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On Hydromagnetic Channel Flow of an Oldroyd-B Fluid induced by Pulses of Longitudinal Impulses

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ABSTRACT:

A flow problem concerning the motion of an incompressible electrically conducting Oldroyd-B fluid in a channel bounded by two infinite rigid non-conducting parallel plates in presence of an external magnetic field acting in a direction normal to the plates has been solved in this paper. The unsteady motion is supposed to generate impulsively from rest in the fluid due to pulses of longitudinal impulses applied periodically on the upper plate in its own plane with the lower plate held fixed. There is no external electric field acting on the system and the magnetic Reynolds number is very small. The operational method is used to obtain the exact solutions for the velocity field and the skin-friction on the walls. The influence of the magnetic field and the fluid elasticity on the flow as well as on the skin-frictions is examined quantitatively. Solutions for the hydromagnetic and hydrodynamic situations are derived as special cases of the present analysis.

Keywords:

Hydromagnetic; impulsive motio; Oldroyd-B fluid.

Mathematics Subject Classification(2000): 76A10.

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1. INTRODUCTION

It is well-known that the fluid flow generated by pulsatile motion of the boundary has important applications in aerodynamics, nuclear technology, astrophysics, geophysics, atmospheric science and cosmic gas dynamics. The investigation in this direction was presented by Chakraborty and Ray [1] who examined the magneto-hydrodynamic Couette flow of an electrically conducting incompressible viscous fluid between two non-conducting parallel plates when the upper plates is set in motion by random pulses. Makar [2] presented the solution of magneto-hydrodynamic flow between two parallel plates when one of the plates is subjected to velocity tooth pulses and the induced magnetic field is neglected. Bestman and Njoku [3] constructed solution of hydrodynamic channel flow of an incompressible, electrically conducting viscous fluid induced by tooth pulses including the effect of induced magnetic field, ignored by the author [2], and using the methodology of Fourier analysis instead of applying the commonly used technique of Laplace transforms which involve complicated inversions. Ghosh and Debnath [4] considered the hydromagnetic channel flow of a two-phase fluid-particle system induced by tooth pulses and obtained solution using the method of Laplace transforms. Datta and Dalal [5,6] discussed the pulsatile flow and heat transfer of a dusty fluid in a channel and in an annular pipe employing the method of perturbation. On the other hand, Hayat et al. [7] have studied some simple flows of an Oldroyd-B fluid using the method of Fourier transforms. Asghar et al. [8] also utilized the same methodology as that of authors [7] to solve the problem concerning Hall effect on unsteady hydromagnetic flows of an Oldroyd-B fluid while Hayat et al. [9] constructed the solution of hydromagnetic Couette flow of an Oldroyd-B fluid in a rotating system following the method of perturbation. Most recently, Ghosh and Sana[10] discussed the hydromagnetic channel flow of an Oldroyd-B fluid induced by recti fied sine pulses with a view to its application in several astrophysical situations. In the present paper, the problem as that of authors [1] has been studied in the case of an Oldroyd-B fluid, which takes into account both the elastic and memory effects exhibited by most polymeric and biological liquids, when the upper plate is set in motion impulsively from rest by pulses of longitudinal impulses of equal strength instead of random pulses of different strengths as considered by authors [1]. It appears that, besides various applications mentioned above, the present investigation is particularly useful in mobile technology, signal processing, seismology and in detecting the e ffect of magnetic field on electrically conducting physiological fluid flow systems. The problem is concerned with the analysis of unsteady hydromagnetic flow of an in- compressible, electrically conducting Oldryod-B fluid in a channel bounded by two in finite rigid non-conducting parallel plates. The motion is suppose to generate impulsively from rest in the fluid due to pulses of longitudinal impulses subjected periodically on the upper plate with the lower plate held fixed. Exact expression for the fluid velocity is obtained using the method of Laplace transforms. The results for the skin-friction on the walls are also obtained. The effects of the magnetic field and the fluid elasticity on the flow and also on the skin-friction on the plates are examined quantitatively. It is observed that the viscoelastic flows neither grow nor decay as fast as the ordinary viscous fluids. The magnetic field has a damping effect on the flow. The increase of fluid elasticity also diminishes the flow since it is retarding in nature. The structure and the magnitude of the flow is found to remain independent with the increase of time period of impulses subjected on the plate. The classical hydromagnetic and hydrodynamic results corresponding to the present problem have been derived. The work of authors [1] has also been compared with the present solution for the velocity field as a special case.

2. BASIC EQUATIONS

The constitutive equations for an Oldroyd-B fluid [7-9] are

$$\mathbf{T} = -p\,\mathbf{I} + \mathbf{S} \tag{2.1}$$

$$\mathbf{S} + \lambda_1 \frac{D\,\mathbf{S}}{D\,t} = \mu\left[1 + \lambda_2 \frac{D}{D\,t}\right]\mathbf{A_1} \tag{2.2}$$

where T = Cauchy stress tensor, p = fluid pressure, I = identity tensor, S = extra stress tensor, $\mu, \lambda_1, \lambda_2$= viscosity coefficient , relaxation time, retardation time (assumed constants).The tensor A_1 is defined as

$$\mathbf{A_1} = \nabla\,\mathbf{V} + (\nabla\,\mathbf{V})^T. \tag{2.3}$$

In a cartesian system, $\frac{D}{Dt}$ (upper convected time derivative) operating on any tensor B_1 is

$$\frac{D\mathbf{B_1}}{D\,t} = \frac{\partial \mathbf{B_1}}{\partial\,t} + (\mathbf{V}\cdot\nabla)\,\mathbf{B_1} - (\nabla\,\mathbf{V})\,\mathbf{B_1} - \mathbf{B_1}\,(\nabla\,\mathbf{V})^T. \tag{2.4}$$

It is to be mentioned here that this model includes the viscous fluid as a particular case for $\lambda_1 = \lambda_2$; the Maxwell fluid when $\lambda_2 = 0$ and an Oldroyd-B fluid when $0 < \lambda_2 < \lambda_1 < 1$.

The stress equations of motion for an incompressible electrically conducting Oldroyd-B fluid in presence of an external magnetic fluid are

$$\nabla \cdot \mathbf{V} = 0, \qquad (2.5)$$

$$\rho \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} \right] = \nabla \cdot \mathbf{T} + \mathbf{J} \times \mathbf{B}, \qquad (2.6)$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \qquad (2.7ab)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \mathbf{J} = \sigma [\mathbf{E} + \mathbf{V} \times \mathbf{B}] \qquad (2.8ab)$$

where V = (u,v,w) = fluid velocity, ρ = fluid density, J = current density, B = magnetic flux density, E = electric field, μ0 = magnetic permeability (assumed constant), σ= electrical conductivity (assumed finite).

3. FORMULATION OF THE PROBLEM

In this problem, we consider the motion of an incompressible electrically conducting Oldroyd-B fluid between two infinite rigid non-conducting parallel plates separated by a distance h. The x-axis is taken in the direction of flow with origin at the lower plate and y-axis perpendicular to the plates. The initial motion is generated in the fluid due to train of longitudinal impulses subjected periodically on the upper plate with the lower plate held fixed. A uniform magnetic field of strength B_0 is acting parallel to y-axis. We assume that no external electric field is acting on the fluid and the magnetic Reynolds number is very small. This implies that the current is mainly due to induced electric field and the applied magnetic field remains essentially unaltered by the electric current flowing in the fluid. We also assume that the induced magnetic field produced by the motion of the fluid is negligible compared to the applied magnetic field so that the Lorentz force term in (2.6) becomes $-\sigma B^2_0 V$.

Since the motion is a plain one and the plates are infinitely long, we assume that all the physical variables are independent of x and z. Then from the equation of continuity (2.5) and from the physical condition of the problem , we take

$$\mathbf{V} = \{u(y,t),\ 0,\ 0\} \quad and \quad \mathbf{S} = \mathbf{S}(y,t). \qquad (3.1ab)$$

The equations of motion in (2.6) then reduces to

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \sigma B_0^2 u, \qquad (3.2)$$

$$\frac{\partial p}{\partial y} = \frac{\partial S_{yy}}{\partial y}, \qquad (3.3)$$

$$\frac{\partial p}{\partial z} = 0. \qquad (3.4)$$

It follows from (2.2) and (3.1ab) that

$$S_{xx} + \lambda_1 \left[\frac{\partial S_{xx}}{\partial t} - 2 S_{xy} \frac{\partial u}{\partial y}\right] = -2\mu\lambda_2 \left(\frac{\partial u}{\partial y}\right)^2, \qquad (3.5)$$

$$S_{xy} + \lambda_1 \left[\frac{\partial S_{xy}}{\partial t} - S_{yy} \frac{\partial u}{\partial y}\right] = \mu \left(\frac{\partial u}{\partial y}\right) + \lambda_2 \mu \left(\frac{\partial^2 u}{\partial y \partial t}\right), \qquad (3.6)$$

$$S_{yy} + \lambda_1 \frac{\partial S_{yy}}{\partial t} = 0. \qquad (3.7)$$

The Equation (3.7) gives

$$S_{yy} = A(y)\, e^{-t/\lambda_1} \qquad (3.8)$$

where A(y) is an arbitrary function of y. But S_{yy} is known to be zero for $t < 0$. This implies that A(y) must be zero. Hence *Syy* is zero always. Consequently, from (3.2) and (3.6) and in absence of pressure gradient along x direction, we get

$$\left(1+ \lambda_1 \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = \nu \left(1+ \lambda_2 \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \left(1+ \lambda_1 \frac{\partial}{\partial t}\right) u \qquad (3.9)$$

which on introducing the dimensionless quantities given by

$$\bar{u} = \frac{u}{U_0},\ \bar{y} = \frac{y}{\sqrt{\nu \lambda_1}},\ \bar{t} = \frac{t}{\lambda_1},\ d = \frac{h}{\sqrt{\nu \lambda_1}},\ k = \frac{\lambda_2}{\lambda_1} (\leq 1) \ and\ M^2 = \frac{\sigma B_0^2 \lambda_1}{\rho}$$

and on dropping the bars, yields

$$(1+\frac{\partial}{\partial t})\frac{\partial u}{\partial t} = (1+k\frac{\partial}{\partial t})\frac{\partial^2 u}{\partial y^2} - M^2(1+\frac{\partial}{\partial t})u. \qquad (3.10)$$

The problem now reduces to solving (3.10) subject to boundary and initial conditions:

$$u(0,t) = 0, \quad u(d,t) = f(t) \quad for\ all\ t > 0 \qquad (3.11)$$

$$and \quad u(y,0) = 0, \quad u_t(y,0) = 0 \quad for\ 0 \le y \le d \qquad (3.12)$$

where f(t) representing the train of longitudinal impulses, as shown in Figure 1, is a periodic function of time with period T and each of strength E_0.

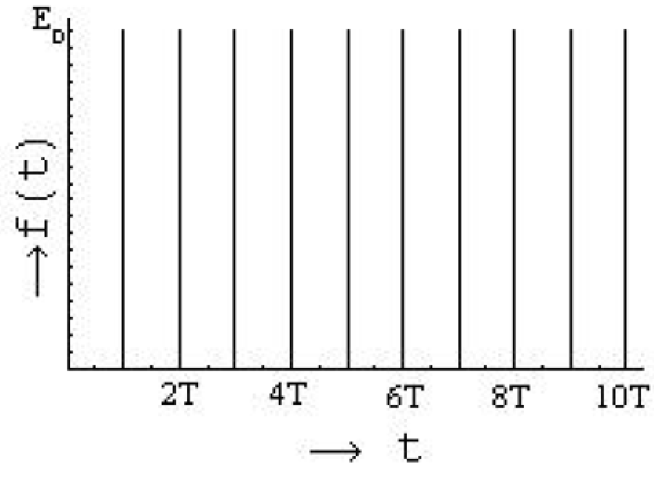

Figure 1: Longitudinal Impulses

4. SOLUTION OF THE PROBLEM

According to the nature of f(t) mentioned above, the mathematical form of u(d,t) may be written as

$$u(d,t) = E_0 \sum_{m=0}^{\infty} \delta(t - mT) \qquad (4.1)$$

where δ(t) is Dirac's Delta function which is defined as

$$\delta(t) = 0, \quad t \neq 0, \quad \delta(t) = \infty, \quad t = 0; \quad \int_{-\infty}^{\infty} \delta(t)\ dt = 1$$
$$\text{and} \quad \int_{-\infty}^{\infty} \delta(t)\ \phi(t)\ dt = \phi(0).$$

The use of Laplace transform method for the solution of the Eqn.(3.10) with initial conditions in Eqn.(3.12), provides the transformed equation for the fluid velocity in the form

$$\frac{d^2\overline{u}}{dy^2} - L^2\overline{u} = 0 \qquad (4.2)$$

with the boundary conditions

$$\overline{u}(0,s) = 0, \quad \overline{u}(d,s) = E_0 \sum_{m=0}^{\infty} e^{-mTs} \qquad (4.3)$$

where s is the Laplace transform variable and $L^2 = \frac{(s+M^2)(1+s)}{1+ks}.$

The transformed solution for the fluid velocity $\overline{u}(y,s)$ becomes

$$\overline{u}(y,s) = E_0 \sum_{m=0}^{\infty} e^{-mTs} \frac{SinhLy}{SinhLd}. \qquad (4.4)$$

The Inversion of Eqn.(4.4) gives

$$u(y,t) = \frac{E_0}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st} \sum_{m=0}^{\infty} e^{-mTs} \frac{SinhLy}{SinhLd}\, ds$$

$$= \frac{E_0}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{e^{st}}{(1-e^{-st})} \frac{SinhLy}{SinhLd}\, ds. \qquad (4.5)$$

The inversion integral has poles at s = 0, a series of poles at $s = \pm i\beta_m$; m = 1; 2; …. and two other poles at s = s1 ; s2. Evaluating Eqn.(4.5) with the help of Cauchy's Residue theorem, we get

$$\frac{u(y,t)}{E_0} = \frac{1}{T}\frac{sinhMy}{sinhMd} + \frac{2}{T} Re\sum_{m=1}^{\infty} \frac{e^{i\beta_m t}\, sinhL_m y}{sinhL_m d} + \frac{2k\pi}{d^2}\sum_{n=1}^{\infty} n\,(-1)^{n+1} B\, sin\frac{n\pi y}{d} \qquad (4.6)$$

where $\beta_m = \frac{2m\pi}{T}$, $L_m = [\ \frac{(1+i\,\beta_m)(M^2+i\,\beta_m)}{1+i\,\beta_m\,k}\ ]^{1/2}$,

$B = B_1 + B_2$, $\qquad B_j = \frac{e^{s_j t}}{(1-e^{-s_j t})\{1+\frac{(1-k)\ (M^2k-1)}{(1+ks_j)^2}\}}$, $j = 1, 2,$

$$2s_1,\ 2s_2 = -\ [\ (1+M^2+\frac{n^2\pi^2 k}{d^2}) \mp \sqrt{(1+M^2+\frac{n^2\pi^2 k}{d^2})^2 - 4(M^2+\frac{n^2\pi^2}{d^2})}\ ].$$

In the event, the number of pulses is finite in number, i.e. $f(t) = E_0 \sum_{m=0}^{N} \delta(t-mT)$, the fluid velocity corresponding to the present problem turns out as

$$\frac{u(y,t)}{E_0} = \frac{2k\pi}{d^2}\sum_{n=1}^{\infty} n\,(-1)^{n+1} \sum_{m=0}^{N} G\, sin\frac{n\pi y}{d} \qquad (4.7)$$

where $G = G_1 + G_2$, $\qquad G_j = \frac{e^{s_j(t-mT)}}{1+\frac{(1-k)\ (M^2k-1)}{(1+ks_j)^2}}$, $j = 1, 2,$

$$2s_1,\ 2s_2 = -\ [\ (1+M^2+\frac{n^2\pi^2 k}{d^2}) \mp \sqrt{(1+M^2+\frac{n^2\pi^2 k}{d^2})^2 - 4(M^2+\frac{n^2\pi^2}{d^2})}\ ].$$

The result (4.7), in the case d=1 and k=1 i.e ($\lambda 1 = \lambda 2 = 0$) which corresponds to Newtonian viscous fluid yields

$$\frac{u(y,t)}{E_0} = 2\,\pi \sum_{n=1}^{\infty} n\,(-1)^{n+1} \sum_{m=0}^{N} e^{-(t-mT)(M^2+n^2\,\pi^2)}\, sin\frac{n\pi y}{d}. \qquad (4.8)$$

The result (4.8) coincides exactly with the finding of the authors [1] provided the variable strength of impulses, *An*, in their result is replaced by E_0. However, the result (4.8) has a drawback because m-series is not convergent. Under the circumstances, we have considered the train of impulses executed periodically on the plate to arrive at the solution (4.6). On the other hand, in the limit $k \to 1$, the hydromagnetic solution corresponding to the present problem as obtained from Eqn.(4.6) is given by

$$\frac{u(y,t)}{E_0} = \frac{1}{T}\frac{sinh\ My}{sinh\ Md} + \frac{2}{T} Re\sum_{m=1}^{\infty} \frac{e^{i\beta_m t}\ sinh\ L_m^* y}{sinh\ L_m^* d}$$

$$+\frac{2\,\pi}{d^2}\sum_{n=1}^{\infty} n\,(-1)^{n+1}\frac{e^{-(M^2+\frac{n^2\,\pi^2}{d^2})t}}{1-e^{(M^2+\frac{n^2\,\pi^2}{d^2})T}}\, sin\frac{n\pi y}{d}, \qquad (4.9)$$

where $L_m^* = \sqrt{M^2 + i\beta_m}$.

Further, when M=0, (4.9) provides the hydrodynamic result

$$\frac{u(y,t)}{E_0} = \frac{1}{T}\frac{y}{d} + \frac{2}{T} Re\sum_{m=1}^{\infty} \frac{e^{i\beta_m t}\ sinh\ \sqrt{i\ \beta_m}\ y}{sinh\ \sqrt{i\ \beta_m}\ d}$$

$$+\frac{2\,\pi}{d^2}\sum_{n=1}^{\infty} n\,(-1)^{n+1}\frac{e^{-\frac{n^2\,\pi^2}{d^2}t}}{1-e^{\frac{n^2\,\pi^2}{d^2}T}}\, sin\frac{n\pi y}{d}\ . \qquad (4.10)$$

The exact expression for the skin-friction at the lower plates (y=0) is given by

$$\frac{\tau_0}{E_0} = \frac{M}{T}\frac{(1-e^{-t})}{\sinh Md} + \frac{2}{T} Re \sum_{m=1}^{\infty} \frac{L_m}{\sinh L_m d} \frac{1+ik\beta_m}{1+i\beta_m} (e^{i\beta_m t} - e^{-t})$$

$$+ \frac{2k\pi^2}{d^3} \sum_{n=1}^{\infty} n^2 (-1)^{n+1} D, \qquad (4.11)$$

where $\beta_m = \frac{2m\pi}{T}, \quad L_m = [\frac{(1+i\beta_m)(M^2+i\beta_m)}{1+i\beta_m k}]^{1/2},$

$$D = D_1 + D_2,$$

$$D_j = \frac{1+ks_j}{1+s_j} \frac{e^{s_j t} - e^{-t}}{(1-e^{-s_j T})[1 + \frac{(1-k)(M^2k-1)}{(1+ks_j)^2}]}, \quad j = 1, 2,$$

$$2s_1, 2s_2 = -[(1+M^2+\frac{n^2\pi^2 k}{d^2}) \mp \sqrt{(1+M^2+\frac{n^2\pi^2 k}{d^2})^2 - 4(M^2+\frac{n^2\pi^2}{d^2})}].$$

Similarly the expression for the skin-friction at the upper plate (y=d) becomes

$$\frac{\tau_d}{E_0} = \frac{M}{T}\frac{\cosh Md}{\sinh Md}(1-e^{-t}) + \frac{2}{T} Re \sum_{m=1}^{\infty} \frac{1+ik\beta_m}{1+i\beta_m} \frac{L_m \cosh L_m d}{\sinh L_m d} (e^{i\beta_m t} - e^{-t})$$

$$- \frac{2k\pi^2}{d^3} \sum_{n=1}^{\infty} n^2 D. \qquad (4.12)$$

When k=1, the hydromagnetic results for the skin-frictions are

$$\frac{\tau_0}{E_0} = \frac{M}{T}\frac{1}{\sinh Md} + \frac{2}{T} Re \sum_{m=1}^{\infty} \frac{\sqrt{M^2+i\beta_m}}{\sinh\sqrt{M^2+i\beta_m}\, d} e^{i\beta_m t}$$

$$+ \frac{2\pi^2}{d^3} \sum_{n=1}^{\infty} n^2 (-1)^{n+1} \frac{e^{-(M^2+\frac{n^2\pi^2}{d^2})t}}{1-e^{(M^2+\frac{n^2\pi^2}{d^2})T}}, \qquad (4.13)$$

$$\frac{\tau_d}{E_0} = \frac{M}{T}\frac{\cosh Md}{\sinh Md}$$

$$+ \frac{2}{T} Re \sum_{m=1}^{\infty} \frac{(\sqrt{M^2+i\beta_m})\cosh\sqrt{M^2+i\beta_m}\, d}{\sinh\sqrt{M^2+i\beta_m}\, d} e^{i\beta_m t}$$

$$- \frac{2\pi^2}{d^3} \sum_{n=1}^{\infty} n^2 \frac{e^{-(M^2+\frac{n^2\pi^2}{d^2})t}}{1-e^{(M^2+\frac{n^2\pi^2}{d^2})T}}. \qquad (4.14)$$

Further, if k=1 and M=0, the hydrodynamic results for the skin-frictions become

$$\frac{\tau_0}{E_0} = \frac{1}{Td} + \frac{2}{T} Re \sum_{m=1}^{\infty} \frac{\sqrt{i\beta_m}}{\sinh\sqrt{i\beta_m}\, d} e^{i\beta_m t}$$

$$+ \frac{2\pi^2}{d^3} \sum_{n=1}^{\infty} n^2 (-1)^{n+1} \frac{e^{-\frac{n^2\pi^2}{d^2}t}}{1-e^{\frac{n^2\pi^2}{d^2}T}}, \qquad (4.15)$$

$$\frac{\tau_d}{E_0} = \frac{1}{Td} + \frac{2}{T} Re \sum_{m=1}^{\infty} \frac{\sqrt{i\beta_m}\cosh\sqrt{i\beta_m}\, d}{\sinh\sqrt{i\beta_m}\, d} e^{i\beta_m t}$$

$$- \frac{2\pi^2}{d^3} \sum_{n=1}^{\infty} n^2 \frac{e^{-\frac{n^2\pi^2}{d^2}t}}{1-e^{\frac{n^2\pi^2}{d^2}T}}. \qquad (4.16)$$

5. NUMERICAL RESULTS

The quantitative analysis of the velocity field, given by Eqn.(4.6),for various values the magnetic field (M) and the fluid elasticity (k) are examined with the progress of time after the first impulse is executed at the upper plate through figures 2(a) to 2(d). It is found that immediately after the impulse is communicated to the plate the fluid in the channel gets a sudden disturbance which tries to settle down after the impulsive action is over. As a result, an unsteady retarding flow exists in the channel which naturally tries to come to rest before receiving the next impulse. The viscous fluid (k=1) comes to rest more quickly than that of viscoelastic fluids. The smaller and smaller the value of k indicates that the fluid is more and more viscoelastic. The increase in the magnetic field (M) and the elasticity (k) of the fluid produce damping effect on the flow. Unlike viscous fluid, viscoelastic fluids come to rest through fluctuation (fig.2(c)) and a small amount of backward flows(fig.2(d)) prevailing in the channel with the progress of time till the next impulse occurs at t=T=1. The execution of the second impulse at t=1.0 on the upper plate provides similar flow structures in the fluid which are shown in figures 3(a) to 3(c).The structure of the velocity profiles with the effects of the magnetic field (M) and the fluid elasticity (k) on them and with the progress of time for the period T=2 are presented in figures 4(a) to 4(e) and figure 5. It is observed that the velocity profiles are independent of time period T of the impulses. In other words, the fluid velocity at time t do not change with the variation of the magnetic field(M) and the fluid elasticity (k) when the time period of impulses changes from T=1 to T=2 as evident from the above figures.

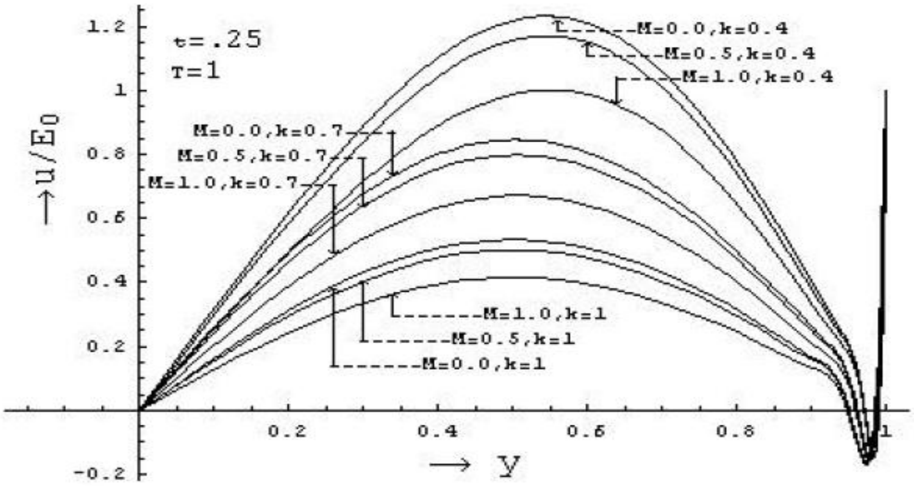

Figure 2(a): Fluid velocity for different values of (M, k) when t= 0.25, T=1 and d=1

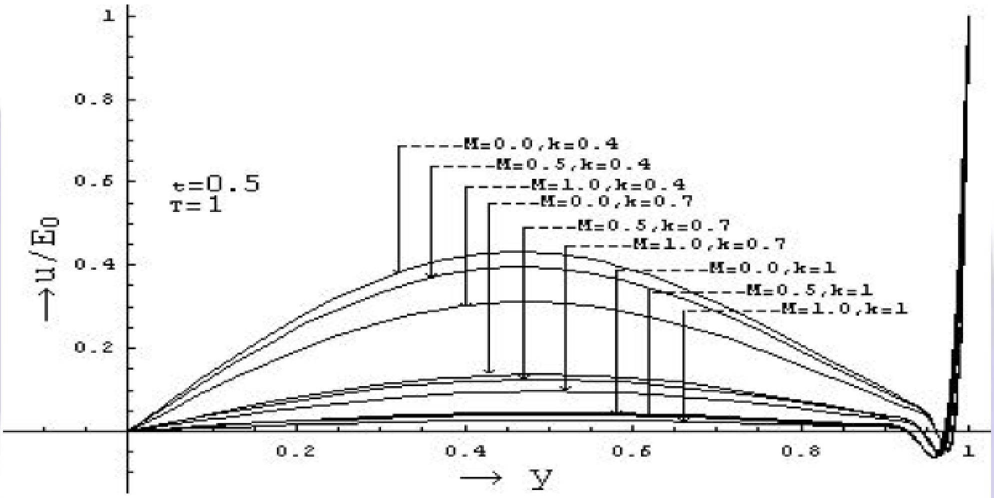

Figure 2(b): Fluid velocity for different values of (M, k) when t=0.5, T=1 and d=1

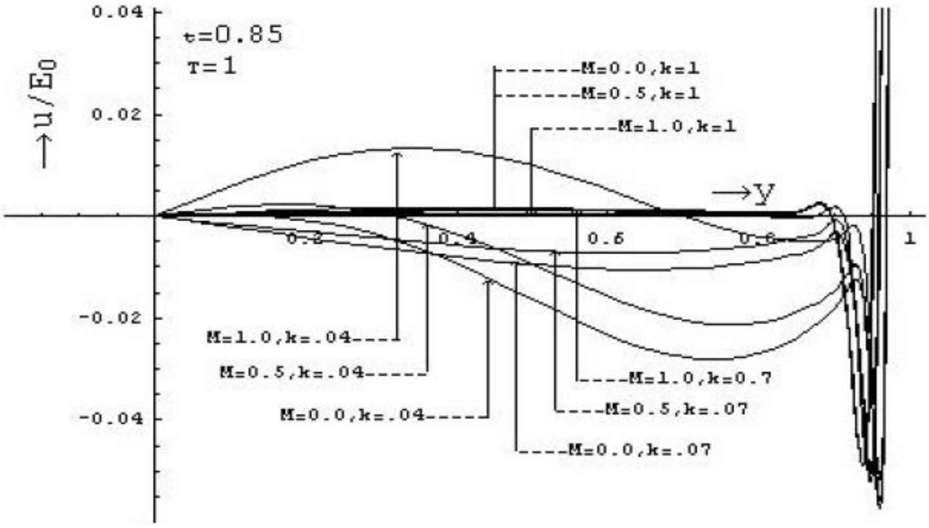

Figure 2(c): Fluid velocity for different values of (M, k) when t=0.85, T=1 and d=1

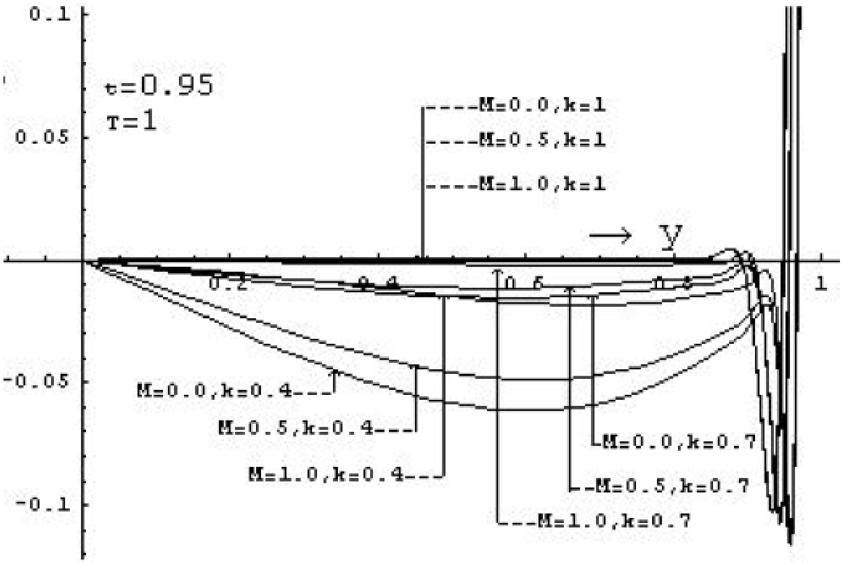

Figure 2(c): Fluid velocity for different values of (M, k) when t=0.95, T=1 and d=1.

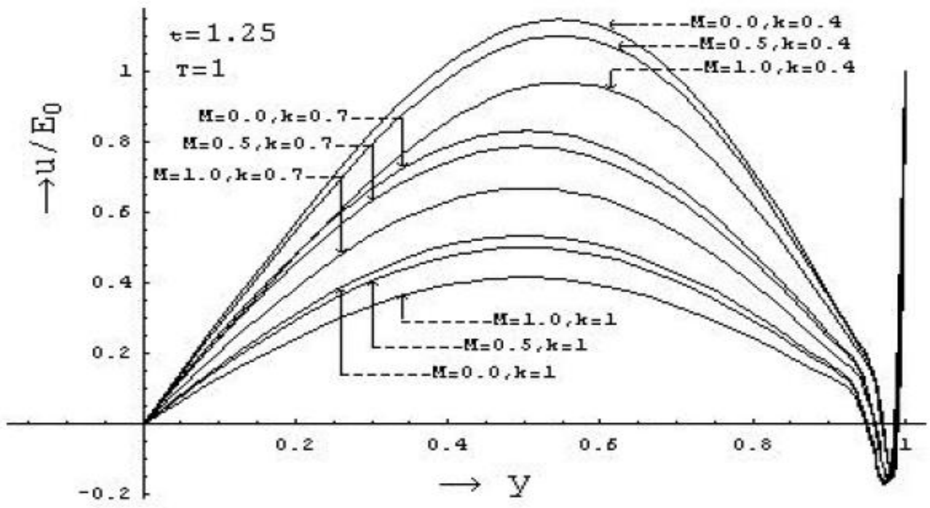

Figure 3(a): Fluid velocity for different values of (M, k) when t=1.25, T=1 and d=1

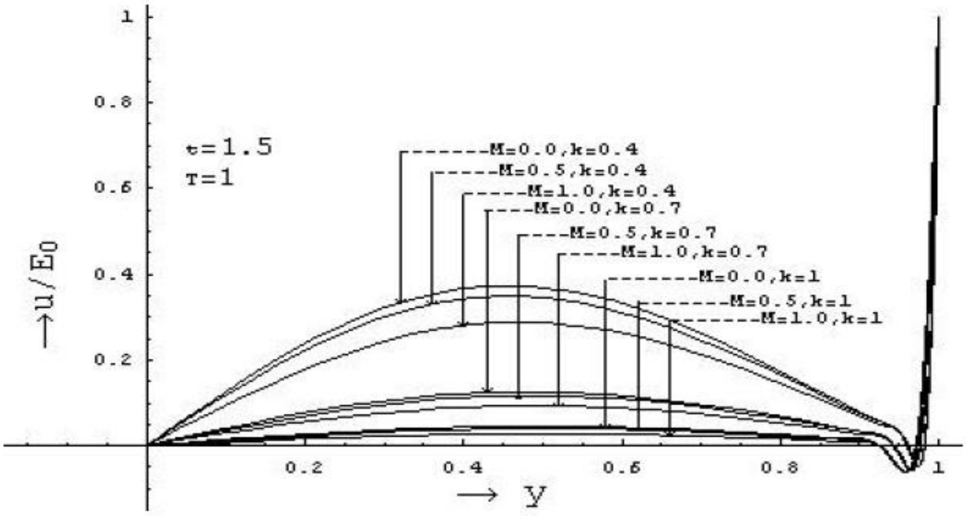

Figure 3(b): Fluid velocity for different values of (M, k) when t=1.5, T=1 and d=1

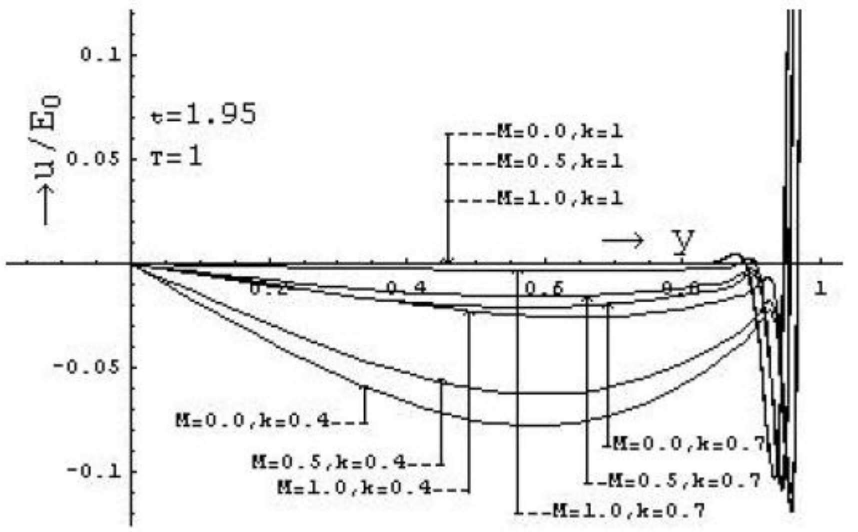

Figure 3(c): Fluid velocity for different values of (M, k) when t=1.95, T=1 and d=1

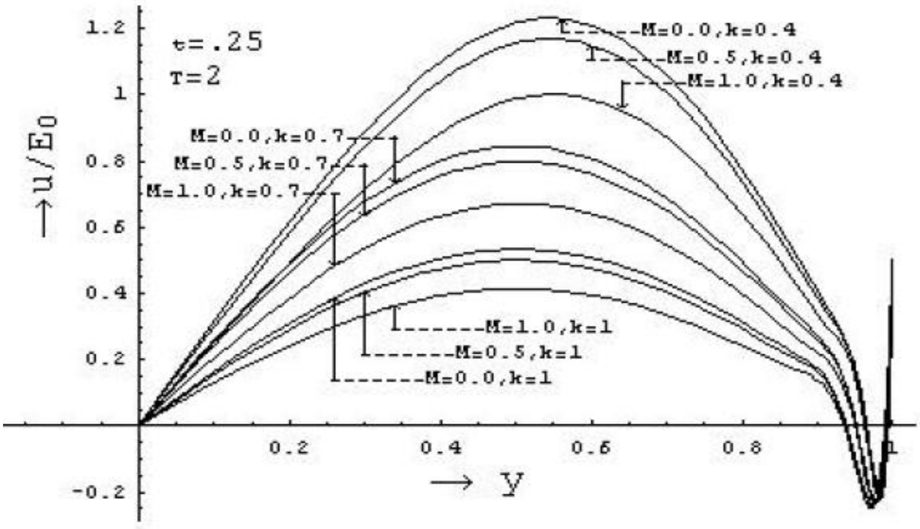

Figure 4(a): Fluid velocity for different values of (M, k) when t=0.25, T=2 and d=1

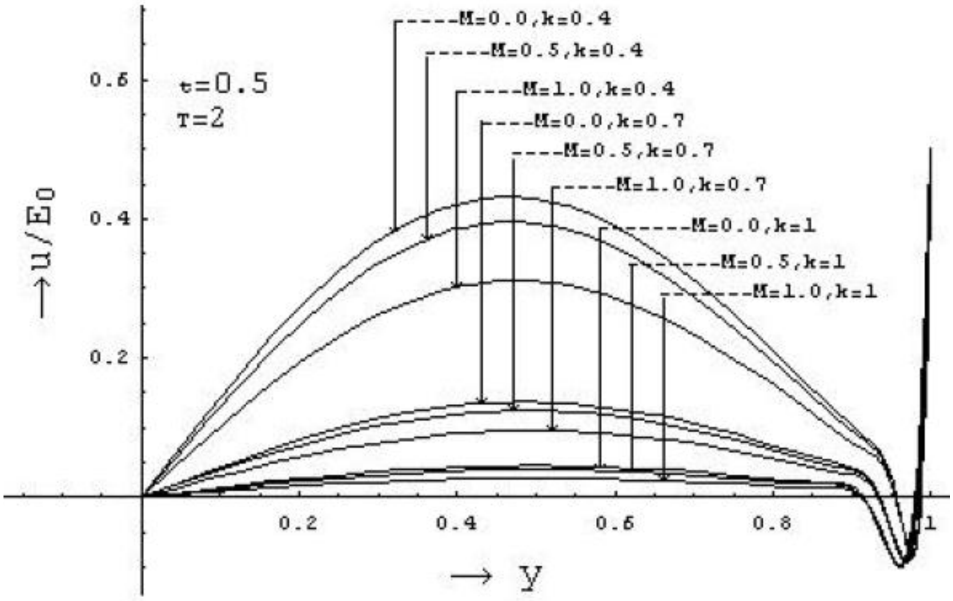

Figure 4(b): Fluid velocity for different values of (M, k) when t=0.5, T=2 and d=1

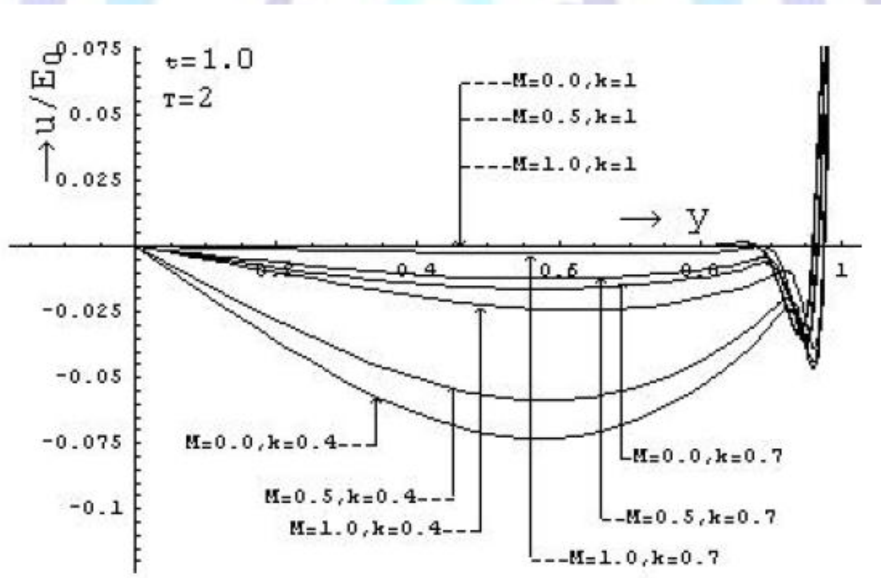

Figure 4(c): Fluid velocity for different values of (M, k) when t=1, T=2 and d=1

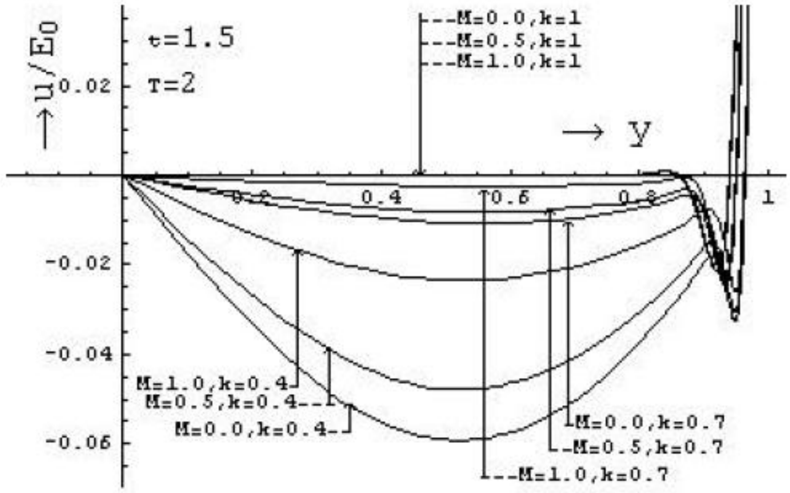

Figure 4(d): Fluid velocity for different values of (M, k) when t=1.5, T=2 and d=1

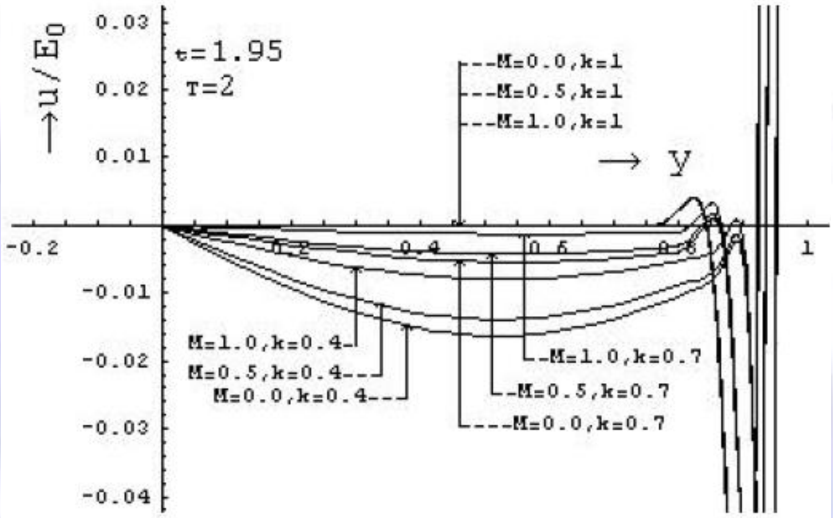

Figure 4(e): Fluid velocity for different values of (M, k) when t=1.95, T=2 and d=1

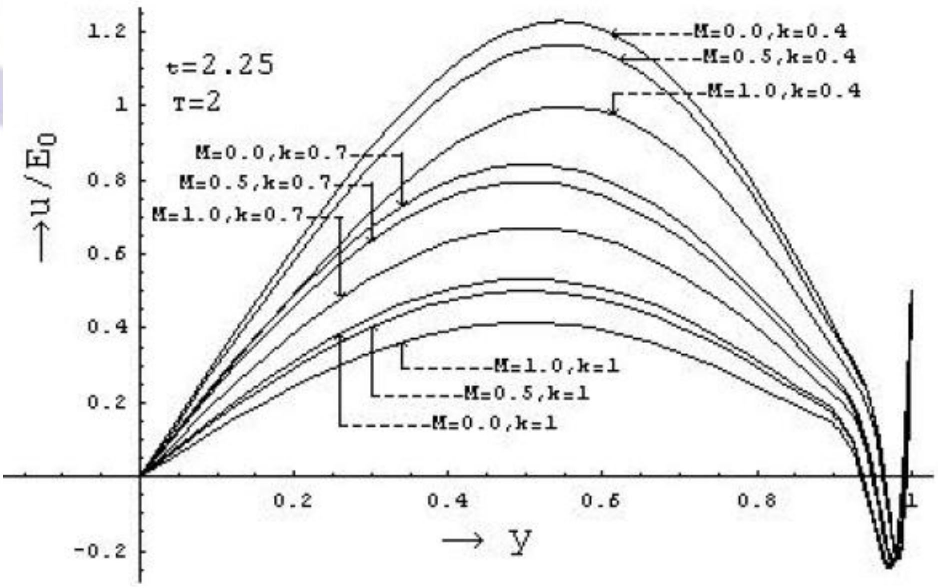

Figure 5: Fluid velocity for different values of (M, k) when t=2.25, T=2 and d=1

The skin-friction at the lower plate(y=0) is presented in figures 6(a) and 6(b) respectively for the cases T=1 and T=2. It is found that for T=1 the skin-friction at the lower plate decreases with the increase of the magnetic field (M)and increases with the increase of fluid elasticity (k). It is further noticed that for all values of the magnetic field(M),the skin- friction at the lower plate for Newtonian fluid remains always higher than that produced by the viscoelastic fluids. Exactly similar situation prevails for T=2 which indicates that skin-friction at the lower plate is also independent of the time period of impulses executed on the upper plate. The skin-friction at the upper plate, presented in figures 7(a) and 7(b), is always negative which is contrary to that found in the lower plate. The magnitude of the skin-friction at the upper plate is more for T=1 than for T=2 which also contradicts the findings at the lower plate. Moreover, the skin-friction at the upper plate is independent of the magnetic field (M) but depends on the fluid elasticity (k) only. Finally, the magnitude of skin-friction at the upper plate for Newtonian viscous fluid remains always greater than that of viscoelastic fluids. This observation is also true for the skin-friction at the lower plate as reported earlier.

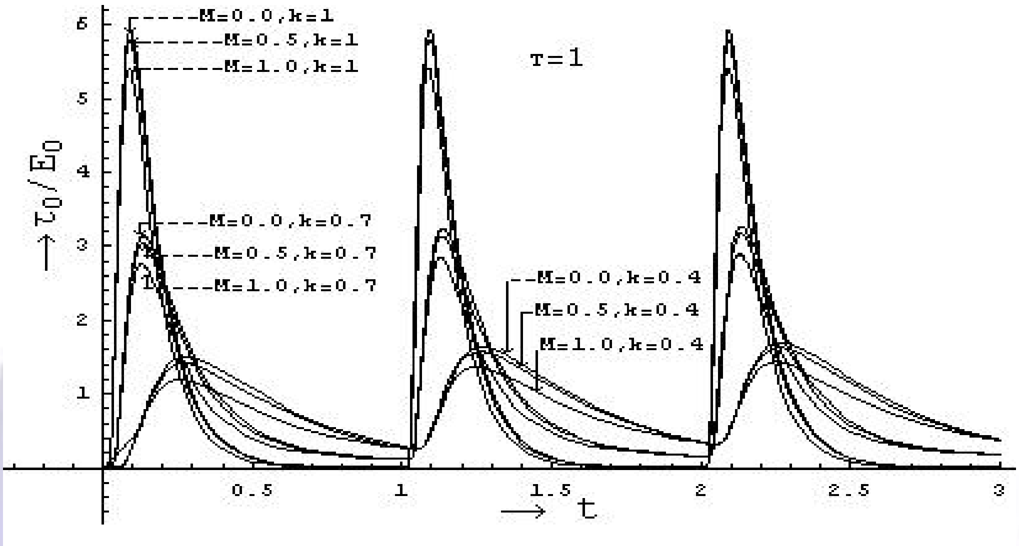

Figure 6(a): Skin- friction (z_0 / E_0) on the lower plate (y=0) for different value of (M, k) when T=1, d=1.

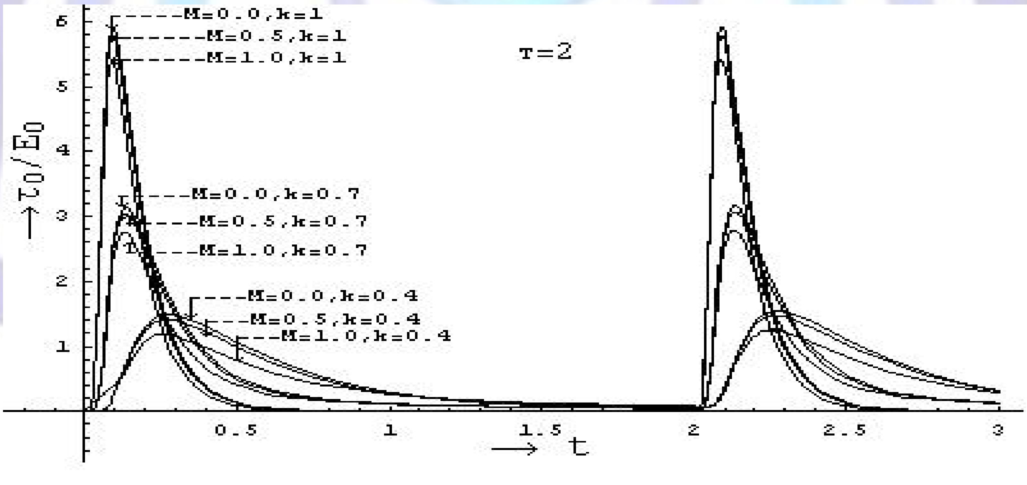

Figure 6(a): Skin- friction (z0 / E0) on the lower plate (y=0) for different value of (M, k) when T=2, d=1.

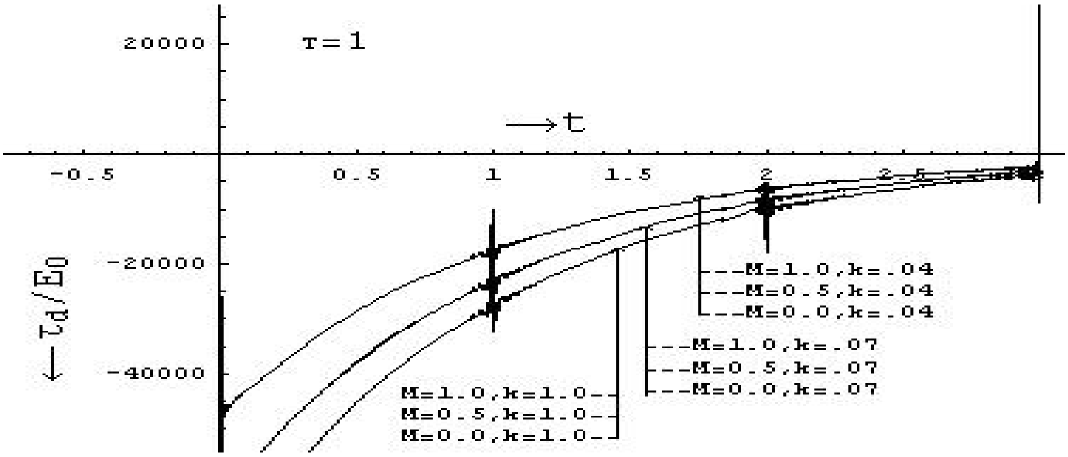

Figure 7(a): Skin- friction (z0 / E0) on the lower plate (y=d) for different value of (M, k) when T=1, d=1.

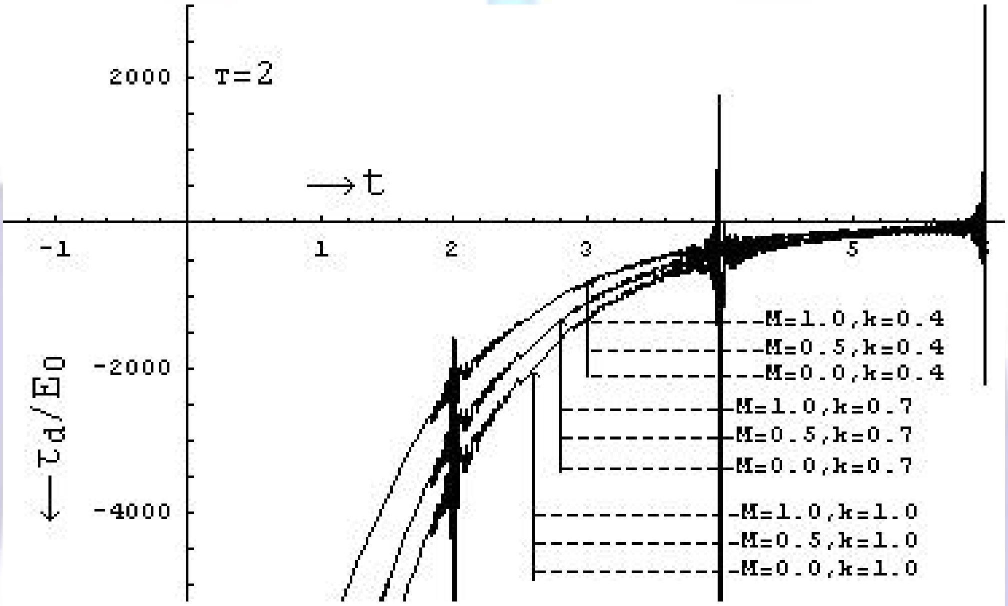

Figure 7(b): Skin- friction (z0 / E0) on the lower plate (y=d) for different value of (M, k) when T=2, d=1.

6. Conclusion

The effect of train of impulses on electrically conducting viscoelastic flow in presence of an external field has been discussed in this paper. The fluid in the channel excited by the impulsive action produces a retarding motion which decreases both with the increase of magnetic field and the elasticity of the fluid. The viscous fluid comes to rest more rapidly than viscoelastic fluids. Contrary to viscous fluids, viscoelastic fluids produce a backward flow, of small magnitude, which persists till the beginning of next impulse. Flows in between successive impulses are of similar kinds and are independent of the time period of occurrence of impulses. The magnitude of skin-frictions on both the plates is greater for viscous fluids than for viscoelastic fluids. The skin-friction is positive at the lower plate and negative at the upper plate which is expected for channel flows induced by excitation of the upper plate. Finally, the skin-friction at the upper plate remains independent of the magnetic field while that at the lower plate decreases with the increase of the magnetic field. The present problem although idealized may provide useful applications in mobile technology, signal processing, seismology and in the analysis of blood flow.

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