) e^{\frac{-1}{2}\left(\frac{x_{rs}-\mathrm{a}}{\alpha_2}\right)^2} dx_{rs} + \int_{x_{rs}=Q^*_{mrs}}^{\infty} \frac{Q_{mrs}}{x_{rs}} \left(\frac{x_{rs}-\mathrm{a}}{{\alpha_2}^2}\right) e^{\frac{-1}{2}\left(\frac{x_{rs}-\mathrm{a}}{\alpha_2}\right)^2} dx_{rs} = \frac{C_{brs}\, N^{\beta_r}}{C_{brs}\, N^{\beta_r} + (1+\lambda_{hrs})\, C_{hr}\, N^{-\beta_r}}$$

5.6 The model with Erlang exponential distribution: If the demand follows the Erlang distribution then,

$$f(x_{rs}) = \frac{(x_{rs}-\mathrm{a})^{\mathrm{m}-1}}{b^m \Gamma[m]} e^{\frac{-(x_{rs}-\mathrm{a})}{b}}, \qquad \mathrm{a} \le x_{rs} < +\infty,$$

m shape parameter (positive integer), b continuous scale parameter $\mathrm{b} > 0$
a continuous location. Hence, the optimal value Q^*_{mrs} can be calculated by the following equation

$$\int_{x_{rs}=\gamma}^{Q^*_{mrs}} \frac{(x_{rs}-\mathrm{a})^{\mathrm{m}-1}}{b^m\, \Gamma[m]} e^{\frac{-(x_{rs}-\mathrm{a})}{b}} dx_{rs} + \int_{x_{rs}=Q^*_{mrs}}^{\infty} \frac{Q_{mrs}}{x_{rs}} \frac{(x_{rs}-\mathrm{a})^{\mathrm{m}-1}}{b^m\, \Gamma[m]} e^{\frac{-(x_{rs}-\mathrm{a})}{b}} dx_{rs} = \frac{C_{brs}\, N^{\beta_r}}{C_{brs}\, N^{\beta_r} + (1+\lambda_{hrs})\, C_{hr}\, N^{-\beta_r}}$$

6. Numerical example

To illustrate the above developed model, consider a hypothetical inventory system with the following parameter values which are given in Table 1. It is desired to determine to optimal value $\mathrm{Q_{mrs}}^*$ which minimize the expected total cost for β_r between $(0,1)$ and $N = 1$ month (0.08333 year). Also, the optimal solutions of the crisp environment and triangular fuzzy number TFN are given in Tables 3, 5, 7, 9, 11 and 13, and show by Figure 4, 5, 6, 7, 8 and 9. When the demand is follows:

1. Two parameter exponential distribution for $\gamma < x_{rs} < \infty$. at $\theta = 0.20755$; $\gamma = 1.6295$
2. Kumaraswamy distribution, $a \le x_{rs} \le b$. at $\alpha_1 = 1.5$; $\alpha_2 = 2$; $\mathrm{a} = 0$; $\mathrm{b} = 1$
3. Gamma distribution, $\delta \le x_{rs} < +\infty$. at $\alpha = 3$; $\delta = 0.041$; $\sigma = 1.3162$
4. Beta distribution, $a \le x_{rs} \le b$. at $\alpha_1 = 1.5$; $\alpha_2 = 2$; $\mathrm{a} = 0$; $\mathrm{b} = 1$
5. Rayleigh distribution, $0 \le x_{rs} \le \infty$. at $\alpha_2 = 2$; $\mathrm{a} = 0$
6. Erlang distribution, $a \le x_{rs} < +\infty$, at ; $\mathrm{m} = 2.5$; $\mathrm{a} = 0$; $\mathrm{b} = 1$

Table 1: Crisp and fuzzy values of the parameters

Parameters	Item	Source 1	Source 2	Source 3
C_{prs}	1	11	10.68	11.45
	2	9	9.5	10.23
	3	14	13.88	–
C_{ors}	1	15	15.43	14.76
	2	13	12.38	13.5
	3	17	15.89	–
C_{brs}	1	2	1.5	2.5
	2	1.7	1	2.2
	3	2.5	2	–
C_{hr}	1	2	2	2
	2	3	3	3
	3	3	3	3
K_{hr} (two Parameters exponential)	1	0.0006	0.00026	0.00097
	2	0.0000865	0.00004	0.0003
	3	0.0006	0.00036	–
K_{hr} (Kumaraswamy)	1	0.00028	0.000136	0.000435
	2	0.000132	0.0000462	0.000225
	3	0.00025	0.00018	–

K_{hr}(Gamma)	1	0.00322	0.00155	0.005
	2	0.00054	0.000265	0.00175
	3	0.00335	0.00215	–
K_{hr} (Beta)	1	0.00025	0.000123	0.000396
	2	0.0000415	0.0000203	0.000135
	3	0.000225	0.000165	–
K_{hr} (Rayleigh)	1	0.0019	0.00092	0.00295
	2	0.00032	0.000156	0.001035
	3	0.001975	0.00126	–
K_{hr} (Erlang)	1	0.0017	0.00088	0.00275
	2	0.00085	0.0003	0.00145
	3	0.00185	0.0012	-
$\tilde{C}_{prs}$	1	(9,11,11.5)	(8.68,10.68,11.18)	(9.45,11.45,11.95)
	2	(7,9,9.5)	(7.5,9.5,10)	(8,23,10.23,10.73)
	3	(12,14,14.5)	(11.88,13.88,14.38)	–
$\tilde{C}_{ors}$	1	(12,15,16.3)	(12.43,15.4316.73)	(11.76,14.76,16.6)
	2	10,13,14.3)	(8.38,12.38,13,68)	(10.5,13.5,14.8)
	3	(14,17,18,3)	12.89,15.89,17,19)	–
$\tilde{C}_{brs}$	1	(1.6,2,2.2)	(1.1,1.5,1.7)	(2.1,2.5,2.7)
	2	(0.6,1,1.2)	(0.3,0.7,0.9)	(1.4,1.8,2)
	3	2.1,2.5,2.7)	(1.6,2,2.2)	–
$\tilde{C}_{hr}$	1	(1.7,2,2.1)	(1.7,2,2.1)	(1.7,2,2.1)
	2	(2.7,3,3.1)	(2.7,3,3.1)	(2.7,3,3.1)
	3	(2.7,3,3.1)	(2.7,3,3.1)	(2.7,3,3.1)
$\widetilde{K}_{hr}$ (two Parameter exponential)	1	0.00057	0.00031	0.00096
	2	0.000075	0.000035	0.00027
	3	0.0006	0.00036	–
$\widetilde{K}_{hr}$ (Kumaraswamy)	1	0.000265	0.00013	0.00041
	2	0.0000424	0.0000195	0.000142
	3	0.00028	0.00015	–
$\widetilde{K}_{hr}$(Gamma)	1	0.003	0.00172	0.0049
	2	0.0005	0.0002	0.00165
	3	0.00325	0.002	–
$\widetilde{K}_{hr}$ (Beta)	1	0.00024	0.000132	0.00035
	2	0.0000382	0.0000178	0.00012
	3	0.000256	0.000162	–
$\widetilde{K}_{hr}$ (Rayleigh)	1	0.0018	0.001035	0.0029
	2	0.000295	0.000138	0.000995
	3	0.00192	0.001235	–
$\widetilde{K}_{hr}$ (Erlang)	1	0.00174	0.00084	0.0027
	2	0.00083	0.000277	0.0014
	3	0.00182	0.00182	-

Table 2: The results of crisp and fuzzy values for two parameters exponential distribution

β_r	S	Item 1			Item 2			Item 3		
		λ_{hr}	Q_{m1}	$E(TC)_1$	λ_{hr}	Q_{m2}	$E(TC)_2$	λ_{hr}	Q_{m3}	$E(TC)_3$
0.1	1	20.2256	0.0155365	20.2377	34.7972	0.0044178	17.4717	20.3643	0.0124822	23.555
	2	20.9304	0.0098981	19.1046	29.1793	0.0029292	15.012	20.6788	0.0094812	21.1403
	3	20.2076	0.0201511	21.2952	24.9585	0.0085965	19.2772	-	-	-
0.2	1	13.4974	0.0135852	19.0887	23.4535	0.0038694	16.4886	13.591	0.0109182	22.1162
	2	13.9773	0.0086608	18.2976	19.6206	0.0025667	14.4332	13.8055	0.0082964	19.9872
	3	13.4877	0.0176127	19.8628	16.7285	0.0075233	18.0082	-	-	-
0.3	1	8.90146	0.0118814	18.1914	15.7056	0.0033896	15.7216	8.96493	0.009552	20.9929
	2	9,22886	0.0075795	17..6677	13.0905	0.0022493	13.9817	9.11159	0.0072611	19.0871
	3	8.89603	0.0153975	18.7439	11.1081	0.0065852	17.0177	-	-	-
0.4	1	5.76228	0.0103934	17.4909	10.4134	0.00296962	15.1232	5.80554	0.0083583	20.116
	2	5.98603	0.0066343	17.1761	8.62923	0.00197143	13.6294	5.90601	0.006356	18.3847
	3	5.02039	0.0134639	17.8679	6.36613	0.00576506	16.2439	-	-	-
0.5	1	3.61827	0.0090936	16.944	6.79838	0.00260207	14.6563	3.64791	0.0073151	19.4316
	2	3.77144	0.005808	16.7924	5.58101	0.0017281	13.3546	3.71685	0.0055646	17.8365
	3	3.61631	0.0117755	17.1876	4.64854	0.00504787	15.6415	-	-	-
0.6	1	2.15406	0.0079577	16.5171	4.32878	0.00228031	14.2921	2.17442	0.0064032	18.8974
	2	2.25903	0.0050854	16.4931	3.49814	0.00151498	13.1403	2.2218	0.0048726	17.4088
	3	2.15274	0.0103009	16.6548	2.85834	0.00442059	15.1708	-	-	-
0.7	1	1.15411	0.006965	16.1839	2.64155	0.00199859	14.0079	1.16815	0.0056059	18.4806

	2	1.22613	0.0044534	16.2594	2.07477	0.00132829	12.973	1.20075	0.0042672	17.075
	3	1.15315	0.0090127	16.2389	1.63568	0.00387183	14.8036	-	-	-
0.8	1	0.471232	0.0060971	15.9239	1.48877	0.00175188	13.7863	0.480927	0.0049087	18.1553
	2	0.520693	0.0039006	16.0771	1.10198	0.00116474	12.8426	0.503382	0.0037377	16.8145
	3	0.4705	0.0078871	15.9143	0.800586	0.00339168	14.517	-	-	-
0.9	**1**	**0.004876**	**0.0053382**	**15.721**	**0.701063**	**0.0015358**	**13.6133**	**0.011587**	**0.0042989**	**17.9014**
	2	**0.03887**	**0.0078871**	**15.9349**	**0.437103**	**0.0010214**	**12.7409**	**0.027067**	**0.0032743**	**16.6113**
	3	**0.004303**	**0.0069032**	**15.6609**	**0.230177**	**0.0029715**	**14.2935**	**-**	**-**	**-**
$\tilde{\beta}_r$		$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$
0.1	1	20.484	0.0153199	19.6822	20.5729	0.0041309	15.0742	20.126	0.0125958	22.9986
	2	20.2135	0.01103	18.8091	20.0671	0.0027529	13.666	20.3276	0.0095672	20.5839
	3	20.1681	0.0203152	20.7391	20.8823	0.0081974	17.6713	-	-	-
0.2	1	13.6738	0.0133961	18.5617	13.7372	0.0036184	14.5248	13.4283	0.0110174	21.5887
	2	13.4878	0.0096496	17.9739	13.3951	0.0024123	13.2897	13.5657	0.0083716	19.4596
	3	13.4608	0.0177559	19.3355	13.9448	0.0071744	16.6616	-	-	-
0.3	1	9.02189	0.0117163	17.6868	9.06821	0.0031699	14.0961	8.85377	0.0096387	20.4879
	2	8.89443	0.0084436	17.3219	8.8369	0.0021142	12.9962	8.94777	0.0073268	18.5821
	3	8.8777	0.0155225	18.239	9.207	0.0062802	15.8735	-	-	-
0.4	1	5.8445	0.0102492	17.0037	5.87896	0.0027774	13.7616	5.72963	0.0084341	19.6286
	2	5.75748	0.0073897	16.813	5.72264	0.0018531	12.7672	5.79411	0.0064134	17.8973
	3	5.74661	0.013573	17.3827	5.97147	0.0054984	15.2586	-	-	-
0.5	1	3.67444	0.0089675	16.4705	3.70034	0.0024338	13.5007	3.59605	0.0073814	18.9579
	2	3.6152	0.0064684	16.4158	3.59471	0.0016245	12.5886	3.64041	0.0056148	17.3628
	3	3.60779	0.0118708	16.714	3.76182	0.0048146	14.7787	-	-	-
0.6	1	2.19242	0.0078476	16.0542	2.21196	0.002133	13.2971	2.139	0.0064611	18.4344
	2	2.15219	0.005663	16.1058	2.14064	0.0014242	12.4492	2.16957	0.0049165	16.9458
	3	2.14691	0.0103842	16.1919	2.25272	0.0042166	14.4042	-	-	-
0.7	1	1.18032	0.0068687	15.7294	1.19508	0.0018696	13.1383	1.14395	0.0056566	18.0259
	2	1.15306	0.0049586	15.8639	1.14691	0.0012488	12.3405	1.16506	0.0043057	16.6203
	3	1.14918	0.0090854	15.7843	1.22202	0.0036933	14.112	-	-	-
0.8	1	0.48914	0.0060129	15.4758	0.518034	0.0032355	13.8841	0.46439	0.004953	17.7072
	2	0.4707	0.0043425	15.6752	0.467747	0.0010951	12.2557	0.478999	0.0037713	16.3664
	3	0.46779	0.0079506	15.4662	0.478999	0.0037713	16.3664	-	-	-
0.9	**1**	**0.017111**	**0.0052646**	**15.278**	**0.02547**	**0.0014368**	**12.9178**	**0.00029**	**0.0043376**	**17.4584**
	2	**0.00466**	**0.0038035**	**15.5279**	**0.003523**	**0.0009604**	**12.1896**	**0.010404**	**0.0033037**	**16.1683**
	3	**0.00245**	**0.0153199**	**15.2179**	**0.037163**	**0.0028348**	**13.7062**	**-**	**-**	**-**

Table 3: The Optimal policy variable for two parameter exponential distribution at $\beta = 0.9$

Item	source	Q^*_{mrs}	Min E(TC)	$\tilde{Q}^*_{mrs}$	Source	Min $E(\widetilde{TC})$
1	3	0.00690321		0.00695875	3	
2	2	0.00102143	45.0131	0.000960373	2	43.5758
3	2	0.00327427		0.0033037	2	

Table 4: The results of crisp and fuzzy values for Kamurswamy distribution

β_r	s	Item 1			Item 2			Item 3		
		λ_{hr}	Q_{m1}	$E(TC)_1$	λ_{hr}	Q_{m2}	$E(TC)_2$	λ_{hr}	Q_{m3}	$E(TC)_3$
0.1	1	18.7367	0.0071865	15.3403	18.9662	0.0039964	13.2932	20.3354	0.0055227	17.4284
	2	18.8417	0.0049818	15.6705	18.8251	0.0023514	12.5537	19.1175	0.0046757	16.234
	3	18.7635	0.0089916	15.1821	18.792	0.0052354	13.8775	-	-	-
0.2	1	12.6023	0.0063344	15.2665	12.7508	0.0035244	13.2292	13.6984	0.0048691	17.3352
	2	12.6678	0.0043927	15.6181	12.65	0.0020744	12.5157	12.8569	0.004123	16.159
	3	12.627	0.0079235	15.0909	12.6343	0.0046161	13.7953	-	-	-
0.3	1	8.37271	0.0055841	15.2086	8.46944	0.0031084	13.1791	9.12467	0.004293	17.2621
	2	8.41382	0.0038736	15.5771	8.39806	0.0018302	12.4859	8.54353	0.003636	16.1003
	3	8.39336	0.0069834	15.0192	8.39119	0.0040705	13.7309	-	-	-
0.4	1	5.45716	0.0049232	15.1632	5.52063	0.0027418	13.14	5.97331	0.0037861	17.205
	2	5.48319	0.0034162	15.545	5.47047	0.0016148	12.4627	5.57224	0.0032068	16.0544
	3	5.4735	0.0061556	14.963	5.46785	0.0035897	13.6805	-	-	-
0.5	1	3.44791	0.0043411	15.1277	3.48984	0.0024186	13.1094	3.80236	0.0033391	17.1602
	2	3.46453	0.0030131	15.5199	3.45476	0.0014249	12.4446	3.5257	0.0028285	16.0185
	3	3.46039	0.0054266	14.919	3.4541	0.003166	13.6411	-	-	-
0.6	1	2.0635	0.0038281	15.0999	2.09138	0.0021337	13.0854	2.30702	0.0029452	17.1253
	2	2.07421	0.0026578	15.5002	2.06697	0.0012574	12.4304	2.11625	0.0024951	15.9904
	3	2.0728	0.0047845	14.8844	2.06712	0.0027926	13.6103	-	-	-
0.7	1	1.10978	0.0033762	15.0781	1.12844	0.0018825	13.0667	1.27714	0.0025979	17.0979
	2	1.11676	0.0023445	15.4849	1.1115	0.0011096	12.4194	1.14565	0.0022011	15.9684
	3	1.11658	0.0042188	14.8574	1.1119	0.0024634	13.5862	-	-	-

0.8	1	0.45285	0.0029778	15.0611	0.46541	0.0016609	13.0521	0.56792	0.0022917	17.0765
	2	0.457436	0.0020684	15.4729	0.45371	0.0009792	12.4107	0.47731	0.0019419	15.9513
	3	0.457763	0.0037204	14.8362	0.45412	0.0021731	13.5674	-	-	-
0.9	**1**	**0.00041**	**0.0026267**	**15.0478**	**0.00891**	**0.0014655**	**13.0407**	**0.07954**	**0.0020218**	**17.0598**
	2	**0.003446**	**0.0018249**	**15.4635**	**0.00084**	**0.0008642**	**12.404**	**0.01829**	**0.0017113**	**15.9379**
	3	**0.003917**	**0.0032812**	**14.8197**	**0.00117**	**0.0019172**	**13.5527**	**-**	**-**	**-**
$\tilde{\beta}_r$		$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$
0.1	1	19.0338	0.0070787	14.9069	18.8231	0.002271	12.7401	18.9211	0.0058995	16.9942
	2	20.2871	0.0049321	15.2541	18.9682	0.0015353	12.0683	20.6673	0.0042999	15.8009
	3	19.2071	0.0088379	14.7489	18.9848	0.0041822	13.3766	-	-	-
0.2	1	12.8068	0.0062395	14.8349	12.6485	0.0020035	12.7039	12.7253	0.005201	16.903
	2	13.6633	0.0043489	15.1998	12.7476	0.0013547	12.0435	13.9232	0.0037918	15.7276
	3	12.9324	0.0077882	14.6594	12.7641	0.0036882	13.3108	-	-	-
0.3	1	8.51334	0.0055005	14.7785	8.39698	0.0017676	12.6757	8.45499	0.0045858	16.8316
	2	9.09936	0.003835	15.1573	8.46484	0.0011955	12.024	9.27731	0.0033441	15.6703
	3	8.6035	0.0068642	14.5892	8.47886	0.0032528	13.2593	-	-	-
0.4	1	5.55394	0.0048496	14.7342	5.46966	0.0015597	12.6536	5.51245	0.0040438	16.7757
	2	5.95527	0.0033822	15.1241	5.51623	0.001055	12.0089	6.07721	0.0029496	15.6254
	3	5.61814	0.0060507	14.5341	5.52727	0.0028691	13.219	-	-	-
0.5	1	3.5145	0.0042762	14.6996	3.45419	0.0013762	12.6364	3.4852	0.0035662	16.7319
	2	3.7896	0.0029831	15.0981	3.48623	0.0009311	11.9971	3.87322	0.0026018	15.5904
	3	3.55993	0.0053342	14.4908	3.4945	0.0025308	13.1875	-	-	-
0.6	1	2.10932	0.003771	14.6724	2.06657	0.0012144	12.6229	2.08873	0.0031453	16.6977
	2	2.29804	0.0026313	15.0777	2.08865	0.0008217	11.9878	2.35544	0.0022952	15.5629
	3	2.14131	0.0047031	14.457	2.09463	0.0022326	13.1629	-	-	-
0.7	1	1.14131	0.0033258	14.6512	1.11124	0.0010717	12.6124	1.12691	0.0027743	16.6709
	2	1.27086	0.0023212	15.0618	1.12647	0.0007253	11.9806	1.31028	0.0020248	15.5415
	3	1.16374	0.0041471	14.4305	1.1307	0.0019697	13.1437	-	-	-
0.8	1	0.47456	0.0029334	14.6346	0.453528	0.0009458	12.6042	0.46451	0.0024473	16.65
	2	0.56354	0.0020478	15.0494	0.46405	0.0006402	11.975	0.59063	0.0017865	15.5248
	3	0.49022	0.0036573	14.4097	0.46698	0.0017378	13.1287	-	-	-
0.9	**1**	**0.01535**	**0.0025875**	**14.6216**	**0.000715**	**0.0008348**	**12.5978**	**0.00836**	**0.0021589**	**16.6336**
	2	**0.07649**	**0.0018067**	**15.0397**	**0.07993**	**0.000565**	**11.9706**	**0.09512**	**0.0015763**	**15.5117**
	3	**0.02626**	**0.0032255**	**14.3935**	**0.00999**	**0.0015334**	**13.1169**	**-**	**-**	**-**

Table 5: The Optimal policy variable for Kamurswamy distribution at $\beta = 0.9$

Item	source	Q^*_{mrs}	Min E(TC)	$\tilde{Q}^*_{mrs}$	Source	Min $E(\widetilde{TC})$
1	3	0.00328123		0.00322554	3	
2	2	0.00086422	43.1616	0.000565047	2	41.8758
3	2	0.001711332		0.00157627	2	

Table 6: The results of crisp and fuzzy values for Gamma distribution

β_r		Item 1			Item 2			Item 3		
		λ_{hr}	Q_{m1}	$E(TC)_1$	λ_{hr}	Q_{m2}	$E(TC)_2$	λ_{hr}	Q_{m3}	$E(TC)_3$
0.1	1	18.3718	0.0824827	17.9883	32.5157	0.0275794	15.6092	18.4852	0.0686927	20.7608
	2	18.7322	0.0572268	17.548	27.2306	0.0193201	13.9211	18.5586	0.055031	18.919
	3	18.283	0.102784	18.4581	22.8952	0.0496486	16.8403	–	–	–
0.2	1	12.3929	0.0728456	17.3429	22.115	0.0243571	15.0385	12.4631	0.0606669	19.946
	2	12.6267	0.0505406	17.0879	18.4633	0.0170627	13.5835	12.5055	0.0486013	18.2607
	3	12.3438	0.0907741	17.6632	15.496	0.0438478	16.1134	–	–	–
0.3	1	8.2554	0.0643345	16.836	14.9397	0.0215113	14.5924	8.2988	0.0535787	19.3068
	2	8.40756	0.0446356	16.7273	12.4176	0.015069	13.3197	8.32308	0.0429229	17.7448
	3	8.22885	0.0801681	17.0377	10.3851	0.0387248	15.544	–	–	–
0.4	1	5.39366	0.0568178	16.4382	9.99044	0.0189979	14.2437	5.42055	0.0473187	18.8057
	2	5.49305	0.0394205	16.4449	8.24915	0.0133082	13.1136	5.43425	0.0379079	17.3409
	3	5.37984	0.0708014	16.5461	6.85585	0.0342003	15.0983	–	–	–
0.5	1	3.41527	0.0501794	16.1262	6.57717	0.0167781	13.9713	3.43195	0.0417902	18.413
	2	3.48044	0.0348147	16.2238	5.37539	0.011753	12.9527	3.43956	0.0334789	17.0246
	3	3.40849	0.0625291	16.16	4.41957	0.0302044	14.7495	–	–	–
0.6	1	2.04815	0.0443166	15.8817	4.22348	0.0148177	13.7584	2.05853	0.0369075	18.1065
	2	2.09103	0.0307471	16.0507	3.39435	0.0103794	12.8271	2.06265	0.0295673	16.7772
	3	2.04515	0.0552234	15.8571	2.73818	0.0266754	14.4767	–	–	–
0.7	1	1.10379	0.0391387	15.6902	2.60068	0.0130862	13.5922	1.11025	0.0325954	17.8649
	2	1.1321	0.0271547	15.9154	2.02883	0.00916609	12.729	1.11242	0.0261127	16.5837
	3	1.10274	0.0487713	15.6196	1.57804	0.0235587	14.2634	–	–	–
0.8	1	0.45167	0.0345659	15.5403	1.48192	0.0115569	13.4624	0.455714	0.028787	17.6767
	2	0.47044	0.023982	15.8095	1.08763	0.0080944	12.6524	0.456796	0.0230618	16.4324

	3	0.45157	0.043073	15.4335	0.7777	0.0208062	14.0966	–	–	–
0.9	**1**	**0.00149**	**0.0305273**	**15.423**	**0.7107**	**0.0102062**	**13.361**	**0.0040254**	**0.0254236**	**17.5294**
	2	**0.01397**	**0.02118**	**15.7268**	**0.43893**	**0.00714771**	**12.5926**	**0.0045259**	**0.0203673**	**16.3141**
	3	**0.001794**	**0.0380405**	**15.2877**	**0.22569**	**0.0183752**	**13.9663**	–	–	–
$\tilde{\beta}_r$		$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$
0.1	1	18.8309	0.0806294	17.4912	18.6344	0.0267623	14.0339	18.5541	0.0682307	20.2615
	2	18.6195	0.0610514	17.1945	20.32	0.0169258	12.958	18.95	0.0535245	18.4204
	3	18.3305	0.103047	17.9587	18.7479	0.0486161	15.7337	–	–	–
0.2	1	12.7093	0.0712088	16.8611	12.5409	0.0236354	13.7148	12.5104	0.0602588	19.4627
	2	12.5512	0.0539184	16.7193	13.6975	0.0149481	12.7381	12.7748	0.0472709	17.7779
	3	12.3768	0.0910066	17.1798	12.6323	0.0429359	15.155	–	–	–
0.3	1	8.47332	0.0628889	16.3663	8.33729	0.0208739	13.4654	8.3313	0.0532183	18.836
	2	8.35685	0.0476187	16.3467	9.13139	0.0132014	12.5664	8.50843	0.0417479	17.2744
	3	8.25177	0.0803734	16.5669	8.40824	0.0379194	14.7018	–	–	–
0.4	1	5.54377	0.05554111	15.978	5.43789	0.018435	13.2704	5.44288	0.0470005	18.3448
	2	5.4589	0.04205511	16.0548	5.98353	0.0116586	12.4323	5.56183	0.0368702	16.8802
	3	5.39574	0.0709827	16.0852	5.49159	0.033489	14.3471	–	–	–
0.5	1	3.51867	0.0490519	15.6735	3.43839	0.016281	13.1181	3.44731	0.0415091	17.9599
	2	3.45739	0.0371415	15.8263	3.81352	0.010296	12.3276	3.52738	0.0325624	16.5717
	3	3.41951	0.0626893	15.7069	3.47824	0.0295763	14.0695	–	–	–
0.6	1	2.11938	0.0433208	15.435	2.05965	0.0143786	12.9991	2.06908	0.0366593	17.6586
	2	2.07545	0.032802	15.6473	2.31772	0.00909249	12.2458	2.12311	0.0287579	16.3303
	3	2.05278	0.0553648	15.4101	2.08882	0.0261206	13.8524	–	–	–
0.7	1	1.15286	0.0382593	15.2481	1.10907	0.0126984	12.9062	1.11752	0.0323761	17.4227
	2	1.12155	0.0289695	15.5074	1.28675	0.00802938	12.182	1.15405	0.0253979	16.1415
	3	1.10802	0.0488962	15.1773	1.13015	0.0230688	13.6827	–	–	–
0.8	1	0.48547	0.0337892	15.1019	0.45375	0.0112144	12.8336	0.460708	0.0285934	17.2382
	2	0.46327	0.0255848	15.3979	0.57618	0.00709028	12.1322	0.485456	0.0224305	15.9939
	3	0.45522	0.0431833	14.995	0.46884	0.0203734	13.55	–	–	–
0.9	**1**	**0.02477**	**0.0298414**	**14.9874**	**0.00201**	**0.00990366**	**12.777**	**0.0074621**	**0.0252526**	**17.0938**
	2	**0.0091**	**0.0225955**	**15.3123**	**0.08646**	**0.00626064**	**12.0933**	**0.0242636**	**0.0198097**	**15.8785**
	3	**0.00432**	**0.0381379**	**14.8521**	**0.01272**	**0.017993**	**13.4463**	–	–	–

Table 7: The Optimal policy variable for Gamma distribution at $\beta = 0.9$

Item	source	Q^*_{mrs}	Min E(TC)	$\tilde{Q}^*_{mrs}$	Source	Min E($\widetilde{TC}$)
1	3	0.0380405		0.0381379	3	
2	2	0.00714771	44.1944	0.00626064	2	42.823
3	2	0.0203673		0.0198097	2	

Table 8: The results of crisp and fuzzy values for Beta distribution

β_r	s	Item 1			Item 2			Item 3		
		λ_{hr}	Q_{m1}	$E(TC)_1$	λ_{hr}	Q_{m2}	$E(TC)_2$	λ_{hr}	Q_{m3}	$E(TC)_3$
0.1	1	18.8835	0.00645563	15.3247	32.7975	0.00211678	13.2814	20.3984	0.00497979	17.4085
	2	20.2663	0.00450279	15.6756	27.376	0.00147583	12.546	18.9877	0.00425484	16.2179
	3	18.7265	0.00815965	15.1627	23.3025	0.00384341	13.8613	-	-	-
0.2	1	12.7003	0.00568945	15.2542	22.2677	0.00186729	13.2197	13.7389	0.00438996	17.3196
	2	13.6464	0.00396985	15.6221	18.5344	0.00130214	12.5096	12.7652	0.00375143	16.1464
	3	12.5981	0.0071892	15.0756	15.7353	0.00338898	13.7824	-	-	-
0.3	1	8.43809	0.0050149	15.1989	15.018	0.00164734	13.1716	9.15068	0.00387046	17.2499
	2	9.0861	0.00350037	15.5802	12.4476	0.00114897	12.4812	8.47885	0.00330794	16.0904
	3	8.37116	0.00633517	15.0072	10.5233	0.00298858	13.7207	-	-	-
0.4	1	5.50087	0.00442089	15.1557	10.027	0.00145339	13.134	5.99004	0.00341281	17.1954
	2	5.94509	0.00308673	15.5474	8.2575	0.00101387	12.459	5.52673	0.00291717	16.0466
	3	5.45678	0.00558341	14.9536	6.93406	0.00263574	13.6725	-	-	-
0.5	1	3.47717	0.0038977	15.1218	6.59116	0.00128235	13.1046	3.81312	0.00300958	17.1527
	2	3.78191	0.00272224	15.5217	5.37298	0.000894706	12.4416	3.49373	0.00257281	16.0124
	3	3.44795	0.00492153	14.9116	4.46246	0.00232477	13.6348	-	-	-
0.6	1	2.05572	0.0034681	15.0952	4.22579	0.00113152	13.0816	2.31394	0.00265425	17.1194
	2	2.2923	0.00240101	15.5017	3.3873	0.000789585	12.4281	2.09384	0.00226929	15.9857
	3	2.06365	0.00433866	14.8786	2.76067	0.00205065	13.6053	-	-	-
0.7	1	1.12295	0.00303073	15.0745	2.59749	0.000998478	13.0637	1.2816	0.00234108	17.0933
	2	1.26663	0.00211786	15.486	2.02034	0.000696848	12.4175	1.12996	0.00200175	15.9647
	3	1.10991	0.00382526	14.8528	1.58898	0.001809	13.5822	-	-	-
0.8	1	0.4617	0.00267289	15.0582	1.47657	0.000881127	13.0497	0.57079	0.00206503	17.0729
	2	0.56045	0.00186824	15.4738	1.07932	0.000615029	12.4093	0.46634	0.00176588	15.9484
	3	0.45293	0.00337299	14.8326	0.78232	0.00159593	13.5642	-	-	-

0.9	**1**	**0.00637**	**0.0023575**	**15.0455**	**0.70492**	**0.000777606**	**13.0388**	**0.08139**	**0.00182167**	**17.057**
	2	**0.07425**	**0.00164817**	**15.4642**	**0.43151**	**0.000542838**	**12.4029**	**0.00945**	**0.00155792**	**15.9356**
	3	**0.00043**	**0.0029745**	**14.8169**	**0.22697**	**0.00140806**	**13.5502**	**-**	**-**	**-**
$\tilde{\beta}_r$		$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$
0.1	1	19.0387	0.00640491	14.8916	18.8488	0.00204737	12.7323	18.8253	0.00536264	16.9747
	2	19.0978	0.00472734	15.2422	18.8576	0.001393	12.063	18.8336	0.00425151	15.7847
	3	19.8316	0.00776139	14.7304	19.6747	0.00365178	13.3627	-	-	-
0.2	1	12.807	0.00564481	14.8229	12.6646	0.0018061	12.6978	12.6566	0.00472712	16.8877
	2	12.8423	0.00416763	15.1905	12.6701	0.0012291	12.0393	12.659	0.0037485	15.715
	3	13.3583	0.00683875	14.6448	13.2366	0.00322014	13.2999	-	-	-
0.3	1	8.51153	0.00497559	14.769	8.40705	0.00159338	12.6709	8.40595	0.00416744	16.8196
	2	8.53275	0.00367461	15.1501	8.41062	0.00108455	12.0208	8.40575	0.00330536	15.6604
	3	8.8942	0.0060267	14.5776	8.80253	0.00283981	13.2507	-	-	-
0.4	1	5.5514	0.00438627	14.7268	5.47593	0.00140581	12.6499	5.47757	0.00367444	16.7663
	2	5.56428	0.00324025	15.1184	5.4784	0.000957047	12.0063	5.47638	0.0029149	15.6177
	3	5.81664	0.00531184	14.5249	5.74911	0.00250462	13.2123	-	-	-
0.5	1	3.51193	0.00386721	14.6937	3.45812	0.00124039	12.6335	3.46048	0.0032401	16.7245
	2	3.51982	0.00285753	15.0936	3.45984	0.00084458	11.9951	3.45908	0.0025708	15.5844
	3	3.69556	0.00468239	14.4837	3.64657	0.00220919	13.1823	-	-	-
0.6	1	2.10704	0.00340994	14.6679	2.06899	0.00109451	12.6206	2.07125	0.0028574	16.6919
	2	2.11193	0.00252024	15.0743	2.07026	0.000745365	11.9863	2.06997	0.00226753	15.5583
	3	2.23403	0.00412803	14.4513	2.19891	0.00194877	13.1588	-	-	-
0.7	1	1.13941	0.00300705	14.6476	1.11273	0.000965835	12.6106	1.11457	0.00252013	16.6664
	2	1.14248	0.00222296	15.0591	1.11367	0.000657834	11.9794	1.11353	0.00200019	15.5379
	3	1.22715	0.00363973	14.426	1.20223	0.00171917	13.1405	-	-	-
0.8	1	0.47033	0.00265202	14.6318	0.45444	0.000852332	12.6028	0.45583	0.00222286	16.6464
	2	0.47499	0.0019609	15.0473	0.45514	0.000580607	11.974	0.45502	0.00176451	15.5219
	3	0.53361	0.00320952	14.4062	0.51605	0.00151673	13.1261	-	-	-
0.9	**1**	**0.04142**	**0.00233912**	**14.6194**	**0.00127**	**0.000752203**	**12.5967**	**0.002265**	**0.00196081**	**16.6308**
	2	**0.01543**	**0.00172987**	**15.0381**	**0.0018**	**0.000512466**	**11.9699**	**0.00167**	**0.00155671**	**15.5095**
	3	**0.05596**	**0.00283045**	**14.3907**	**0.04367**	**0.00133821**	**13.1149**	**-**	**-**	**-**

Table 8: The results of crisp and fuzzy values for Beta distribution

Item	source	Q^*_{mrs}	Min E(TC)	$\tilde{Q}^*_{mrs}$	Source	Min $E(\widetilde{TC})$
1	3	0.0029745		0.00283045	3	
2	2	0.000542838	43.1554	0.000512466	2	41.8701
3	2	0.00155792		0.00155671	2	

Table 10: The results of crisp and fuzzy values for Raylieph distribution

β_r	Source	Item 1			Item 2			Item 3		
		λ_{hr}	Q_{m1}	$E(TC)_1$	λ_{hr}	Q_{m2}	$E(TC)_2$	λ_{hr}	Q_{m3}	$E(TC)_3$
0.1	1	18.4856	0.0487883	16.8822	18.6994	0.0163127	13.9652	18.586	0.0405919	19.3678
	2	20.1581	0.0339157	16.858	18.7981	0.011386	13.0583	18.6972	0.0324043	17.7964
	3	18.43	0.0608416	17.0907	18.5892	0.029363	15.2198	-	-	-
0.2	1	12.4609	0.0430716	16.4752	12.5827	0.014405	13.7541	12.5237	0.0358379	18.8543
	2	13.6034	0.0299452	16.5476	12.6469	0.0100548	12.9096	12.5939	0.0286111	17.3817
	3	12.4321	0.0537074	16.589	12.5171	0.0259263	14.8453	-	-	-
0.3	1	8.29582	0.0380264	16.1557	8.36421	0.0127205	13.589	8.33523	0.0316418	18.4516
	2	9.07707	0.0264403	16.3043	8.40605	0.00887934	12.7935	8.37972	0.0252626	17.0569
	3	8.28179	0.0474124	16.1944	8.32519	0.0228926	14.5521	-	-	-
0.4	1	5.41761	0.0335735	15.9051	5.45524	0.0112332	13.46	5.44238	0.027938	18.136
	2	5.95222	0.0233463	16.1139	5.48266	0.00784136	12.7028	5.47067	0.0223066	16.8026
	3	5.41147	0.0418573	15.8844	5.43219	0.0202142	14.3226	-	-	-
0.5	1	3.42939	0.0296431	15.7086	3.44957	0.00991985	13.3593	3.44503	0.0246684	17.8888
	2	3.79551	0.0206148	15.9648	3.46758	0.00692478	12.632	3.46308	0.0196969	16.6036
	3	3.42732	0.0369547	15.6411	3.43597	0.0178497	14.1431	-	-	-
0.6	1	2.05644	0.0261736	15.5546	2.06683	0.00876017	13.2805	2.06634	0.021782	17.6953
	2	2.30733	0.0182033	15.8481	2.07871	0.00611538	12.5767	2.07789	0.0173929	16.4479
	3	2.05632	0.0326276	15.4503	2.05887	0.015762	14.0026	-	-	-
0.7	1	1.10862	0.0231109	15.4341	1.11365	0.00773613	13.2191	1.11491	0.0192338	17.5439
	2	1.28065	0.0160742	15.7569	1.12151	0.00540062	12.5336	1.12233	0.0153587	16.3261
	3	1.1-933	0.0288082	15.3007	1.10902	0.0139187	13.8928	-	-	-
0.8	1	0.45446	0.020407	15.3397	0.45663	0.00683185	13.1711	0.458474	0.016984	17.4254
	2	0.57247	0.0141944	15.6855	0.46185	0.00476943	12.4999	0.463264	0.0135626	16.2309
	3	0.45544	0.0254366	15.1835	0.454	0.0122911	13.807	-	-	-
0.9	**1**	**0.00309**	**0.0180198**	**15.2659**	**0.00379**	**0.00603332**	**13.1336**	**0.00565**	**0.0149977**	**17.3328**

	2	**0.08408**	**0.0125346**	**15.6298**	**0.00727**	**0.00421203**	**12.4736**	**0.00876**	**0.0119767**	**16.1565**
	3	**0.00406**	**0.0224602**	**15.0918**	**0.00227**	**0.010854**	**13.74**	**-**	**-**	**-**
$\tilde{\beta}_r$		$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$
	1	18.7746	0.0480898	16.4111	18.6608	0.0157942	13.4923	18.6338	0.0403598	18.8959
0.1	2	18.5042	0.0364375	16.3828	18.7169	0.0107989	12.5851	18.5623	0.0323519	17.3238
	3	18.447	0.0610933	16.6186	18.5913	0.0290323	14.7475	-	-	-
	1	12.66	0.0424551	16.014	12.5557	0.0139471	13.2916	12.5566	0.035633	18.3925
0.2	2	12.464	0.0321711	16.0836	12.5904	0.00953643	12.447	12.5008	0.0285648	16.9195
	3	12.4441	0.0539295	16.1271	12.5182	0.0256344	14.3833	-	-	-
	1	6.43297	0.0374823	15.7023	8.3453	0.0123163	13.1347	8.35778	0.031461	17.9978
0.3	2	8.29197	0.0284052	15.849	8.3684	0.0084216	12.3391	8.31548	0.0252217	16.6028
	3	8.29019	0.0476084	15.7405	8.32582	0.0226349	14.0981	-	-	-
	1	5.51208	0.0330933	15.4577	5.4421	0.0108762	13.0122	5.45788	0.0277784	17.6884
0.4	2	5.41134	0.0250808	15.6653	5.45546	0.00743715	12.2549	5.42634	0.0222705	16.3548
	3	5.41734	0.0420303	15.4368	5.43253	0.0199867	13.8749	-	-	-
	1	3.49447	0.0292192	15.266	3.44041	0.00960466	12.9164	3.45568	0.0245275	17.4461
0.5	2	3.42286	0.0221461	15.5214	3.44873	0.00656784	12.1891	3.43248	0.0196651	16.1608
	3	3.43142	0.0371074	15.1984	3.43615	0.0176488	13.7003	-	-	-
	1	2.10127	0.0257994	15.1159	2.06046	0.00848184	12.8416	2.07366	0.0216576	17.2564
0.6	2	2.05058	0.0195553	15.4089	2.06566	0.00580018	12.1377	2.05679	0.0173648	16.0089
	3	2.05918	0.0327623	15.0114	2.05896	0.0155846	13.5637	-	-	-
	1	1.1395	0.0227805	14.9983	1.10922	0.00749035	12.7831	1.11994	0.019124	17.108
0.7	2	1.10378	0.0172678	15.3208	1.11248	0.00512227	12.0976	1.10777	0.0153339	15.8902
	3	1.11132	0.0289271	14.8648	1.10905	0.0137621	13.457	-	-	-
	1	0.47574	0.0201153	14.9062	0.45356	0.00661482	12.7375	0.461931	0.0168871	16.9919
0.8	2	0.45063	0.0152483	15.252	0.4556	0.00452362	12.0663	0.453224	0.0135407	15.7974
	3	0.45683	0.0255416	14.75	0.45397	0.0121529	13.3735	-	-	-
	1	**0.01775**	**0.0177623**	**14.8342**	**0.00166**	**0.00584166**	**12.7019**	**0.00803**	**0.0149121**	**16.9011**
0.9	**2**	**0.00014**	**0.0134651**	**15.1981**	**0.00295**	**0.00399495**	**12.0419**	**0.00183**	**0.0119574**	**15.7249**
	3	**0.00502**	**0.0225529**	**14.6601**	**0.00227**	**0.0107319**	**13.3083**	**-**	**-**	**-**

Table 11: The Optimal policy variable for Raylieph distribution at $\beta = 0.9$

Item	source	Q^*_{mrs}	Min E(TC)	$\tilde{Q}^*_{mrs}$	Source	Min $E(\widetilde{TC})$
1	3	0.0224602		0.0225529	3	
2	2	0.00421203	43.7219	0.00399495	2	42.4269
3	2	0.0119767		0.0119574	2	

Table 12: The results of crisp and fuzzy values for Erlang distribution

β_r	s	Item 1			Item 2			Item 3		
		λ_{hr}	Q_{m1}	$E(TC)_1$	λ_{hr}	Q_{m2}	$E(TC)_2$	λ_{hr}	Q_{m3}	$E(TC)_3$
	1	18.8942	0.0446332	16.8831	18.7561	0.0257575	14.6244	18.5436	0.0380102	19.3661
0.1	2	18.5063	0.0321026	16.7612	18.6905	0.0152993	13.3434	18.5023	0.0306076	17.7939
	3	18.4067	0.056787	17.0916	18.4789	0.0336475	15.5893	-	-	-
	1	12.7482	0.0394131	16.4753	12.6346	0.0227467	14.2703	12.4997	0.0335656	18.8525
0.2	2	12.4683	0.0283495	16.4716	12.5792	0.0135114	13.1326	12.4641	0.0270294	17.3795
	3	12.4228	0.0501425	16.589	12.4508	0.0297136	15.1351	-	-	-
	1	8.49751	0.0348043	16.1553	8.40787	0.020088	13.9931	8.32196	0.0296413	18.4499
0.3	2	8.29683	0.0250355	16.2448	8.36334	0.0119325	12.9678	8.29304	0.0238699	17.055
	3	8.27971	0.0442769	16.1939	8.28569	0.02624	14.7792	-	-	-
	1	5.55889	0.030735	15.9045	5.49015	0.0177403	13.7762	5.4353	0.0261762	18.1344
0.4	2	5.41583	0.0221092	16.0672	5.45567	0.0105381	12.839	5.4127	0.0210799	16.801
	3	5.41277	0.0390985	15.8837	5.4087	0.0231728	14.5005	-	-	-
	1	3.52821	0.0271419	15.7079	3.47659	0.015667	13.6066	3.44144	0.0231164	17.8874
0.5	2	3.42671	0.0195251	15.9282	3.45004	0.0093076	12.7385	3.42424	0.0186161	16.6022
	3	3.42995	0.0345265	15.6402	3.42213	0.0204643	14.2824	-	-	-
	1	2.12543	0.0239692	15.5539	2.08727	0.0138361	13.474	2.06467	0.0204145	17.6941
0.6	2	2.05373	0.0172431	15.8195	2.06781	0.0082193	12.6599	2.0518	0.0164405	16.4467
	3	2.05921	0.0304898	15.4494	2.05077	0.0180725	14.1117	-	-	-
	1	1.15674	0.0211675	15.4334	1.12883	0.0122193	13.3704	1.11426	0.0180286	17.5428
0.7	2	1.10623	0.015228	15.7345	1.11455	0.0072589	12.5986	1.10478	0.0145192	16.3252
	3	1.11202	0.0269254	15.2999	1.10432	0.0159604	13.9783	-	-	-
	1	0.48798	0.0186935	15.3391	0.46778	0.0107914	13.2894	0.458843	0.0159216	17.4245
0.8	2	0.45251	0.0134484	15.668	0.4574	0.00641072	12.5506	0.451431	0.0128225	16.2302
	3	0.45773	0.023778	15.1827	0.45126	0.0140953	13.8739	-	-	-
	1	**0.0264**	**0.0165088**	**15.2653**	**0.01189**	**0.00953041**	**13.2262**	**0.00576**	**0.014061**	**17.332**
0.9	**2**	**0.00155**	**0.0118769**	**15.6161**	**0.00441**	**0.00566166**	**12.5132**	**0.00077**	**0.0113241**	**16.1559**
	3	**0.0059**	**0.0209987**	**15.0911**	**0.00074**	**0.0124481**	**13.7923**	**-**	**-**	**-**

$\tilde{\beta}_r$		$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$\tilde{\lambda}_{hr}$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$
	1	18.4037	0.0457318	16.4095	18.5695	0.0256674	14.1518	18.4728	0.0380189	18.8938
0.1	2	19.9463	0.0317638	16.384	18.6373	0.0148251	12.8706	18.5062	0.0303469	17.3217
	3	18.436	0.0569856	16.6196	18.5403	0.0333411	15.1173	-	-	-
	1	12.4103	0.040383	16.0124	12.5058	0.0226671	13.808	12.4508	0.0335733	18.3905
0.2	2	13.4621	0.0280504	16.084	12.542	0.0130926	12.6702	12.4665	0.0267993	16.9175
	3	12.4433	0.0503178	16.1271	12.493	0.029443	14.6733	-	-	-
	1	8.26471	0.0356606	15.7008	8.31889	0.0200178	13.5389	8.28819	0.0296481	17.9959
0.3	2	8.98264	0.0247714	15.849	8.3374	0.0115627	12.5136	8.29459	0.0236666	16.6011
	3	8.29395	0.0444317	15.74	8.31459	0.0260011	14.3254	-	-	-
	1	5.39851	0.0314911	15.4564	5.42874	0.0176783	13.3284	5.41199	0.0261822	17.6867
0.4	2	5.88902	0.0218759	15.665	5.43762	0.0102115	12.3912	5.41366	0.0209004	16.3533
	3	5.4227	0.0392351	15.4361	5.42855	0.0229618	14.0529	-	-	-
	1	3.41772	0.0278095	15.2649	3.43423	0.0156122	13.1638	3.42536	0.0231217	17.4447
0.5	2	3.75311	0.0193191	15.521	3.4379	0.00901832	12.2956	3.42484	0.0184576	16.1595
	3	3.43684	0.0346472	15.1976	3.43576	0.020278	13.8397	-	-	-
	1	2.04932	0.0245587	15.1149	2.05802	0.0137878	13.0351	2.05357	0.0204192	17.2552
0.6	2	2.27884	0.0170612	15.4084	2.0591	0.00796455	12.221	2.05219	0.0163005	16.0079
	3	2.06401	0.0305962	15.0106	2.06013	0.017908	13.6729	-	-	-
	1	1.1043	0.0216881	14.9974	1.10866	0.0121766	12.9345	1.10661	0.0180327	17.1069
0.7	2	1.26146	0.0150674	15.3203	1.10851	0.00703392	12.1627	1.10502	0.0143956	15.8893
	3	1.11535	0.0270194	14.864	1.11074	0.0158152	13.5424	-	-	-
	1	0.45185	0.0191533	14.9055	0.45386	0.0107537	12.8559	0.453059	0.0159253	16.991
0.8	2	0.55954	0.0133066	15.2515	0.45321	0.00621204	12.1171	0.451591	0.0127133	15.7967
	3	0.46003	0.023861	14.7493	0.45568	0.013967	13.4404	-	-	-
	1	**0.00152**	**0.0169148**	**14.8337**	**0.0023**	**0.00949709**	**12.7945**	**0.00212**	**0.0140642**	**16.9004**
0.9	**2**	**0.07535**	**0.0117516**	**15.1977**	**0.0015**	**0.00548621**	**12.0816**	**0.00087**	**0.0112277**	**15.7243**
	3	**0.0075**	**0.021072**	**14.6594**	**0.0038**	**0.0123348**	**13.3607**	**-**	**-**	**-**

Table 13: The Optimal policy variable for Erlang distribution at $\beta = 0.9$

Item	Source	Q^*_{mrs}	Min E(TC)	$\tilde{Q}^*_{mrs}$	Source	Min $E(\widetilde{TC})$
1	3	0.0209987		0.021072	3	
2	2	0.00566166	43.7602	0.00548621	2	42.4653
3	2	0.0113241		0.0112277	2	

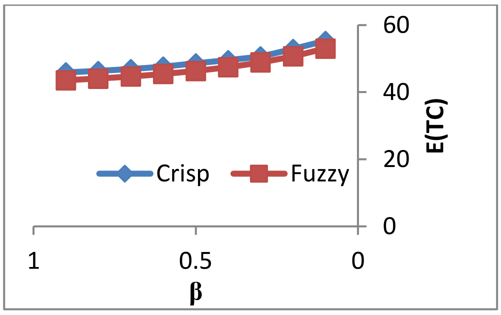

Figure 4: Crisp and fuzzy value for two-parameter exponential distribution

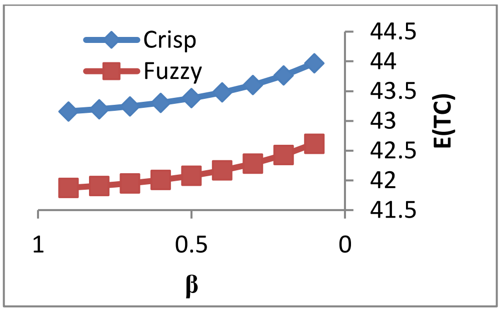

Figure 5: Crisp and fuzzy value for Kumaraswamy distribution

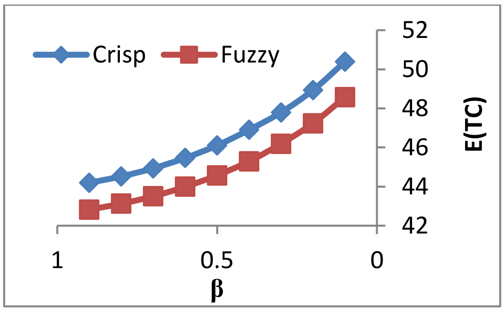

Figure 6: Crisp and fuzzy value for Gamma distribution

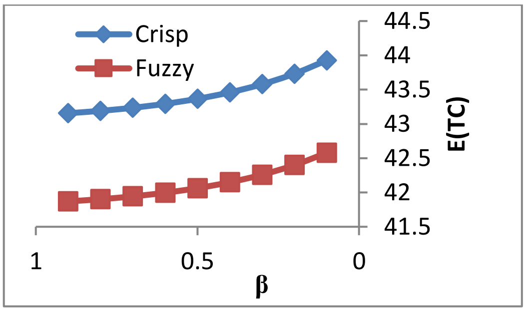

Figure 7: Crisp and fuzzy value for Beta distribution

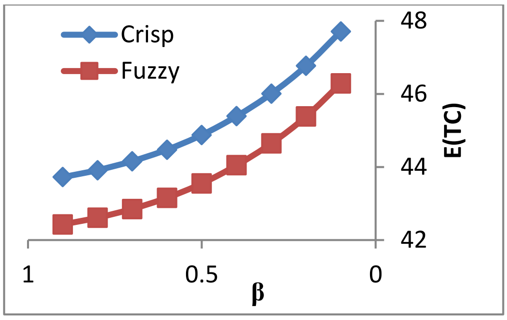

Figure 8: Crisp and fuzzy value for Raylieph distribution

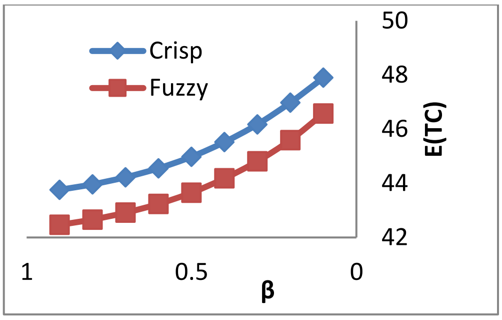

Figure 9: Crisp and fuzzy value for Erlang distribution

7. Conclusion

This paper concerns with a multi-item multi-source (MIMS) constrained probabilistic periodic review inventory model. We determine the optimal maximum inventory level for a given N that minimized the expected annual total cost under varying holding cost constraint using Lagrange multiplier technique for crisp and TFN environment. And we conclude that: the fuzzy environment is more closed to the practical situation than crisp number. Also, When β convergence to 1, the solution approaching to the optimal solution. Furthermore, under our assumption and from selected distributions: Raylieph distribution give the optimum expected annual total cost.

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