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On R^G-Homeomorphisms in Topological Spaces

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ABSTRACT:

This paper deals with r^g open and closed maps. Also we introduce a new class of maps namely r^g* - homeomorphism which form a subclass of r^g - homeomorphism.

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1. INTRODUCTION

Generalized closed mappings were introduced and studied by Malghan [5], Regular closed maps, gpr-closed maps and rg-closed maps have been introduced and studied by Long [4], Gnanambal [2] and Arockiarani [1] respectively. Recently rw closed maps and open maps were introduced by Karpagadevi [3].

The purpose of this paper is to introduce the concept of a new-class of maps called r^g-closed maps and r^g open maps. Further we introduce r^g- homeomorphism, r^g*- homeomorphism and discuss their properties.

2. PRELIMINARIES

In this section, we recollect some definitions which are used in this paper.

Definition 2.1:

A subset A of (X,τ) is said to be

i) an α-open set if $A \subseteq int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A))) \subseteq A$

ii) a generalized closed (briefly g-closed)[2]set iff $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open inX.

iii) a generalized *closed (briefly g*-closed) [11] set iff $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

iv) a weakly generalized semi closed (briefly wg – closed) [11] if $cl(int(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X

v) a semi generalized closed (briefly sg – closed)[4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X.

vi) a generalized semi closed (briefly gs – closed)[4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

vii) a regular weakly generalized semi closed (briefly rwg – closed)[11] if $cl(int(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

viii) a regular generalized weakly semi closed (briefly rgw – closed)[11] if $cl(int(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X.

ix) a semi weakly generalized closed (briefly swg – closed)[11] if $cl(int(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X.

x) a regular^ generalized closed (briefly r^g closed) [8] if $gcl(A) \subset U$, whenever $A \subset U$ and U is regular open in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.2: A map $f: X \to Y$ is said to be

i) a continuous function[1]if $f^{-1}(V)$ is closed in X for every closed set V in Y.

ii) a wg - continuous [4]if f^{-1} (V) is wg- closed in X for every closed set V in Y.

iii) a sg -continuous [1]if $f^{-1}(V)$ is rg closed in X for every closed set V in Y.

iv) a gs -continuous [1]if $f^{-1}(V)$ is gs closed in X for every closed set V in Y.

v) an rw-continuous [11] if $f^{-1}(V)$ is rw- closed in X for every closed set V in Y.

vi) an rwg-continuous [11] if $f^{-1}(V)$ is rwg- closed in X for every closed set V in Y.

vii) an rgw-continuous [11] if $f^{-1}(V)$ is rgw- closed in X for every closed set V in Y.

viii) a swg –continuous[11] if $f^{-1}(V)$ is swg- closed in X for every closed set V in Y.

ix) an r^g -continuous [11] if $f^{-1}(V)$ is r^g- closed in X for every closed set V in Y.

Definition 2.3:

A topological space (X,τ) is said to be

(i) a $T_{1/2}$ space if every gclosed set is closed.

(ii) an $T_{r\hat{}g}$ space if every r^g closed set is closed.

Definition 2.3

A bijective function $f: (X,\tau) \to (Y,\sigma)$ is called

i) homeomorphism if both f and f^{-1} are continuous.

ii) wg - homeomorphism if both f and f^{-1} are wg-continuous.

iii) sg - homeomorphism if both f and f^{-1} are sg-continuous.

iv) gs - homeomorphism if both f and f^{-1} are gs-continuous.

v) rw - homeomorphism if both f and f^{-1} are rw-continuous.

vi) swg - homeomorphism if both f and f^{-1} are swg-continuous.

vii) rwg - homeomorphism if both f and f^{-1} are rwg-continuous.

viii) rgw - homeomorphism if both f and f^{-1} are rgw-continuous.

ix) r^g - homeomorphism if both f and f^{-1} are r^g-continuous

3. R^G CLOSED MAPS

Definition 3.1:

A map f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be regular ^ generalized (briefly – r^g) closed map if the image of every closed set in (X,τ) is r^g closed in (Y,σ).

Theorem 3.2:

(i) Every closed map is r^g closed map.

(ii) Every rg-closed map is r^g closed map.

(iii) Every g-closed map is r^g closed maps.

(iv) Every g*-closed map is r^g closed maps.

Proof:

Follows from the definition.

Remark 3.3:

The converse of the above theorem need not be true as seen from the following examples.

Example 3.4:

- Let X = {a,b,c}, τ = {X,φ,{a,b},{c}}, σ = {Y,φ,{a},{b},{a,b}}. Let f be the identity map such that f : X $\rightarrow$Y. then f is r^g closed but it is not a closed map.
- Let X = {a,b,c,d,e}, τ = {X,φ,{b,c,d,e}}, σ = {Y,φ,{a},{b},{c}, {a,b}, {a,c}, {b,c}, {a,b,c}}. Then define f: X$\rightarrow$Y, the identity map, then f is r^g closed map but it is not rg closed map.
- Let X = {a,b,c,d} = Y, τ = {X,φ,{a,b},{c,d}}, σ = {Y,φ,{a},{b,c},{a,b,c}}. Define a map f:X$\rightarrow$Y by f(a) = b, f(b) = a, f(c) = c, f(d) = a, then f is r^g closed map but it is not g-closed map.
- Let X = Y = {a,b,c,d}, τ = {X,φ,{b},{b,c},{a,d},{a,b,d}}, σ = {Y,φ,{a,b}, {c}, {a,b,c}}. Define a map f: X $\rightarrow$ Y by f(a) = a, f(b) = b, f(c) = d, f(d) = c, then f is r^g closed map but it is not g* closed map.

Theorem 3.5:

A map f: $(X,\tau) \rightarrow (Y,\sigma)$ is r^g closed if and only if for each subset S of (Y,σ) and each open set U containing $f^{-1}(S)$ there is an r^g open V set of (Y,σ) such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof:

Suppose f is r^g closed set of (X,τ). Let $S \subseteq Y$ and U be an open set of (X,τ) such that $f^{-1}(S) \subseteq U$. Now X-U is closed set in (X,τ). Since f is r^g closed, f(X-U) is an r^g closed set in (Y,σ). Then V = Y-f(X-U) is r^g-open set in (Y,σ) . $f^{-1}(S) \subseteq U$ implies $S \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X-U)) \subset X-(X-U) = U$, i.e., $f^{-1}(V) \subseteq U$.

Conversely, let F be a closed set of (X,τ). Then $f^{-1}(f(F)^c) \subset F^c$ is an open set in (X,τ). By hypothesis, there exists an r^g open set V in (Y,σ) such that $f(F)^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f(((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) \subseteq V^c$. Since V^c is r^g closed, f(F) is r^g closed. That is f(F) is r^g closed in (Y,σ). Therefore f is r^g closed map.

Remark 3.6:

The composition of two r^g closed maps need not be r^g closed map in general and this is shown by the following example.

Example 3.7:

Let X = Y = {a,b,c,d},τ = {X,φ,{a},{b},{a,b},{a,b,c}}, σ = {Y,φ,{b},{b,c},{a,d},{a,b,d}},

η = {Z,φ,{a},{b,c},{a,b,c}}. Define f : $(X,\tau) \rightarrow (Y,\sigma)$, the identity map and g : $(Y,\sigma) \rightarrow (Z,\eta)$ by g(a) = a, g(b) =c, g(c) = d, g(d) = b. Then f and g are r^g closed maps. But $g \circ f(d) = g(f(d)) = g(d)$ = {b} is not r^g closed in (Z,η). Hence $g^\circ f$ is not r^g closed map.

Theorem 3.8:

If f: $(X,\tau) \to (Y,\sigma)$ is closed map and g: $(Y,\sigma) \to (Z,\eta)$ is r^g closed map, then the composition $g \circ f$: $(X,\tau) \to (Z,\eta)$ is r^g closed map.

Proof:

Let F be any closed set in (X,τ). Since f is a closed map, f(F) is closed set in (Y,σ). Since g is r^g closed map, g(f(F)) = $g \circ f(F)$ is r^g closed set in (Z,η). Thus $g \circ f$ is r^g closed map.

Remark 3.9:

If f:$(X,\tau) \to (Y,\sigma)$ is r^g closed map and g:$(Y,\sigma) \to (Z,\eta)$ is closed map, then the composition need not be an r^g closed map as seen from the following example.

Example 3.10:

Let X = Y = Z = {a,b,c,d}, τ = {X,φ,{a},{c},{d},{a,c},{a,d},{c,d},{a,c,d}}, σ = {Y,φ, {a},{c,d},{a,c,d}}, η = {Z,φ,{a},{b,c},{a,b,c}}. Define f : $(X,\tau) \to (Y,\sigma)$, the identity map and g : $(Y,\sigma) \to (Z,\eta)$ by g(a)=a, g(b)=d, g(c)=c, g(d)=b. Then f is r^g closed map and g is a closed map. But $g \circ f : (X,\tau) \to (Z,\eta)$ is not an r^g closed map, since $g \circ f${a,b} = g(f{a,b}) = g(a,b) = {a,d} is not an r^g closed set in (Z,η).

Theorem 3.11:

Let f: $(X,\tau) \to (Y,\sigma)$ and g : $(Y,\sigma) \to (Z,\eta)$ be two r^g closed maps where (Y,σ) is $T_{r\text{^}g}$ space. Then the composition $g \circ f$: $(X,\tau) \to (Z,\eta)$ is r^g closed.

Proof:

Let A be a closed set of (X,τ). Since f is r^g closed, f(A) is r^g closed in (Y,σ). By hypothesis, f(A) is closed. Since g is r^g closed, g(f(A)) is r^g closed in (Z,η) and g(f(A)) = $g \circ f(A)$. Therefore $g \circ f$ is r^g closed.

Theorem 3.12:

If f: $(X,\tau) \to (Y,\sigma)$ is g-closed, g: $(Y,\sigma) \to (Z,\eta)$ be r^g closed and (Y,σ) is $T_{1/2}$ space then the composition $g \circ f : (X,\tau) \to (Z,\eta)$ is r^g closed.

Proof:

Let A be a closed set of (X,τ). Since f is g-closed, f(A) is g-closed in (Y,σ). By hypothesis f(A) is closed. Since g is r^g closed, g(f(A)) = $g \circ f(A)$ is r^g closed in (Z,η). Thus $g \circ f$ is r^g closed.

Theorem 3.13:

Let f: $(X,\tau) \to (Y,\sigma)$, g : $(Y,\sigma) \to (Z,\eta)$ be two mappings such that their composition $g \circ f : (X,\tau) \to (Z,\eta)$ be r^g closed map. Then the following statements are true.

(i) If f is continuous and surjective, then g is r^g closed.
(ii) If g is r^g irresolute and injective, then f is r^g closed.
(iii) If f is g-continuous, surjective and (X,τ) is a $T_{1/2}$ space then g is r^g closed.

Proof:

(i) Let A be a closed set of (Y,σ). Since f is continuous, $f^{-1}(A)$ is closed in (X,τ). $g \circ f$ is r^g closed, therefore $g \circ f(f^{-1}(A))$ is r^g closed in (Z,η). That is g(A) is r^g closed in (Z,η),since f is surjective. Therefore g is r^g closed.

(ii) Let A be a closed set of (X,τ). Since $g \circ f$ is r^g closed, $g \circ f(B)$ is r^g closed set in (Z,η). g is r^g irresolute, $g^{-1}(g \circ f(B))$ is r^g closed set in (Y,σ). That is f(B) is r^g closed in (Y,σ), since f is injective. Hence f is r^g closed.

(iii) Let C be a closed set of (Y,σ). Since f is g-continuous $f^{-1}(C)$ is g-closed in (X,τ). Since (X,τ) is a $T_{1/2}$ space, $f^{-1}(C)$ is closed. By hypothesis, $g \circ f^{-1}(f(C))$ = g(C) is r^g closed in (Z,η), since f is surjective. Therefore g is r^g closed.

4. R^G OPEN MAPS:

Definition4.1:

A map f: $(X,\tau) \to (Y,\sigma)$ is called an r^g open map if the image f(A) is r^g open in (Y,σ) for each open set A in (X,τ).

Theorem 4.2:

Every open map is r^g open.

Proof: Obvious.

Remark 4.3: The converse of the above theorem need not be true as seen from the following example.

Example 4.4:

Let X = Y = {a,b,c}, τ = {X,φ,{b,c}}, σ = {Y,φ,{a},{b},{a,b}}. Define f : (X,τ) $\to$ (Y,σ) the identity map, then f is r^g open but it is not an open map.

Theorem 4.5:

For any bijection map f : (X,τ) $\to$ (Y,σ), the following statements are equivalent.

(i) f^{-1} : (Y,σ)$\to$(X,τ) is r^g continuous.
(ii) f is r^g open map and
(iii) f is r^g closed map.

Proof:

(i) $\to$ (ii) : Let U be an open set of (X,τ). By assumption, $(f^{-1})^{-1}$(U) = f(U) is r^g open in (Y,σ) and so f is r^g open

(ii) $\to$ (iii) : Let F be a closed set of (X,τ). Then F^c is open set on (X,τ). By hypothesis, f(F^c) is r^g open in (Y,σ). That is f(F^c) = $f(F)^c$ is r^g open in (Y,σ). Thus f(F) is r^g closed in Y. Hence f is r^g closed.

(iii) $\to$ (i) : Let F be a closed set in X. By hypothesis, f(F) is r^g closed in Y. That is f(F) = $(f^{-1})^{-1}$(F) and therefore f^{-1} is continuous.

Theorem 4.6:

A map f: (X,τ) $\to$ (Y,σ) is r^g open iff for any subset S of (X,τ) containing f^{-1}(S), there exists an r^g closed set K of (Y,σ) containing S such that $f^{-1}(K) \subset F$.

Proof: Obvious.

5. R^G HOMEOMORPHISMS

Definition 5.1:

A bijection f : (X,τ) $\to$ (Y,σ) is called regular ^ generalized (briefly r^g) homeomorphism if both f and f^{-1} are r^g continuous.

We say that the spaces (X,τ) and (Y,σ) are r^g-homeomorphic if there exists an r^g – homeomorphism from (X,τ) onto (Y,σ).

Definition 5.2:

A bijection f: (X,τ) $\to$ (Y,σ) is said to be r^g*-homeomorphism if both f and f^{-1} are r^g irresolutes.

We say that the spaces (X,τ) and (Y,σ) are r^g*-homeomorphic if there exists an r^g*- homeomorphism from (X,τ) onto (Y,σ).

We denote the family of all r^g*- homeomorphisms of a topological spaces of (X,τ) itself by r^g*- h(X,τ).

Theorem 5.3:

(i) Every homeomorphism is an r^g homeomorphism.

(ii) Every g-homeomorphism is an r^g homeomorphism.

(iii) Every r^g* – homeomorphism is an r^g homeomorphism.

(iv) Every rwg homeomorphism is an r^g homeomorphism.

(v) Every rgw homeomorphism is an r^g homeomorphism.

Proof:

Follows from the definition.

Remark 5.4:

The converse of the above theorem need not be true as seen from the following examples.

Example 5.5:

- Let X = Y = {a,b,c,d}, τ = {X,φ,{a},{b},{a,b},{a,b,c}}, σ = {Y,φ,{a},{c,d},{a,c,d}}. Define a map f : (X,τ) $\to$ (Y,σ) by f(a) = a, f(b) = d, f(c) = c, f(d) = b. Then f is r^g homeomorphism but not homeomorphism.
- Let X = Y = {a,b,c}, τ = {X,φ,{a},{b},{a,b}}, σ = {Y,φ,{a,b},{c}}. Define an identity map f: (X,τ) $\to$ (Y,σ). Then f is r^g homeomorphism but not g homeomorphism.
- Let X = Y = {a,b,c,d}, τ = {X,φ,{a},{b},{a,b},{a,b,c}}, σ = {Y,φ,{a},{c,d},{a,c,d}}. Define a map f : (X,τ) $\to$ (Y,σ) by f(a) = a, f(b) = d, f(c) = c, f(d) = b. Then f is r^g homeomorphism but not r^g* homeomorphism

- Let X = Y = {a,b,c,d}, τ = {X,φ,{a},{c},{a,c},{a,b},{a,b,c}}, σ = {Y,φ,{c},{d},{b,d}, {c,d},{a,c,d},{b,c,d}}. Define a map g: (X,τ) $\rightarrow$ (Y,σ) by g(a) = b, g(b) = d, g(c) = c, g(d) = a, then g is rgw and rwg homeomorphisms but not r^g homeomorphism.

Remark 5.6:

The composition of two r^g homeomorphism need not be r^g homeomorphism in general as seen from the following example.

Example 5.7:

Let X = Y = Z = {a,b,c,d}, τ = {X,φ ,{a},{b},{a,b},{a,b,c}}, σ = {Y,φ,{a}, {c,d}, {a,c,d}}, η = {Z,φ,{a},{c},{d},{a,c},{a,d},{c,d},{a,c,d}}.

Define f: (X,τ) $\rightarrow$ (Y,σ), g: (Y,σ) $\rightarrow$ (Z,η), the identity mappings, then f and g are r^g homeomorphisms. But g$\circ$f: (X,τ) $\rightarrow$ (Y,σ) is not an r^g homeomorphism, because for the closed set {b} in (Z,η), $(g\circ f)^{-1}$\{b\} = f^{-1}\{g^{-1}\{b\}\} = f^{-1}\{b\} = {b} is not an r^g closed set in (X,τ).

Remark 5.8:

The converse of the above theorem need not be true as seen from the following example.

Remark 5.9:

The concept of r^g - homeomorphism is independent with the concept of wg- homeomorphism as seen from the following example.

Example 5.10:

* X = Y = {a,b,c,d},τ = {X,φ,{a},{c},{a,c},{a,b},{a,b,c}}, σ = {Y,φ,{a},{b},{a,b}, {a,b,c}. Define the identity map f: (X,τ) $\rightarrow$ (Y,φ). Then f is wg homeomorphism but not r^g homeomorphism.

* X = Y {a,b,c}, τ = {X,φ,{a},{b},{a,b}}, σ = {Y,φ,{c},{a,b}}. The identity map f: (X,τ) $\rightarrow$ (Y,σ) is r^g homeomorphism but not wg homeomorphism.

Remark 5.11:

r^g - homeomorphism is independent with the concepts of sg- homeomorphism and gs- homeomorphism as seen from the following example.

Example 5.12:

***** Let X = Y = {a,b,c}, τ = {X,φ,{a},{b},{a,b}}, σ = {Y,φ,{a,b},{c}}. Define an identity map f: (X,τ) $\rightarrow$ (Y,σ). Then f is r^g homeomorphism but not sg- homeomorphism and gs- homeomorphism.

* Let X = Y = {a,b,c}, τ = {X, ,φ,{a,b},{c}}, σ = {Y,φ,{a},{b},{a,b}}. Define f: (X,τ) $\rightarrow$ (Y,σ) by f(a) = a, f(b) = c, f(c) = b , then f is sg and gs homeomorphism but not r^g homeomorphism.

Remark 5.13:

The concept of r^g - homeomorphism is independent with the concept of α homeomorphism as seen from the following example.

Example 5.14:

***** X = Y = {a,b,c}, τ = {X,φ,{a},{b},{a,b}}, σ = {Y,φ,{a},{a,b}}. Define the identity map f: (X,τ) $\rightarrow$ (Y,φ), then f is r^g homeomorphism but not α homeomorphism.

* X = Y = {a,b,c,d}, τ = {X,φ,{a},{c},{a,c},{c,d},{a,c,d}}, σ = {Y,φ,{a},{c},{a,c},{a,b}, {a,b,c}}. Define a map f: (X,τ) $\rightarrow$ (Y,σ) by f(a) = c, f(b) = d, f(c) = a, f(d) = b. Then f is α homeomorphism but it is not r^g homeomorphism.

Remark 5.15:

r^g - homeomorphism is independent with the concepts of swg- homeomorphism as seen from the following example.

Example 5.16:

***** X = Y = {a,b,c,d}, τ = {X, φ,{a},{b},{a,b}, {a,b,c}}, σ = {Y, φ,{a},{c},{a,c},{a,b}, {a,b,c}}. Define f: (X,τ) $\rightarrow$ (Y,σ), then f is r^g homeomorphism but not swg homeomorphism.

* X = Y = {a,b,c,d}, τ = {X,φ,{a},{c},{a,c},{a,b},{a,b,c}}, σ = {Y,φ,{a},{c},{a,c}, {c,d}, {a,c,d}}. Define a map f: (X,τ) $\rightarrow$ (Y,σ) by f(a) = c, f(b) = d, f(c) = a, f(d) = b. Then f is swg homeomorphism but it is not r^g homeomorphism.

The above discussions are implicated as shown below:

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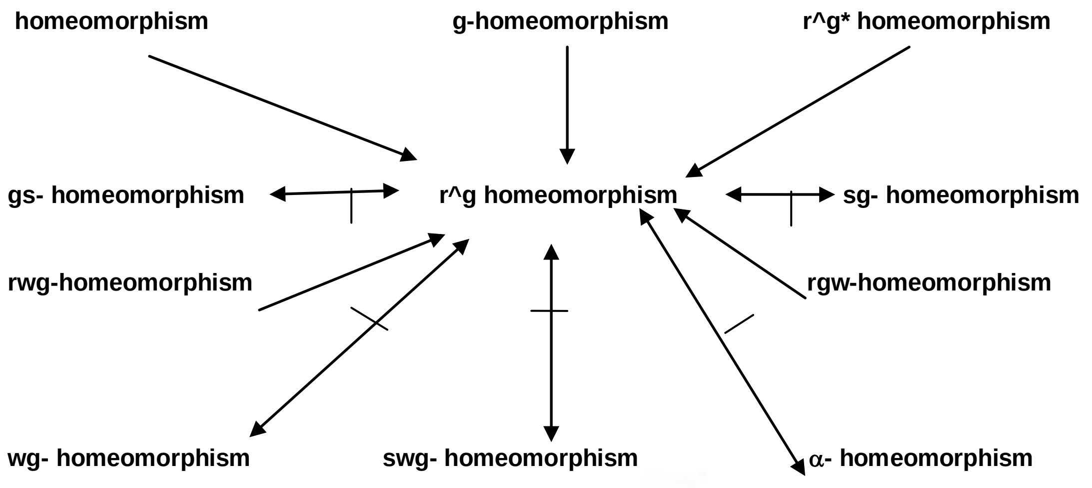

where A $\longrightarrow$ B represents A implies B but not conversely, and A $\nleftrightarrow$ B represents A and B are independent.

Theorem 5.17:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ and g: $(Y,\sigma) \rightarrow (Z,\eta)$ be r^g*-homeomorphisms. Then their composition $g \circ f : (X,\tau) \rightarrow (Y,\sigma)$ is also r^g*-homeomorphism.

Proof:

Suppose f and g are r^g homeomorphisms. Then f and g are r^g irresolutes. Let U be r^g closed set in (Z,η). Since g is r^g irresolute, $g^{-1}(U)$ is r^g – closed in (Y,σ). This implies that $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is r^g closed in (X,τ), since f is r^g irresolute. Hence $(g \circ f)$ is r^g irresolute. Also for an r^g closed set V in (X,τ) we have $g \circ f(V) = g(f(V))$. By hypothesis, f(V) is r^g closed set in (Y,σ), this implies that g(f(V)) is r^g closed set in (Z,η) i.e., $g \circ f(V)$ is r^g closed set in (Z,η) implies that $(g \circ f)^{-1}$ is r^g irresolute. Also $g \circ f$ is a bijection. This proves $g \circ f$ is r^g homeomorphism.

Theorem 5.18:

The set r^g*-h(X,τ) from (X,τ) onto itself is a group under the composition of functions.

Proof:

Let f,g $\in$ r^g*-h(X,τ). Then by theorem 5.13, $g \circ f \in$ r^g*-h(X,τ). The composition of functions is associative and the identity element I: $(X,\tau) \rightarrow (X,\tau)$ belonging to r^g*-h(X,τ) serves as the identity element. If f $\in$ r^g*-h(X,τ) then $f^{-1} \in$ r^g*-h(X,τ). This proves r^g*-h(X,τ) is a group under the operation of functions.

Theorem 5.19:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an r^g*-homeomorphism. Then f induces an isomorphism from the group r^g*- h(X,τ) onto the group r^g*-h(Y,σ).

Proof:

Let f $\in$ r^g*-h(X,τ). We define a function Ψ_f : r^g*-h$(X,\tau) \rightarrow$ r^g*-h(Y,σ) by $\Psi_f(h) = (f \circ g) \circ f^{-1}$, for every h $\in$ r^g*-h(X,τ). Then Ψ_f is a bijection. Further for all $h_1,h_2 \in$ r^g*-h(X,τ) $\Psi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \Psi_f(h_1) \circ \Psi_f(h_2)$. Therefore Ψ_f is a homeomorphism and so it is an isomorphism induced by f.

Theorem 5.20:

Let f : $(X,\tau) \rightarrow (Y,\sigma)$ be a bijective r^g- continuous map. Then the following are equivalent.

(i) f is an r^g – open map.
(ii) f is an r^g – homeomorphism.
(iii) f is an r^g – closed map.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a bijective r^g – continuous map.

(i) → (ii): Let F be a closed set in (X,τ). Then X\F is open in (X,τ). Since f is r^g- open, then f(X\F) is r^g-open in (Y,σ) i.e., f(F) is r^g closed in (Y,σ). Thus f is r^g continuous. Further $(f^{-1})^{-1}(F) = f(F)$ is r^g closed in (Y,σ). Thus f^{-1} is r^g continuous. Hence (i) → (ii).

(ii) → (iii):

Suppose f is an r^g homeomorphism. Then f is bijective, f and f^{-1} are r^g continuous. Let f be an r^g closed set in (X,τ). Since f^{-1} is r^g continuous, $(f^{-1})^{-1}(F) = f(F)$ is r^g closed in (Y,σ). Thus f is r^g closed. Thus (ii) $\rightarrow$ (iii).

(iii) $\rightarrow$ (i):

Let f be an r^g closed map. Let V be r^g open in X. Then X\V is r^g closed in (X,τ). Since f is r^g closed, f(X\V) is r^g closed in (Y,σ).This implies Y\f(V) is r^g closed in (Y,σ). Therefore f(V) is r^g open in (Y,σ). This proves (iii) $\rightarrow$ (i).

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