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On Interval-Valued Intuitionistic Fuzzy Hyper BCK-Ideals of Hyper BCK-Algebras

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Abstract: In this paper, we apply the concept of an interval-valued intuitionistic fuzzy set to hyper BCK-ideals in hyper BCK-algebras. The notion of an interval-valued intuitionistic fuzzification of (strong, weak, s-weak) hyper BCK-ideals is introduced, and related properties are investigated. Characterizations of an interval-valued intuitionistic fuzzification of hyper BCK-ideals are established.

Keywords: Hyper BCK-algebras; Interval-valued intuitionistic fuzzification of (strong, weak, s-weak) hyper BCK-ideals.

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1. Introduction:

Algebraic structures play an important role in mathematics with wide range of applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory etc. On the other hand, in handling information regarding various aspects of uncertainty, non-classical logic (a great extension and development of classical logic) is considered to be more powerful technique than the classical logic one. The non-classical logic, therefore, has now a day become a useful tool in computer science. Moreover, non-classical logic deals with the fuzzy information and uncertainty. In 1965, Zadeh [15] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. Extending the concept of fuzzy sets (FSs), many scholars introduced various notions of higher order FSs. Among them, interval-valued fuzzy sets (i-v FSs) provides with a flexible mathematical framework to cope with imperfect and imprecise information. Moreover, Attanssov [2,4] introduced the concept of intuitionistic fuzzy sets (IFSs) and the interval-valued intuitionistic fuzzy sets (i-v IFSs), as a generalization of an ordinary FSs. The hyper structure theory (called also multialgebras) was introduced in 1934 by Marty [13] at the 8th congress of Scandinavian Mathematicians. In [12], Jun et al. applied the hyper structures to BCK-algebras, and introduced the concept of a hyper BCK-algebra which is a generalization of a BCK-algebra. In [6], Borzooei and Jun, using the Atanassov's idea, established the intuitionistic fuzzification of notion of (strong, weak, s-weak) hyper BCK-ideals of hyper BCK-algebras, and investigated some of their properties.

In this paper, we use Attanssov's idea and establish the interval-valued intuitionistic fuzziffication of notion of (strong, weak, s-weak) hyper BCK-ideals of hyper BCK-algebras and related properties are investigated. We give Characterizations of an interval- valued intuitionistic fuzzy hyper BCK-ideals are established.

2. Preliminaries:

In this section, we include some elementary aspects of hyper BCK-algebras that are necessary for this paper, and more details we refer to [9], [10], and [11].

Let H be a non-empty set endowed with hyper operation that is, "$\circ$" is a function from $H\times H$ to $P^*(H)=P(H)\backslash\{\Phi\}$. For any two sub-sets A and B of H, denoted by $A\circ B$ the set $\bigcup_{a\in A,b\in B} a\circ b$. We shall use $x\circ y$ instead of $x\circ\{y\}$, $\{x\}\circ y$, or $\{x\}\circ\{y\}$.

By a hyper BCK-algebra, we mean a non-empty set H endowed with a hyperoperation "$\circ$" and a constant 0 satisfying the following axioms:

(HK-1) $(x\circ z)\circ(y\circ z) << x\circ y$,

(HK-2) $(x\circ y)\circ z=(x\circ z)\circ y$,

(HK-3) $x\circ H << \{x\}$,

(HK-4) $x << y$ and $y << x \Rightarrow x=y$ for all $x, y, z\in H$.

We can define a relation "<<" on H by letting $x << y$ if and only if $0\in x\circ y$ and for every $A, B\subseteq H$, $A << B$ is defined by $\forall a\in A, \exists b\in B$ such that $a << b$. In such case, we call "<<" the hyper order in H.

Note that the condition (HK3) is equivalent to the condition: (p1) $x\circ y << \{x\}$, for all $x, y\in H$

In any hyper BCK-algebra H the following hold.

(p2) $x\circ 0 << \{x\}$, $0\circ x << \{x\}$ and $0\circ 0 << \{0\}$,

(p3) $(A\circ B)\circ C=(A\circ C)\circ B$, $A\circ B << A$ and $0\circ A << \{0\}$, (p4) $0\circ 0=\{0\}$, (p5) $0 << x$,

(p6) $x << x$, (p7) $A << A$, (P8) $A\subseteq B$ implies $A << B$, (p9) $0\circ x=\{0\}$, (p10) $x\circ 0=\{x\}$,
(p11) $0\circ A=\{0\}$, (p12) $A << \{0\}$ implies $A=\{0\}$, (p13) $A\circ B << A$. (p13) $x\in x\circ 0$,

(p14) $x\circ 0 << \{y\}$ implies $x << y$, (p15) $y << z$ implies $x\circ z << x\circ y$,

(p16) $x\circ y=\{0\}$ implies $(x\circ z)\circ(y\circ z)=\{0\}$ and $x\circ z << y\circ z$,

(P17) $A\circ\{0\}=\{0\}$ implies $A=\{0\}$ for all $x, y, z\in H$ and for any non-empty sub-sets A, B, C of H

Let I be a non-empty subset of a hyper BCK-algebra H and $0\in I$. Then I is called

- a weak hyper BCK-ideal of H if $x \circ y \subseteq I$ and $y \in I$ imply $x \in I$ for all $x, y \in H$,
- a hyper BCK-ideal of H, if $x \circ y << I$ and $y \in I$ imply $x \in I$ for all $x, y \in H$,
- a strong hyper BCK-ideal of H, if $x \circ y \bigcap I \neq \Phi$ and $y \in I$ imply $x \in I$ for all $x, y \in H$,
- I is said to be reflexive if $x \circ x \subseteq I$ for all $x \in H$,
- S-reflexive if it satisfies $(x \circ y) \bigcap I \neq \Phi \quad x \circ y << I$ for all $x, y \in H$, ,
- closed if $x << y$ and $y \in I$ implies that $x \in I$ for all $x \in H$,

It is easy to see that every S-reflexive sub-set of H is reflexive.

The determination of maximum and minimum between two real numbers is very simple, but it is not simple for two intervals. Biswas [7] described a method to find max/sup and min/inf between two intervals and set of intervals. By an interval number $\tilde{a}$ on $[0,1]$, we mean (cf. [5]) an interval $\left[a^{-}, a^{+}\right]$, where $0 \leq a^{-} \leq a^{+} \leq 1$. The set of all closed subintervals of $[0,1]$ is denoted by $D[0,1]$. The interval $[a, a]$ is identified with the number $a \in [0,1]$.

For an interval numbers $\tilde{a}_i = \left[a_i^{-}, b_i^{+}\right] \in D[0,1], i \in I$. We define

$$\inf \tilde{a}_i = \left[\min_{i \in I} a_i^{-}, \min_{i \in I} b_i^{+}\right], \qquad \sup \tilde{a}_i = \left[\max_{i \in I} a_i^{-}, \max_{i \in I} b_i^{+}\right]$$

And put

(i) $\tilde{a}_1 \cap \tilde{a}_2 = \min(\tilde{a}_1, \tilde{a}_2) = \min\left(\left[a_1^{-}, b_1^{+}\right], \left[a_2^{-}, b_2^{+}\right]\right) = \left[\min\left\{a_1^{-}, a_2^{-}\right\}, \min\left\{b_1^{+}, b_2^{+}\right\}\right]$

(ii) $\tilde{a}_1 \cup \tilde{a}_2 = \max(\tilde{a}_1, \tilde{a}_2) = \max\left(\left[a_1^{-}, b_1^{+}\right], \left[a_2^{-}, b_2^{+}\right]\right) = \left[\max\left\{a_1^{-}, a_2^{-}\right\}, \max\left\{b_1^{+}, b_2^{+}\right\}\right]$

(iii) $\tilde{a}_1 + \tilde{a}_2 = \left[a_1^{-} + a_2^{-} - a_1^{-}.a_2^{-}, \; b_1^{+} + b_2^{+} - b_1^{+}.b_2^{+}\right]$

(iv) $\tilde{a}_1 \leq \tilde{a}_2 \Leftrightarrow a_1^{-} \leq a_2^{-}$ and $b_1^{+} \leq b_2^{+}$

(v) $\tilde{a}_1 = \tilde{a}_2 \Leftrightarrow a_1^{-} = a_2^{-}$ and $b_1^{+} = b_2^{+}$,

(vi) $mD = m[a_1^{-}, b_1^{+}] = [ma_1^{-}, mb_1^{+}]$, where $0 \leq m \leq 1$.

Obviously $(D[0,1], \leq, \vee, \wedge)$ form a complete lattice with $[0, 0]$ as its least element and $[1,1]$ as its greatest element.

The people observed that the determination of membership value is a difficult task for a decision maker. In [15], Zadeh defined another type of fuzzy set called interval-valued fuzzy sets (i-v FSs). The membership value of an element of this set is not a single number, it is an interval and this interval is an sub-interval of the interval [0,1]. Let D [0, 1] be the set of a subintervals of the interval [0, 1].

Let X be a given nonempty set. An interval-valued fuzzy set (briefly, i-v fuzzy set) B on X is defined by $B = \left\{\left(x, [\mu_B^{-}(x), \mu_B^{+}(x)]\right) : x \in X\right\}$, where $\mu_B^{-}(x)$ and $\mu_B^{+}(x)$ are fuzzy sets of X such that $\mu_B^{-}(x) \leq \mu_B^{+}(x)$ for all $x \in X$. Let $\tilde{\mu}_B(x) = \left[\mu_B^{-}(x), \mu_B^{+}(x)\right]$, then $B = \{(x, \tilde{\mu}_B(x)) : x \in X\}$ where $\tilde{\mu}_B : X \to D[0,1]$.

Combining the idea of intuitionistic fuzzy set and interval-valued fuzzy sets, Atanassov and Gargov [3] defined a new class of fuzzy sets called interval-valued intuitionistic fuzzy sets (IVIFSs) defined below.

An i-v IFS "A" over X is an object having the form $\tilde{A} = \{(x, \tilde{\mu}_A, \tilde{\lambda}_A) : x \in X\}$, where $\tilde{\mu}_A(x) : X \to D[0,1]$ and $\tilde{\lambda}_A(x) : X \to D[0,1]$, the intervals $\tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x)$ denotes the intervals of the degree of membership and the degree of non-membership of the element x to the set $\tilde{A}$, where $\tilde{\mu}_A(x) = [\mu_A^-(x), \mu_A^+(x)]$ and $\tilde{\lambda}_A(x) = [\lambda_A^-(x), \lambda_A^+(x)]$ for all $x \in X$ with the condition $[0,0] \le \tilde{\mu}_A(x) + \tilde{\lambda}_A(x) \le [1,1]$ for all $x \in X$. For the sake of simplicity, we use the symbol $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$

3. Interval-valued intuitionistic fuzzy hyper BCK-ideals of hyper BCK- Algebras

In what follows, let H denote a hyper BCK-algebra, unless otherwise specified.

Definition 3.1 An i-v IFS $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ in H is called an interval-valued intuitionistic fuzzy hyper BCK-ideal of H if it satisfies

(i-v k1) $x << y$ implies $\tilde{\mu}_A(x) \ge \tilde{\mu}_A(y)$ and $\tilde{\lambda}_A(x) \le \tilde{\lambda}_A(y)$

(i-v k2) $\tilde{\mu}_A(x) \ge \min\{\inf_{a \in x \circ y} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\}$

(i-v k3) $\tilde{\lambda}_A(x) \le \max\{\sup_{b \in x \circ y} \tilde{\lambda}_A(a), \tilde{\lambda}_A(y)\}$ all $x, y \in H$.

Example 3.2. Consider a set $H = \{0, a, b\}$ and "$\circ$" is defined on H as follows:

o	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
A	$\{a\}$	$\{0, a\}$	$\{0, a\}$
B	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$

Then $(H, \circ)$ is a hyper BCK-algebra [6]. Define an i-v IFS $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ in H by

$\tilde{\mu}_A(0) = [0.65, 0.7], \tilde{\mu}_A(a) = [0.35, 0.4], \tilde{\mu}_A(b) = [0.1, 0.2]$ and

$\tilde{\lambda}_A(0) = [0.07, 0.08], \tilde{\lambda}_A(a) = [0.5, 0.55]$ and $\tilde{\lambda}_A(b) = [0.6, 0.65]$.

It is easily verified that $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an interval-valued intuitionistic fuzzy hyper BCK-ideal of H.

Definition 3.3. An i-v IFS $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ in H is called an interval-valued intuitionistic fuzzy strong hyper BCK-ideal of H if it satisfies

$\inf_{a \in x \circ x} \tilde{\mu}_A(a) \ge \tilde{\mu}_A(x) \ge \min\{\sup_{b \in x \circ y} \tilde{\mu}_A(b), \tilde{\mu}_A(y)\}$ and

$$\sup_{c\in x\circ x} \tilde{\lambda}_A(c) \le \tilde{\lambda}_A(x) \le \max\{\inf_{d\in x\circ y} \tilde{\lambda}_A(d), \tilde{\lambda}_A(y)\}$$ for all $x, y \in H$.

Example 3.4. Consider a set $H = \{0, a, b\}$ and "$\circ$" is defined on H as follows:

$\circ$	**0**	**a**	**B**
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0, b\}$

Then $(H, \circ)$ is a hyper BCK-algebra [6]. Define an IFS $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ in H by

$$\tilde{\mu}_A(0) = [0.85, 0.9], \tilde{\mu}_A(a) = [0.5, 0.6], \tilde{\mu}_A(b) = [0.25, 0.3]$$ and

$$\tilde{\lambda}_A(0) = [0.09, 0.10], \tilde{\lambda}_A(a) = [0.16, 0.20] \text{ and } \tilde{\lambda}_A(b) = [0.23, 0.28].$$

It is routine to cheek that $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an interval-valued intuitionistic fuzzy strong hyper BCK-ideal of H.

Definition 3.5. An i-v IFS $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ in H is called an interval-valued intuitionistic fuzzy s-weak hyper BCK-ideal of H if it satisfies

(i-v s1) $\tilde{\mu}_A(0) \ge \tilde{\mu}_A(y)$ and $\tilde{\lambda}_A(0) \le \tilde{\lambda}_A(y)$, for all $x, y \in H$.

(i-v s2) for every $x, y \in H$ there exists $a, b \in x \circ y$ such that

$$\tilde{\mu}_A(x) \ge \min\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\} \text{ and } \tilde{\lambda}_A(x) \le \max\{\tilde{\lambda}_A(b), \tilde{\lambda}_A(y)\}.$$

Definition 3.6. An i-v IFS $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ in H is called an interval-valued intuitionistic fuzzy weak hyper BCK-ideal of H if it satisfies

$$\tilde{\mu}_A(0) \ge \tilde{\mu}_A(x) \ge \min\{\inf_{a\in x\circ y} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\}$$ and

$$\tilde{\lambda}_A(0) \le \tilde{\lambda}_A(x) \le \max\{\sup_{b\in x\circ y} \tilde{\lambda}_A(b), \tilde{\lambda}_A(y)\}$$, for all $x, y \in H$.

Theorem 3.7. Every interval-valued intuitionistic fuzzy s-weak hyper BCK-ideal of H is an interval-valued intuitionistic fuzzy weak hyper BCK-ideal.

Proof: Let an i-v IFS $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ in H be an interval-valued intuitionistic fuzzy s-weak hyper BCK-ideal of H and let $x, y \in H$. Then there exist $a, b \in x \circ y$ such that

$$\tilde{\mu}_A(x) \ge \min\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\} \text{ and } \tilde{\lambda}_A(x) \le \max\{\tilde{\lambda}_A(b), \tilde{\lambda}_A(y)\}$$

Since $\tilde{\mu}_A(a) \geq \inf_{c \in x \circ y} \tilde{\mu}_A(c)$ and $\tilde{\lambda}_A(a) \leq \sup_{d \in x \circ y} \tilde{\lambda}_A(d)$, it follows that

$$\tilde{\mu}_A(x) \geq \min\{\inf_{c \in x \circ y} \tilde{\mu}_A(c), \tilde{\mu}_A(y)\} \text{ and } \tilde{\lambda}_A(x) \leq \max\{\sup_{d \in x \circ y} \tilde{\lambda}_A(d), \tilde{\lambda}_A(y)\}.$$

Definition 3.8. An i-v IFS $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ in H is said to satisfy the "inf-sup" property if for any sub-set T of H there exist $x_0, y_0 \in T$ such that $\tilde{\mu}_A(x_0) = \inf_{x \in T} \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x_0) = \sup_{y \in T} \tilde{\lambda}_A(y)$.

It is not easy to find an example of an interval-valued intuitionistic fuzzy s-weak hyper BCK-ideal of H which is not an interval-valued intuitionistic fuzzy weak hyper BCK-ideal of H.

Proposition 3.9. Let $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be an interval-valued intuitionistic fuzzy weak hyper BCK-ideal of H. If $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ satisfies the "inf-sup" property, then $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an interval-valued intuitionistic fuzzy s-weak hyper BCK-ideal of H.

Proof: Assume that $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an interval-valued intuitionistic fuzzy weak hyper BCK-ideal of H and satisfies the "inf-sup" property. There exists $a_0, b_0 \in x \circ y$ such that

$$\tilde{\mu}_A(a_0) = \inf_{a \in x \circ y} \tilde{\mu}_A(a) \text{ and } \tilde{\lambda}_A(b_0) = \sup_{b \in x \circ y} \tilde{\lambda}_A(b).$$

It follows that

$$\tilde{\mu}_A(x) \geq \min\{\inf_{a \in x \circ y} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\} = \min\{\tilde{\mu}_A(a_0), \mu_A(y)\} \text{ and}$$

$$\tilde{\lambda}_A(x) \leq \max\{\sup_{b \in x \circ y} \tilde{\lambda}_A(b), \tilde{\lambda}_A(y)\} = \max\{\tilde{\lambda}_A(b_0), \tilde{\lambda}_A(y)\}$$

This completes the proof.

Note that, in a finite hyper BCK-algebra, every interval-valued intuitionistic fuzzy set satisfies the **"**inf-sup". Hence the concept of interval-valued intuitionistic fuzzy weak hyper BCK-ideals and interval-valued intuitionistic fuzzy s-weak hyper BCK-ideals coincide in a hyper BCK-algebra.

Proposition 3.10. Let $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be an interval-valued intuitionistic fuzzy strong hyper BCK-ideal of H and let $x, y \in H$. Then

(i) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(y)$ and $\tilde{\lambda}_A(0) \leq \tilde{\lambda}_A(y)$

(ii) $x << y$ implies $\tilde{\mu}_A(x) \geq \tilde{\mu}_A(y)$ and $\tilde{\lambda}_A(x) \leq \tilde{\lambda}_A(y)$

(iii) $\tilde{\mu}_A(x) \geq \min\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\}$ and $\tilde{\lambda}_A(x) \leq \max\{\tilde{\lambda}_A(b), \tilde{\lambda}_A(y)\}$, for all $a, b \in x \circ y$.

Proof: Let $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be an interval-valued intuitionistic fuzzy strong hyper BCK-ideal of H and let $x, y \in H$.

(i) Since $0 \in x \circ x$, for all $x \in H$,

We have

$$\tilde{\mu}_A(0) \geq \inf_{a \in x \circ x} \tilde{\mu}_A(a) \geq \mu_A(x) \text{ and}$$

$$\tilde{\lambda}_A(0) \le \sup_{b \in x \circ x} \tilde{\lambda}_A(b) \le \tilde{\lambda}_A(x) \text{ for all } x \in H.$$

Therefore, $\tilde{\mu}_A(0) \ge \tilde{\mu}_A(y)$ and $\tilde{\lambda}_A(0) \le \tilde{\lambda}_A(y)$ for all $x, y \in H$

(ii) Let $x, y \in H$ be such that $x << y$ then $0 \in x \circ y$ and so

$$\tilde{\mu}_A(0) \le \sup_{c \in x \circ y} \tilde{\mu}_A(c) \text{ and } \tilde{\lambda}_A(0) \ge \inf_{d \in x \circ y} \tilde{\lambda}_A(d)$$

It follows from (i) that

$$\tilde{\mu}_A(x) \ge \min\{\sup_{c \in x \circ y} \tilde{\mu}_A(c), \tilde{\mu}_A(y)\} \ge \min\{\tilde{\mu}_A(0), \tilde{\mu}_A(y)\} = \tilde{\mu}_A(y) \text{ and}$$

$$\tilde{\lambda}_A(x) \le \max\{\inf_{d \in x \circ y} \tilde{\lambda}_A(d), \tilde{\lambda}_A(y)\} \le \max\{\tilde{\lambda}_A(0), \tilde{\lambda}_A(y)\} = \tilde{\lambda}_A(y)$$

Therefore, if $x << y$ implies $\tilde{\mu}_A(x) \ge \tilde{\mu}_A(y)$ and $\tilde{\lambda}_A(x) \le \tilde{\lambda}_A(y)$

(iii) Let $x, y \in H$ since

$$\tilde{\mu}_A(x) \ge \min\{\sup_{c \in x \circ y} \tilde{\mu}_A(c), \tilde{\mu}_A(y)\} \ge \min\{\tilde{\mu}_A(a), \mu_A(y)\} \text{ and}$$

$$\tilde{\lambda}_A(x) \le \max\{\inf_{d \in x \circ y} \tilde{\lambda}_A(d), \tilde{\lambda}_A(y)\} \le \max\{\lambda_A(b), \lambda_A(y)\}$$

for all $a, b \in x \circ y$.

Theorem 3.11. Le $\tilde{A} = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an interval-valued intuitionistic fuzzy strong hyper BCK-ideal of H, then $\tilde{\mu}_A(x) \ge \min\{\inf_{a \in x \circ y} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\}$ and $\tilde{\lambda}_A(x) \le \max\{\sup_{b \in x \circ y} \tilde{\lambda}_A(b), \tilde{\mu}_A(y)\}$ for all $x, y \in H$.

Proof: For any $x, y \in H$, We have

$$\sup_{a \in x \circ y} \tilde{\mu}_A(a) \ge \inf_{a \in x \circ y} \tilde{\mu}_A(a) \text{ and } \inf_{b \in x \circ y} \tilde{\lambda}_A(b) \le \sup_{b \in x \circ x} \tilde{\lambda}_A(b) \text{ for all } x \in H.$$

It follows from the definition, we get

$$\tilde{\mu}_A(x) \ge \min\{\sup_{a \in x \circ y} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\} \ge \min\{\inf_{a \in x \circ y} \tilde{\mu}_A(a), \mu_A(y)\}$$

and

$$\tilde{\lambda}_A(x) \le \max\{\inf_{b \in x \circ y} \tilde{\lambda}_A(b), \tilde{\lambda}_A(y)\} \le \max\{\sup_{b \in x \circ y} \tilde{\lambda}_A(b), \tilde{\lambda}_A(y)\}$$

for all $x, y \in H$.

Corollary 3.12. Every interval-valued intuitionistic fuzzy strong hyper BCK-ideal is both an interval-valued intuitionistic fuzzy s-weak hyper BCK-ideal (and hence an interval-valued intuitionistic fuzzy weak hyper BCK-ideal) and an interval-valued intuitionistic fuzzy hyper BCK-ideal.

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