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Spectral Continiuty : (p, k) - Quasihyponormal and Totally p − (α, β) normal operators

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ABSTRACT

An operator $T \in B(H)$ is said to be p - (α, β) - normal operators for 0 < p ≤ 1 if $\alpha^2 (T^*T)^p \leq (TT^*)^p \leq \beta^2 (T^*T)^p$, $0 \leq \alpha \leq 1 \leq \beta$. In this paper, we prove that continuity of the set theoretic functions spectrum, Weyl spectrum, Browder spectrum and essential surjectivity spectrum on the classes consisting of (p, k) - quasihyponormal operators and totally p - (α, β) - normal operators.

Indexing terms/Keywords

Weyl's theorem; Single valued extension property; Continuity of spectrum; Fredholm; B – Fredholm; generalized a - Weyl's theorem; B – Fredholm; B - Weyl.

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INTRODUCTION

Let H be an infinite dimensional complex Hilbert space and $B(H)$ denotes the algebra of all bounded linear operators acting on H. Every operator T can be decomposed into $T=U|T|$ with a partial isometry U, where $|T|=\sqrt{TT^*}$. In this paper, T=U|T| denotes the polar decomposition satisfying the kernel condition N(U) = N(|T|).

An operator $T \in B(H)$ is said to be normal if $TT^*=T^*T$ and hyponormal if $T^*T \geq TT^*$. An operator T is said to be Dominant if ran $(T-\lambda I) \subseteq$ ran $(T-\lambda I)^*$ for all $\lambda \in$ C or equivalently there exists a real number M_λ for each $\lambda \in$ C such that $\|(T-\lambda I)^* x\| \leq M_\lambda \|(T-\lambda I)x\|$ for each $x \in H$. If there exists a constant M such that $M_\lambda \subseteq M$ for all $\lambda \in$ C, then T is called M – hyponormal and if M = 1, T is hyponormal. The class of hyponormal operators has been studied by many authors. In recent years this class has been generalized , in some sense , to the larger sets of so called p - hyponormal , log hyponormal ,Posinormal,etc $[31],[32],[29],[26]$ *and* $[27]$.

An operator $T \in B(H)$ is said to be

- p -hyponormal for $0<p<1$ iff $\left(TT^*\right)^p \leq \left(T^*T\right)^p$,
- p -posinormal for $0<p<1$ iff $\left(TT^*\right)^p \leq c^2 \left(T^*T\right)^p$,
- (α,β) - normal operators if $\alpha^2 T^*T \leq TT^* \leq \beta^2 T^*T,\ 0 \leq \alpha \leq 1 \leq \beta$ [30].

The example of an M - hyponormal operator given by Wadhwa [35], the weighted shift operator defined by $Te_1=e_2,\ Te_2=2e_3$ and $Te_i=e_{i+1}$ for $i \geq 0$, is not an p - (α,β) - normal, which is neither normal nor hyponormal. So it is clear that the class of p - (α,β) - normal lies between hyponormal and M - hyponormal operators. Now the inclusion relation becomes

Normal $\subseteq$ Hyponormal $\subseteq$ (α,β) - normal

$\subseteq$ p - (α,β) - normal $\subseteq$ M - hyponormal $\subseteq$ Dominant

S.S Dragomir and M.S.Moslehian [28] and [30] has given various inequalities between the operator norm and numerical radius of (α,β) - normal operators . Weyl type theorems and composition operators of (α,β) have been studied by D.SenthilKumar and Sherin Joy.S.M [33, 34]. As a generalisation of (α,β) - normal operators, we introduce p - (α,β) - normal operators. When p = 1, this coincide with (α,β) - normal operators. An operator T is called totally p - (α,β) - normal, if the translate $T-\lambda$ is p - (α,β) - normal for all $\lambda \in C$.

An operator $T \in B(H)$ is said to be (p,k)-quasihyponormal operator, for some $0<p\leq 1$ and integer $k \geq 1$ if $T^{*k}\left(|T|^{2p}-|T^*|^{2p}\right)T^k \geq 0$. Evidently,

a $(1,k)$ -quasihyponormal operator is *k*-quasihyponormal;

a $(1,1)$ -quasihyponormal operator is quasihyponormal;

a $(p,1)$ -quasihyponormal operator is k -quasihyponormal or quasi- p -hyponormal [8, 10],

a $(p,0)$ -quasihyponormal operator is p -hyponormal if $0<p<1$ and hyponormal if $p=1$.

If $T \in B(H)$, we shall write $N(T)$ and $R(T)$ for the null space and the range of T respectively. Let $\alpha(T)=\dim N(T)=\dim(T^{-1}(0))$, $\beta(T)=\dim N(T^*)=\dim(H/T(H))$, $\sigma(T)$ denote the spectrum and

$\sigma_a(T)$ denote the approximate point spectrum. Then $\sigma(T)$ is a compact subset of the set C of complex numbers. The function σ viewed as a function from $B(H)$ into the set of all compact subsets of C, with its Hausdroff metric, is known to be an upper semi-continuous function by [15, Problem 103], but it fails to be continuous by [15, Problem 102]. Also we know that σ is continuous on the set of normal operators in $B(H)$ extended to hyponormal operators [15, Problem 105]. The continuity of σ on the set of quasihyponormal operators (in $B(H)$) has been proved by Djordjevic [10], the continuity of σ on the set of p-hyponormal has been proved by Duggal [13] and Djordjevic [9], and the continuity of σ on the set of G_1-operators has been proved by Luecke [18].

An operator $T \in B(H)$ is called Fredholm if it has closed range, finite dimensional null space and its range has finite co - dimension. The index of a Fredholm operator is given by i(T) = $\alpha(T)-\beta(T)$. The ascent of T, asc$T$, is the least non - negative integer n such that $T^{-n}(0)=T^{-(n+1)}(0)$ and the descent of T, dsc$T$, is the least non - negative integer n such that $T^n(H)=T^{n+1}(H)$. We say that T is of finite ascent (resp., finite descent) if asc$(T-\lambda I)<\infty$ (resp., dsc$(T-\lambda I)<\infty$) for all complex numbers λ. An operator T is said to be left semi - Fredholm (resp., right semi - Fredholm), $T \in \Phi_+(H)$ (resp., $T \in \Phi_-(H)$) if $T\ H$ is closed and the deficiency index $\alpha(T)=\dim(T^{-1}(0))$ is finite (resp., the deficiency index $\beta(T)=\dim(H/T(H))$ is finite); T is semi - Fredholm if it is either left semi - Fredholm or right semi - Fredholm, and T is Fredholm if it is both left and right semi - Fredholm. The semi - Fredholm index of T, ind (T), is the number ind (T) = $\alpha(T)-\beta(T)$. An operator T is called Weyl if it is Fredholm of index zero and Browder if it is Fredholm of finite ascent and descent. Let C denote the set of complex numbers. The Weyl spectrum $\sigma_w(T)$ and the Browder spectrum $\sigma_b(T)$ of T are the sets $\sigma_w(T)=\{\lambda \in C: T-\lambda$ is not Weyl$\}$ and $\sigma_b(T)=\{\lambda \in C: T-\lambda$ is not Browder$\}$.

Let $\pi_0(T)$ denote the set of Riesz points of T (i.e., the set of $\lambda \in C$ such that $T-\lambda$ is Fredholm of finite ascent and descent [7] and let $\pi_{00}(T)$ and iso $\sigma(T)$ denotes the set of eigen values of T of finite geometric multiplicity and isolated points of the spectrum. The operator $T \in B(H)$ is said to satisfy Browder's theorem if $\sigma(T)\setminus\sigma_w(T)=\pi_0(T)$ and T is said to satisfy Weyl's theorem if $\sigma(T)\setminus\sigma_w(T)=\pi_{00}(T)$. In [16], Weyl's theorem for T implies Browder's theorem for T, and Browder's theorem for T is equivalent to Browder's theorem for T^*.

Berkani [5] has called an operatot $T \in B(H)$ as B - Fredholm if there exists a natural number n for which the induced operator $T_n: T^n(X)\to T^n(X)$ is Fredholm. We say that the B - Fredholm operator T has stable index if ind $(T-\lambda)$ ind $(T-\mu)\geq 0$ for every $\lambda,\ \mu$ in the B - Fredholm region of T.

The essential spectrum $\sigma_e(T)$ of $T \in B(H)$ is the set $\sigma_e(T)=\{\lambda \in C: T-\lambda \text{ is not Fredholm}\}$. Let $acc\ \sigma(T)$ denote the set of all accumulation points of $\sigma(T)$, then $\sigma_e(T)\subseteq\sigma_w(T)\subseteq\sigma_b(T)\subseteq\sigma_e(T)\cup acc\sigma(T)$. Let $\pi_{a0}(T)$ be the set of $\lambda \in C$ such that λ is an isolated point of $\sigma_a(T)$ and $0<\alpha(T-\lambda)<\infty$, where $\sigma_a(T)$ denotes the approximate point spectrum of the operator T. Then $\pi_0(T)\subseteq\pi_{00}(T)\subseteq\pi_{a0}(T)$. We say that *a*-Weyl's theorem holds for T if

$$\sigma_{aw}(T)=\sigma_a(T)\setminus\pi_{a0}(T)$$

where $\sigma_{aw}(T)$ denotes the essential approximate point spectrum of T (i.e., $\sigma_{aw}(T)=\bigcap\{\sigma_a(T+K): K \in K(H)\}$ with $K(H)$ denoting the ideal of compact operators on H).

Let $\Phi_+(H)=\{T\in B(H):\alpha(T)<\infty \text{ and } T(H) \text{is closed}\}$ and $\Phi_-(H)=\{T\in B(H):\beta(T)<\infty\}$ denotes the semigroup of upper semi-Fredholm and lower semi-Fredholm operators in $B(H)$ and let $\Phi_+^-(H)=\{T\in\Phi_+(H):ind(T)\le 0\}$. Then $\sigma_{aw}(T)$ is the complement in C of all those λ for which $(T-\lambda)\in\Phi_+^-(H)$ [20]. The concept of *a*-Weyl's theorem was introduced by Rakocevic [21]. The concept states that *a*-Weyl's theorem holds for $T \Rightarrow$ Weyl's theorem holds for T, but converse is generally false. Let $\sigma_{ab}(T)$ denote the Browder essential approximate point spectrum of T.

$$\sigma_{ab}(T)=\bigcap\{\sigma_a(T+K):TK=KT \text{ and } K\in K(H)\}$$

$$=\{\lambda\in C:T-\lambda\notin\Phi_+^-(H) \text{ or } asc(T\text{-}\lambda)=\infty\}$$

then $\sigma_{aw}(T)\subseteq\sigma_{ab}(T)$. We say that T satisfies *a*-Browder's theorem if $\sigma_{ab}(T)=\sigma_{aw}(T)$ [20].

An operator $T\in B(H)$ is said to have the Single Valued Extension Property at $\lambda_0\in C$, if for every open disc D_{λ_0} centered at λ_0, the only analytic function $f:D_{\lambda_0}\to H$ which satisfies the equation

$$(T-\lambda)f(\lambda)=0\text{; for all } \lambda\in D_{\lambda_0}$$

is the function $f\equiv 0$. Trivially, every operator T has SVEP at points of the resolvent $\rho(T)=C/\sigma(T)$. Also T has SVEP at $\lambda\in$ iso $\sigma(T)$. We say that T has SVEP if it has SVEP at every $\lambda\in C$. In this paper, we prove that if $\{T_n\}$ is a sequence of operators in the class (p, k) - quasihyponormal operator or totally p - (α,β) - normal operators which converges in the operator norm topology to an operator T in the same class, then the functions spectrum, Weyl spectrum, Browder spectrum and essential surjectivity spectrum are continuous at T. Note that if an operator T has finite ascent, then it has SVEP and $\alpha(T-\lambda)\le\beta(T-\lambda)$ for all λ [1, Theorem 3.8 and Theorem 3.4]. For a subset S of the set of complex numbers, let $\overline{S}=\{\overline{\lambda}:\lambda\in S\}$ where λ denotes the complex number and $\overline{\lambda}$ denotes the conjugate.

MAIN RESULTS

Lemma 2.1

Let $T\in$ *totally p* - (α,β) - *normal* operator, if $\overline{\lambda}\in\pi_{00}(T^*)$, then it is a pole of the resolvent of T^*.

Proof

If $0\ne\overline{\lambda}\in\pi_{00}(T^*)$, then $\lambda\in iso\ \sigma(T)$ implies that λ is a normal eigenvalue of T [22, Lemma 2.4] and hence a simple pole of the resolvent of T [22, Theorem 2.5]. If instead, $\lambda=0$ then $\dim\ker(T^*)<\infty$ implies that $ran\ T^*$ is closed and hence $T^*\in\Phi_+(H)$. Since both T and T^* has SVEP at 0, it follows that, $asc(T)=dsc(T)<\infty$ [2, Theorem 2.3]. Hence 0 is a pole of the resolvent of T implies 0 is the pole of the resolvent of T^*.

Lemma 2.2

(i) If $T\in(p,k)$-quasihyponormal operator, then $asc(T-\lambda)\le k$ for all λ.

(ii) If $T\in$ *totally p* - (α,β) - *normal* operator, then T has SVEP.

Proof

(i) Proof follows [13, page 146] or [25].

(ii) Proof follows from [22, Lemma 2.1].

Lemma 2.3

If $T \in (p,k)$-quasihyponormal $\bigcup$ *totally p* - (α, β) - *normal* operator and $\lambda \in iso\ \sigma(T)$, then λ is a pole of the resolvent of T.

Proof

Proof follows from [25, Theorem 6] and Lemma 2.1.

Lemma 2.4

If $T \in (p,k)$-quasihyponormal $\bigcup$ *totally p* - (α, β) - *normal* operator, then T^* satisfies *a*-Weyl's theorem.

Proof

If $T \in (p,k)$-quasihyponormal, then T has SVEP, which implies that $\sigma(T^*) = \sigma_a(T^*)$ by [1, Corollary 2.45]. Then T satisfies Weyl's theorem i.e., $\sigma(T) \setminus \sigma_w(T) = \pi_0(T) = \pi_{00}(T)$ [13, Corollary 3.7].

Since $\pi_{00}(T) = \overline{\pi_{00}(T^*)} = \overline{\pi_{a0}(T^*)}$, $\sigma(T) = \overline{\sigma(T^*)} = \overline{\sigma_a(T^*)}$ and $\sigma_w(T) = \overline{\sigma_w(T^*)} = \overline{\sigma_{ea}(T^*)}$ by [3, Theorem 3.6 (ii)], $\sigma_a(T^*) \setminus \sigma_{ea}(T^*) = \pi_{a0}(T^*)$. Hence if $T \in (p,k)$-quasihyponormal, then T^* satisfies a-Weyl's theorem.

If $T \in$ *totally p* -(α, β) - *normal* operator, then by [22, Theorem 2.9] T^* satisfies *a*-Weyl's theorem.

Corollary 2.5

If $T \in (p,k)$-quasihyponormal $\bigcup$ *totally p* -(α, β) - *normal*, then $\lambda \in \sigma_a(T^*) \setminus \sigma_{ea}(T^*) \Rightarrow \lambda \in iso\ \sigma_a(T^*)$.

Lemma 2.6

If $T \in (p,k)$-quasihyponormal $\bigcup$ *totally p* -(α, β) - *normal*, then $asc(T-\lambda) < \infty$ for all λ.

Proof

Since $T-\lambda$ is lower semi-Fredholm, it has SVEP. We know that from [1, Theorem 3.16] that SVEP implies finite ascent. Hence the proof.

Lemma 2.7 [6, Proposition 3.1]

If σ is continuous at a $T^* \in B(H)$, then σ is continuous at T.

Lemma 2.8 [12, Theorem 2.2]

If an operator $T \in B(H)$ has SVEP at points $\lambda \notin \sigma_w(T)$, then σ is continuous at $T \Leftrightarrow \sigma_w$ is continuous at $T \Leftrightarrow \sigma_b$ is continuous at T.

Lemma 2.9

If $\{T_n\}$ is a sequence in (p,k)-qusaihyponormal or *totally p* -(α, β) - *normal* which converges in norm to T, then T^* is a point of continuity of σ_{ea}.

Proof

We have to prove that the function σ_{ea} is both upper semi-continuous and lower semi-continuous at T^*. But by [11, Theorem 2.1], we have that the function σ_{ea} is upper semi-continuous at T^*. So we have to prove that σ_{ea} is lower semi-continuous at T^* i.e., $\sigma_{ea}(T^*) \subset \liminf \sigma_{ea}(T_n^*)$.

Assume the contradiction that σ_{ea} is not lower semi-continuous at T^*. Then there exists an $\varepsilon > 0$, an integer n_0, a $\lambda \in \sigma_{ea}\left(T^*\right)$ and an ε-neighbourhood $(\lambda)_\varepsilon$ of λ such that $\sigma_{ea}\left(T_n^*\right) \cap (\lambda)_\varepsilon = 0$ for all $n \geq n_0$. Since $\lambda \notin \sigma_{ea}\left(T_n^*\right)$ for all $n \geq n_0$ implies $T_n^* - \lambda \in \Phi_+^-(H)$ for all $n \geq n_0$, the following implications holds:

$ind\left(T_n^* - \lambda\right) \leq 0$, $\alpha\left(T_n^* - \lambda\right) < \infty$ and $\left(T_n^* - \lambda\right)H$ is closed

$$\Rightarrow ind\left(T_n - \bar{\lambda}\right) \geq 0,\ \beta\left(T_n - \bar{\lambda}\right) < \infty$$

$$\Rightarrow ind\left(T_n - \bar{\lambda}\right) = 0,\ \alpha\left(T_n - \bar{\lambda}\right) = \beta\left(T_n - \bar{\lambda}\right) < \infty$$

$$\Rightarrow ind\left(T_n - \bar{\lambda}\right) \leq 0$$

(Since $T_n \in (p,k)$-quasihyponormal $\bigcup$ *totally p* - (α, β) - *normal* by Lemma 2.2 and Lemma 2.6)

for all $n \geq n_0$. The continuity of the index implies that $ind\left(T - \bar{\lambda}\right) = \lim_{n\to\infty} ind\left(T_n - \bar{\lambda}\right) = 0$, and hence that $\left(T - \bar{\lambda}\right)$ is Fredholm with $ind\left(T - \bar{\lambda}\right) = 0$. But then $T^* - \bar{\lambda}$ is Fredholm with $ind\left(T^* - \bar{\lambda}\right) = 0 \Rightarrow T^* - \lambda \in \Phi_+^-(H)$, which is a contradiction. Therefore σ_{ea} is lower semi-continuous at T^*. Hence the proof.

Theorem 2.10

If $\{T_n\}$ is sequence in (p,k)-quasihyponormal or *totally p* - (α, β) - *normal* which converges in norm to T, then σ is continuous at T.

Proof

Since *T* has SVEP by Lemma 2.2, we have $\sigma\left(T^*\right) = \sigma_a\left(T^*\right)$. Evidently, it is enough if we prove that $\sigma_a\left(T^*\right) \subset \liminf \sigma_a\left(T_n^*\right)$ for every sequence $\{T_n\}$ of operators in (p,k)-quasihyponormal or *totally p* - (α, β) - *normal* such that T_n converges in norm to T. Let $\lambda \in \sigma_a\left(T^*\right)$. Then either $\lambda \in \sigma_{ea}\left(T^*\right)$ or $\lambda \in \sigma_a\left(T^*\right) \setminus \sigma_{ea}\left(T^*\right)$.

If $\lambda \in \sigma_{ea}\left(T^*\right)$, then proof follows, since

$$\sigma_{ea}\left(T^*\right) \subset \liminf \sigma_{ea}\left(T_n^*\right) \subset \liminf \sigma_a\left(T_n^*\right),$$

If $\lambda \in \sigma_a\left(T^*\right) \setminus \sigma_{ea}\left(T^*\right)$, then $\lambda \in iso\ \sigma_a\left(T^*\right)$ by Corollary 2.5. Consequently, $\lambda \in \liminf \sigma_a\left(T_n^*\right)$ i.e., $\lambda \in \liminf \sigma\left(T_n^*\right)$ for all n by [17, Theorem IV 3.16] and there exists a sequence $\{\lambda_n\}$, $\lambda_n \in \sigma_a\left(T_n^*\right)$, such that λ_n converges to λ.

Evidently $\lambda \in \liminf \sigma_a\left(T_n^*\right)$. Hence $\lambda \in \liminf \sigma\left(T_n^*\right)$. By applying Lemma 2.7, we obtain the result.

Corollar 2.11

If $\{T_n\}$ is a sequence in (p,k)-quasihyponormal or *totally p* - (α, β) - *normal* which converges in norm to T, then σ, σ_w and σ_b are continuous at T.

Proof

Combining Theorem 2.10 with Lemma 2.8 and Lemma 2.9, we obtain the results.

Let $\sigma_s(T)=\{\lambda : T-\lambda \text{ is not surjective}\}$ denote the surjective spectrum of T and let $\Phi_+^-(H)=\{\lambda : T-\lambda \in \Phi_-(H),\ ind(T-\lambda)\geq 0\}$. Then the essential surjectivity spectrum of T is the set $\sigma_{es}(T)=\{\lambda : T-\lambda \notin \Phi_+^-(H)\}$.

Corollary 2.12

If $\{T_n\}$ is a sequence in (p,k)-quasihyponormal or *totally p -* (α,β) *- normal* which converges in norm to T, then σ_{es} is continuous at T.

Proof

Since T has SVEP by Lemma 2.2, $\sigma_{es}(T)=\sigma_{ea}(T^*)$ by [1, Theorem 3.65(ii)]. Then by applying Lemma 2.9, we obtain the result.

Let $K\subset B(H)$ denote the ideal of compact operators, $B(H)/K$ the Calkin algebra and let $\pi : B(H)\to B(H)/K$ denote the quotient map. Then $B(H)/K$ being a C^*-algebra, there exists a Hilbert space H_1, and an isometric *-isomorphism $v : B(H)/K \to B(H_1)$ such that the essential spectrum $\sigma_e(T)=\sigma(\pi(T))$ of $T\in B(H)$ is the spectrum of $v\circ\pi(T)$ $(\in B(H_1))$. In general, $\sigma_e(T)$ is not a continuous function of T.

Corollary 2.13

If $\{\pi(T_n)\}$ is a sequence in (p,k)-quasihyponormal or *totally p -* (α,β) *- normal* which converges in norm to $\pi(T)$, then σ_e is continuous at T.

Proof

If $T_n\in B(H)$ is essentially (p,k)-quasihyponormal or *totally p -* (α,β) *- normal* that is if $\pi(T_n)\in(p,k)$-quasihyponormal or *totally p -* (α,β) *- normal*, and the sequence $\{T_n\}$ converges in norm to T, then $v\circ\pi(T)\in B(H_1)$ is a point of continuity of σ by Theorem 2.10. Hence σ_e is continuous at T, since $\sigma_e(T)=\sigma(v\circ\pi(T))$.

Let $H(\sigma(T))$ denote the set of functions f that are non-constant and analytic on a neighbourhood of $\sigma(T)$.

Lemma 2.14

Let $T\in B(H)$ be a *totally p -* (α,β) *- normal* and let $f\in H(\sigma(T))$. Then $\sigma_{bw}(f(T))\subset f(\sigma_{bw}(T))$, and if the B-Fredholm operator T has stable index, then $\sigma_{bw}(f(T))=f(\sigma_{bw}(T))$.

Proof

Let $T\in B(H)$ be a *totally p -* (α,β) *- normal*, let $f\in H(\sigma(T))$, and let $g(T)$ be an invertible function such that $f(\mu)-\lambda=(\mu-\alpha_1).....(\mu-\alpha_n)g(\mu)$. If $\lambda\notin f(\sigma_{bw}(T))$, then $f(T)-\lambda=(T-\alpha_1).....(T-\alpha_n)g(T)$ and $\alpha_i\notin\sigma_{bw}(T),\ i=1,2,...,n$. Consequently, $T-\alpha_i$ is a B-Fredholm operator of zero index for all $i=1,2,...,n$, which, by [5, Theorem 3.2], implies that $f(T)-\lambda$ is a B-Fredholm operator of zero index. Hence $\lambda\notin\sigma_{bw}(f(T))$, Suppose that T has stable index, and that $\lambda \notin \sigma_{bw}\left(f\left(T\right)\right)$. Then $f\left(T\right)-\lambda=\left(T-\alpha_1\right).....\left(T-\alpha_n\right)g\left(T\right)$ is a B-Fredholm operator of zero index. Hence, by [4, Corollary 3.3], the operator $g\left(T\right)$ and $T-\alpha_i,\ i=1,2,...,n$, are B-Fredholm and

$$0=ind\left(f\left(T\right)-\lambda\right)$$
$$=ind\left(T-\alpha_1\right)+....+ind\left(T-\alpha_n\right)+ind\ g\left(T\right).$$

Since $g\left(T\right)$ is an invertible operator, $ind\left(g\left(T\right)\right)=0$; also $ind\left(T-\alpha_i\right)$ has the same sign for all $i=1,2,...,n$. Thus $ind\left(T-\alpha_i\right)=0$, which implies that $\alpha_i \notin \sigma_{bw}\left(T\right)$ for all $i=1,2,...,n$, and hence $\lambda \notin f\left(\sigma_{bw}\left(T\right)\right)$.

Lemma 2.15

Let $T \in B\left(H\right)$ be a *totally p* -(α,β) - *normal* has Single Valued Extension Property. Then $ind\left(T-\lambda\right)\le 0$ for every $\lambda \in C$ such that $T-\lambda$ is B-Fredholm.

Proof

An operator $T \in$ *totally p* - (α,β) - *normal* has SVEP by [22, Theorem 2.1]. Then $T|_M$ has SVEP for every invariant subspaces $M \subset X$ of T.

From [4, Theorm 2.7] , we know that if $T-\lambda$ is a B-Fredholm operator, then there exists $T-\lambda$ invariant closed subspaces M and N of X such that $X=M\oplus N$, $\left(T-\lambda\right)|_M$ is a Fredholm operator with SVEP and $\left(T-\lambda\right)|_N$ is a Nilpotent operator. Since $ind\left(T-\lambda\right)|_M \le 0$ by [19, Proposition 2.2], it follows that $ind\left(T-\lambda\right)\le 0$.

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- On p- (α, β) - normal operators, Communicated.
- Spectral Continuity : p- (α, β) - normal operators and (p, k) – Quasihyponormal operators. Communicated.
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