Inverse System in The Category of Intuitionistic Fuzzy Soft Modules

S.E. Abdullayev, Sadi Bayramov

Department of Algebra and Geometry, Baku State University, 23, Z.Khalilov str., AZ1148, Baku, Azerbaijan

sebuhi_abdullaye@mail.ru, baysadi@gmail.com

Abstract.

This paper begins with the basic concepts of soft module. Later, we introduce inverse system in the category of intutionistic fuzzy soft modules and prove that its limit exists in this category. Generally, limit of inverse system of exact sequences of intutionistic fuzzy soft modules is not exact ([12]). Then we define the notion $\varprojlim^{(1)}$ which is first derived functor of the inverse limit functor. Finally, using methods of homology algebra, we prove that the inverse system limit of exact sequence of intutionistic fuzzy soft modules is exact.

Keywords:

Soft Set, Soft Module, Fuzzy Soft Module, Inverse System, Inverse Limit, Perivative Factor of İnverse Limit.

Language: English

Date of Submission: 2018-02-14

Date of Acceptance: 2018-02-20

Date of Publication: 2018-03-18

ISSN: 2347-1921

Volume: 14 Issue: 01

Journal: Journal of Advances in Mathematics

Publisher: CIRWORLD

Website: https://cirworld.com

1. Introduction

Many practical problems in economics, engineering, environment, social science, medical science etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. Probability theory, fuzzy sets rough sets, and other mathematical tools have their inherent difficulties ([14, 19, 20]). The reason for these difficulties may be due to the inadequacy of the theories of parameterization tools.

Molodtsov [12] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Later, work on the soft set theory is progressing rapidly. Maji et al. [10, 11] have published a detailed theoretical study on soft sets. After Molodtsov's work, some different applications of soft sets were studied in [16]. H. Aktaş and N. Cagman [2] has established a connection betwen soft sets and fuzzy sets and they introduced soft groups. At the same time, they gave a definition of soft groups, soft rings and derived their basic properties ([1, 7, 8]). Qiu-Mei Sun et al. [20] defined soft modules and investigated their basic properties.

U. Acar and F. Koyuncu introduced soft rings. Qiu-Mei Sun and his friends introduced soft modules [15].

L. Jin-Liang [1, 16] presented intuitionistic fuzzy soft sets and intuitionistic fuzzy soft groups. C. Gunduz and S. Bayramov [11] presented fuzzy soft and intuitionistic fuzzy soft modules.

The problem which obtained in new categories are closed according to algebraic operations is very important. Since inverse limit and direct limit contain most of the operations, the proof of presence of the limits is actual problem.

The inverse (direct) limit is not only an important concept in category theory, but also plays an important role in topology, algebra, homology theory etc. To the date, inverse and direct saystems and their limits were defined in different categories. Furthermore, some of their properties were investigated [5, 8, 10, 12].

In this paper begins with the basic concepts of soft module. We introduce inverse system in the category of intuitionistic fuzzy soft modules and prove that its limit exists in this category. Generally, limit of inverse system of exact sequences of intuitionistic fuzzy soft modules is not exact. Then we define the notion $\varprojlim^{(1)}$ which is first derived functor of the inverse limit functor. Finally, using methods of homology algebra, we prove that the inverse system limit of exact sequence of intuitionistic fuzzy soft modules is exact.

2. Preminilaries

In this section, we recall necessary information commonly used in intuitionistic fuzzy soft module.

Definition 2.1. ([17]). Let X be an initial universe set and E be a set of parameters. A pair (F,E) is called a soft set over X if only if F is a mapping from E into the set of all subsets of the set X, i.e., $F:E\to P(X)$, where $P(X)$ is the power set of X.

In other words, the soft set is a parameterized family of subsets of the set X. Every set $F(e)$, for every $e\in E$, may be considered as the set of $e-$elements of the soft set (F,E), or as the set of $e-$approximate elements of the soft set.

According to this manner, a soft set (F,E) is given as consisting of collection of approximations:

$$(F,E)=\{F(e):\ e\in E\}.$$

Definition 2.2 ([4, 11]). Let I^X denote the set of all fuzzy sets on X and $A \subset E$. A pair (f, A) is called a intutionistic fuzzy soft set over X, where f is a mapping from A into I^X. That is, for each $a \in A$, $f(a) = f_a : X \to I$, is a fuzzy set on X.

Definition 2.3 ([4, 11]). Union of two fuzzy soft sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A \cup B$ and

$$h(c) = \begin{cases} f(c), & \text{if } c \in A - B \\ g(c), & \text{if } c \in B - A \\ f(c) \vee g(c), & \text{if } c \in A \cap B \end{cases}, \quad \forall c \in C.$$

It is denoted as $(f, A) \cup (g, B) = (h, C)$.

Definition 2.4 ([4, 11]). Intersection of two fuzzy soft sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A \cap B$ and $h(c) = f(c) \wedge g(c)$, $\forall c \in C$.

It is written as $(f, A) \cap (g, B) = (h, C)$.

Definition 2.5 ([4, 11]). If (f, A) and (g, B) are two soft sets, then (f, A) and (g, B) is denoted as $(f, A) \wedge (g, B)$. $(f, A) \wedge (g, B)$ is defined as $(h, A \times B)$ where $h(a, b) = f(a) \wedge g(b)$, $\forall (a, b) \in A \times B$.

Now, let M be a left $R-$module, A be any nonemply set. $F : A \to P(M)$ refer to a set-valued function and the pair (F, A) is a soft set over M.

Definition 2.6 ([20]). Let (F, A) be a soft set over M. (F, A) is said to be a soft module over M if and only if $F(x) < M$ for all $x \in A$.

Definition 2.7 ([20]). Let (F, A) and (G, B) be two soft modules over M and N respectively. Then $(F, A) \times (G, B) = (H, A \times B)$ is defined as $H(x, y) = F(x) \times G(y)$ for all $(x, y) \in A \times B$.

Proposition 2.8 ([20]). Let (F, A) and (G, B) be two soft modules over M and N respectively. Then $(F, A) \times (G, B)$ is soft module over $M \times N$.

Definition 2.9 ([20]). Let (F, A) and (G, B) be two soft modules over M and N respectively, $f : M \to N$, $g : A \to B$ be two functions. Then we say that (f, g) is a soft homomorphism if the following conditions are satisfied:

(1) f is a homomorphism from M onto N,

(2) g is a mapping from A onto B, and

(3) $f(F(x)) = G(g(x))$ for all $x \in A$.

Definition 2.10 ([11]). Let (F, A) be a intutionistic fuzzy soft set over M. Then (F, A) is said to be a intutionistic fuzzy soft module over M iff for each $a \in A$, $F(a)$ is a intutionistic fuzzy submodule of M and denoted as $F_a = (F_a, F^a)$.

Definition 2.11 ([11]). Let (F,A) and (H,B) be two intuitionistic fuzzy soft modules over M and N respectively, and let $f:M\to N$ be a homomorphism of modules, and let $g:A\to B$ be a mapping of sets. Then we say that $(f,g):(F,A)\to(H,B)$ is a fuzzy soft homomorphism of intutionistic fuzzy soft modules, if the following condition is satisfied:

$$f(F(a))=f(F_a,F^a)=H(g(a))=(H_{g(a)},H^{g(a)}).$$

Theorem 2.16 ([11]). If $\{(F_i,A_i)\}_{i\in I}$ is a family of intuitionistic fuzzy soft modules over $\{M_i\}_{i\in I}$, then $\prod_{i\in I}(F_i,A_i)$ is a intutionistic fuzzy soft module over $\prod_{i\in I}M_i$.

Theorem 2.17 ([11]). If $\{(F_i,A_i)\}_{i\in I}$ is a family of intuitionistic fuzzy soft modules over $\{M_i\}_{i\in I}$, then $\bigoplus_{i\in I}(F_i,A_i)$ is an intuitionistic fuzzy soft module over $\bigoplus_{i\in I}M_i$.

3. Inverse system of intuitionistic fuzzy soft modules

This category of intuitionistic fuzzy soft modules denoted as IFSM.

Definition 3.1. Any factor $D:\Lambda^{op}\to IFSM$, where Λ is a directed set, is called an inverse system of intuitionistic fuzzy soft modules.

Now we consider the following any inverse system

$$\left(\{(F_\alpha,A_\alpha)\}_{\alpha\in\Lambda},\ \left\{(p_\alpha^{\alpha'},q_\alpha^{\alpha'}):(F'_{\alpha'},A'_{\alpha'})\to(F_\alpha,A_\alpha)\right\}_{\alpha\pi\alpha'}\right). \tag{3.1}$$

It is clear that parameter sets in (3.1) consist of the following inverse system of sets

$$\left(\{A_\alpha\}_{\alpha\in\Lambda},\ \{q_\alpha^{\alpha'}:A'_\alpha\to A_\alpha\}_{\alpha\pi\alpha'}\right). \tag{3.2}$$

Similarly, $\{M_\alpha\}_{\alpha\in\Lambda}$ in (3.1) consist of the following inverse system of modules

$$\left(\{M_\alpha\}_{\alpha\in\Lambda},\ \{p_\alpha^{\alpha'}:M'_\alpha\to M_\alpha\}_{\alpha\pi\alpha'}\right). \tag{3.3}$$

Let $A=\varprojlim_\alpha A_\alpha$ be inverse limit of (3.2) and $M=\varprojlim_\alpha M_\alpha$ be inverse limit of (3.3). Since $p_\alpha^{\alpha'}(a_{\alpha'})=a_\alpha$ for all $a=\{a_\alpha\}\in A$,

$$\left(\{(M_\alpha,(F_\alpha)_{a_\alpha})\}_{\alpha\in\Lambda},\ \left\{p_\alpha^{\alpha'}:\left(M_{\alpha'},(F_{\alpha'})_{a_{\alpha'}}\right)\to(M_\alpha,(F_\alpha)_{a_\alpha})\right\}_{\alpha\pi\alpha'}\right) \tag{3.4}$$

is an inverse system of intuitionistic fuzzy modules?

We denote inverse limit of (3.4) as (M,F_α). We define $F:A\to PF(M)$ as $F(\alpha)=F_\alpha$. Then (F,A) is an intuitionistic fuzzy soft module over M.

If $\pi_\alpha:\lim M_\alpha\to M_\alpha$ and $q_\alpha:\varprojlim A_\alpha\to A_\alpha$ are projection mappings, then $(\pi_\alpha,q_\alpha):(F,A)\to(F_\alpha,A_\alpha)$ is a homomorphism of intuitionistic fuzzy soft modules, and, for $\alpha\,\pi\,\alpha'$, the following diagram is commutative:

$$(F,A)\xrightarrow{(\pi_\alpha,q_\alpha)}(F_\alpha,A_\alpha)$$
$$(\pi_\alpha,q_\alpha)] \qquad\qquad Z\ p_\alpha^{\alpha'} \quad .$$
$$(F'_\alpha,A'_\alpha)$$

Theorem 3.2. Every inverse system of intuitionistic fuzzy soft modules has limit. This limit is unique and this limit is equal to (F,A).

Proof. We get inverse system (3.1). Let (G,B) be an intuitionistic fuzzy soft module over N. For $\{(h_\alpha,\varphi_\alpha):(G,B)\to(F_\alpha,A_\alpha)\}_{\alpha\in\Lambda}$ be a family of intuitionistic fuzzy soft homomorphisms of intuitionistic fuzzy soft modules, the conditions $\alpha\,\pi\,\alpha'$, $(p_\alpha^{\alpha'},q_\alpha^{\alpha'})(h_{\alpha'},\varphi_{\alpha'})=(h_\alpha,\varphi_\alpha)$. Now we define intuitionistic fuzzy soft homomorphism $(\psi,\gamma):(G,B)\to(F,A)$, where $\gamma:B\to A=\varprojlim_{\alpha} A_\alpha$, $\gamma(b)=\{\varphi_\alpha(b)\}$ and $\psi:N\to M=\varprojlim_{\alpha} M_\alpha$, $\psi(x)=\{h_\alpha(x)\}$. Then $(\psi,\gamma):(G,B)\to(F,A)$ is an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy soft modules. It is clear that for all $\alpha\in\Lambda$, the following diagram is commutative:

$$(G,B)\xrightarrow{(h_\alpha,\phi_\alpha)}(F_\alpha,A_\alpha)$$
$$(\psi,\gamma)] \qquad\qquad Z\ (\pi_\alpha,q_\alpha) \quad .$$
$$(F,A)$$

The proof is completed.

Now we consider the following inverse system of intuitionistic fuzzy soft modules over $\{N_\beta\}_{\beta\in\Lambda'}$

$$(G,B)=\left(\{(G_\beta,B_\beta)\}_{\beta\in\Lambda'},\left\{\left(r_\beta^{\beta'},\ \chi_\beta^{\beta'}\right):(G_{\beta'}B_{\beta'})\to(G_\beta,B_\beta)\right\}_{\beta\,\pi\,\beta'}\right). \qquad (3.5)$$

Let $\varphi:\Lambda'\to\Lambda$ be an isotone mapping and following mapping

$$(f_\beta,g_\beta):\left(F_{\varphi(\beta)},A_{\varphi(\beta)}\right)\to(G_\beta,B_\beta)$$

be an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy soft modules, for all $\beta\in\Lambda'$.

Definition 3.3. If for all $\beta\,\pi\,\beta'$, the condition

$$\left(r_\beta^{\beta'},\ \chi_\beta^{\beta'}\right)\mathrm{o}(f'_\beta,g'_\beta)=(f_\beta,g_\beta)\,\mathrm{o}\left(p_{\varphi(\beta)}^{\varphi(\beta')},q_{\varphi(\beta)}^{\varphi(\beta')}\right)$$

is satisfied, then the family $\left(\varphi,\ \{(f_\beta,g_\beta)\}_{\beta\in\Lambda'}\right)$ is said to by morphism of inverse systems.

It is clear that inverse systems of intuitionistic fuzzy soft modules and morphisms of them consist of a category. This category is denoted as *Inv*(*IFSM*).

Let $\left(\varphi,\ \{(f_\beta,g_\beta)\}_{\beta\in\Lambda'}\right):(\underline{F},\underline{A})\to(\underline{G},\underline{B})$ be a morphism of inverse systems of intuitionistic fuzzy soft modules. Here $\underline{B}=\left(\{B_\beta\}_{\beta\in\Lambda'},\{\chi_\beta^{\beta'}\}_{\beta\,\pi\,\beta'}\right)$ is an inverse system of sets and

$\left(\varphi, \{(g_\beta)\}_{\beta\in\Lambda'}\right): \underline{A} \to \underline{B}$ is a morphism of inverse systems of sets. Then the mapping $g = \lim\limits_{\leftarrow}\left(\varphi, \{g_\beta\}_{\beta\in\Lambda'}\right): \lim\limits_{\alpha} A_\alpha = A \to \lim\limits_{\beta} B_\beta = B$ is a mapping of limit sets of this inverse systems. Similarly,

$$\left(\varphi, \{(f_\beta)\}_{\beta\in\Lambda'}\right): \{M_\alpha\}_{\alpha\in\Lambda} \to \{N_\beta\}_{\beta\in\Lambda'}$$

is a morphism of inverse systems of modules?

Proposition 3.4. Let $\lim\limits_{\leftarrow}\left(\varphi, \{f_\beta\}_{\beta\in\Lambda'}\right) = f$. Then

$$(f, g): \lim\limits_{\alpha}(F_\alpha, A_\alpha) \to \lim\limits_{\beta}(G_\beta, B_\beta)$$

is a morphism of limits of inverse systems of intuitionistic fuzzy soft modules?

Proof. Since the product operation of intuitionistic fuzzy soft modules is a factor, the following diagram is commutative:

$$\begin{array}{ccc} \prod\limits_{\beta} A_{\varphi(\beta)} & \xrightarrow{\Lambda F_\beta} & \prod\limits_{\beta} M_{\varphi(\beta)} \\ \prod g_\beta \downarrow & & \downarrow \prod f_\beta \\ \prod\limits_{\beta} B_\beta & \xrightarrow{\Lambda G_\beta} & \prod\limits_{\beta} N_\beta \end{array}.$$

For all $\{\alpha_{\varphi(\beta)}\} \in \prod\limits_{\beta} A_{\varphi(\beta)}$

$$(\varphi, \{f_\beta\}_{\beta\in\Lambda'}): \{(M_{\varphi(\beta)}, F_{a_{\varphi(\beta)}})\} \to \left\{\left(N_\beta, G_{g\beta(a_{\varphi(\beta)})}\right)\right\}_{\beta\in\Lambda'}$$

is a morphism of inverse systems of intuitionistic fuzzy modules? Then

$$\lim\limits_{\leftarrow}\left(\varphi, \{f_\beta\}_{\beta\in\Lambda'}\right): \lim\limits_{\leftarrow}\{(M_{\varphi(\beta)}, F_{a_{\varphi(\beta)}})\} \to \lim\limits_{\leftarrow}\left\{\left(N_\beta, G_{g\beta(a_{\varphi(\beta)})}\right)\right\}_{\beta\in\Lambda'}$$

is an intuitionistic fuzzy soft homomorphism of intuitionistic fuzzy modules and the following diagram being commutative:

$$\begin{array}{ccc} A & \xrightarrow{F} & \lim M_{\phi(\beta)} \\ g\downarrow & & f\downarrow \\ B & \xrightarrow{G} & \lim N_\beta \end{array}.$$

Theorem 3.5. The corresponding

$$\{(F_\alpha, A_\alpha)\}_{\alpha\in\Lambda} \to \lim\limits_{\alpha}(F_\alpha, A_\alpha)$$

is a covariant factor from the category Inv (IFSM) to the category of IFSM.

Theorem 3.6. If $\{(F,A)\}_{j\in J}$ is a family of inverse systems of intuitionistic fuzzy soft modules, then

$$\varprojlim \prod_j (F,A)_j = \prod_j \varprojlim (F,A)_j .$$

Proof. The proof of the theorem is straightforward.

4. Derivative Factor of $\varprojlim$ factor

Let us review the problem of exact limit for inverse system of exact sequence of intuitionistic fuzzy soft modules

Example 4.1. Let $M_n = Z$, $M'_n = Z$, $M''_n = Z_2$ be modules a ring. Then

$$\underline{M} = \left(\{M_n\}_{n\in N}, \{p_n^{n+1}(m) = 3m\}\right)$$

$$\underline{M'} = \left(\{M'_n\}_{n\in N}, \{q_n^{n+1}(m) = 3m\}\right)$$

$$\underline{M''} = \left(\{M''_n\}_{n\in N}, \{r_n^{n+1}([m]) = [m]\}\right)$$

are inverse systems of modules and?

$$f = \{f_n : M'_n \to M_n \, f_n(m) = 2m\}$$

$$g = \{g_n : M_n \to M''_n g_n(m) = [m]\}$$

are morphisms of inverse systems. The following sequence

$$0 \to \underline{M'} \xrightarrow{f} \underline{M} \xrightarrow{g} \underline{M^n} \to 0$$

is short exact sequence of inverse systems of Z- modules.

Let A be a parameter set

$$F'_n : A \to IFSM(M'_n), \quad F_n : A \to IFSM(M_n), \quad F''_n : A \to IFSM(M''_n)$$

intuitionistic fuzzy soft modules defined by the formula

$$\forall a \in A, \quad F'_{na} = (\chi(0))_{M'_n}, \quad F'^{a}_n = 1-(\chi(0))_{M'_n}, \quad F_{na} = (\chi(0))_{M_n},$$

$$F^a_n = 1-(\chi_a(0))_{M'_n}, \quad F''_{na} = (\chi(0))_{M''_n}, F''^{a}_n = 1-(\chi(0))_{M''_n}.$$

The sequence

$$0 \to (M'_n, F'_{na}, F'^{a}_n) \to (M_a, F_{na}, F^a_n) \to (M''_n, F''_{na}, F''^{a}_n) \to 0$$

is also short exact sequence of intuitionistic fuzzy modules for each $a \in A$. Then the sequence

$$0 \to (F', A) \to (F, A) \to (F'', A) \to 0$$

is short exact sequence of inverse systems of intuitionistic fuzzy soft modules. Taking the limits of this sequence is not exact.

As it seen the limit of inverse system of exact sequence of intuitionistic fuzzy soft modules is not exact. So, it is necessary to define derivative factor of inverse limit factor in category of intuitionistic fuzzy soft modules.

We get inverse system in (3.1). We define the following homomorphism of modules

$$d : \prod_{\alpha} M_{\alpha} \to \prod_{\alpha} M_{\alpha}$$

by the formula:

$$d(\{x_{\alpha}\}) = \{x_{\alpha} - p_{\alpha}^{\alpha'}(x'_{\alpha})\}_{\alpha \pi \alpha'}.$$

We demonstrate that $\forall a \in A \ d$ is a homomorphism of intuitionistic fuzzy modules. Indeed,

$$F_{Aa}(d(\{x_{\alpha}\})) = F_{Aa}(\{x_{\alpha} - p_{\alpha}^{\alpha'}(x_{\alpha'})\}) = \wedge_{\alpha} F_{\alpha a}(x_{\alpha} - p_{\alpha}^{\alpha'}(x_{\alpha'}))$$

$$\geq \wedge_{\alpha} \min\{F_{\alpha a}(x_{\alpha}), F_{\alpha a}(p_{\alpha}^{\alpha'}(x_{\alpha'}))\}.$$

$$F_A^a(d(\{x_{\alpha}\})) = F_A^a(\{x_{\alpha} - p_{\alpha}^{\alpha'}(x_{\alpha'})\}) = \vee_{\alpha} F_{\alpha}^a(x_{\alpha} - p_{\alpha}^{\alpha'}(x_{\alpha'}))$$

$$\leq \vee_{\alpha} \max\{F_{\alpha}^a(x_{\alpha}), F_{\alpha}^a(p_{\alpha}^{\alpha'}(x_{\alpha'}))\}.$$

Since $F_{\alpha a}(p_{\alpha}^{\alpha'}(x_{\alpha'})) \geq F_{\alpha' a}(x_{\alpha'})$, and $F_{\alpha}^a(p_{\alpha}^{\alpha'}(x_{\alpha'})) \leq F_{\alpha'}^a(x_{\alpha'})$

$$F_{Aa}(d(\{x_{\alpha}\})) \geq \wedge_{\alpha} \min\{F_{\alpha a}(x_{\alpha}), F_{\alpha' a}(x_{\alpha'})\}$$

$$= \wedge_{\alpha}(F_{\alpha a}(x_{\alpha}) \Lambda F_{\alpha'' a}(x_{\alpha'}))$$

$$= \wedge_{\alpha} F_{\alpha a}(x_{\alpha}) = F_{Aa}(\{x_{\alpha}\})$$

and

$$F_A^a(d(\{x_{\alpha}\})) \leq \vee_{\alpha} \max\{F_{\alpha}^a(x_{\alpha}), F_{\alpha'}^a(x_{\alpha'})\}$$

$$= \vee_{\alpha}(F_{\alpha}^a(x_{\alpha}) \vee F_{\alpha'}^a(x_{\alpha'})) = \vee_{\alpha} F_{\alpha}^a(x_{\alpha}) = F_A^a(\{x_{\alpha}\}).$$

Then $\bar{d}$ is a homomorphism of intuitionistic fuzzy modules. Therefore $(\ker d, F_{Aa}|\ker d)$ and $(co \ker d, (F_{Aa})_p)$ are defined.

For inverse system of modules $\left(\{M_\alpha\}_{\alpha\in\Lambda}, \{p_\alpha^{\alpha'}\}_{\alpha\pi\alpha'}\right)$, $\varprojlim^{(1)} M_\alpha = \prod_\alpha M_\alpha \big/ \mathrm{Im}\, d$ is derivative factor.

If $\pi = \prod_\alpha M_\alpha \to \varprojlim^{(1)} M_\alpha$ is the canonical homomorphism, we can define intuitionistic fuzzy modules by $\left(\varprojlim^{(1)} M_\alpha, (F_A)_\alpha^\pi, (F_A)_\pi^a\right)$. Then $(F_A^\pi, F_\pi^A): A \to \prod_\alpha M_\alpha$ is intuitionistic fuzzy soft module.

Definition 4.2. $\left((F_A)^\pi, (F^A)_\pi\right)$ is called "first derived factor" of the inverse system of intuitionistic fuzzy soft modules given (3.1).

Proposition 4.3. $\varprojlim^{(1)}$ is a factor.

Proof. For this reason, it suffices to show that for each the morphism

$$\underline{\overline{f}} = \left(\rho : B \to A, \left\{\left(\overline{f}_\beta, g_\beta\right) : \left(F_{\rho(\beta)}, A_{\rho(\beta)}\right) \to (G_\beta, B_\beta)\right\}_{\beta\in B}\right),$$

$\varprojlim^{(1)} \underline{\overline{f}} : \left((F_A)^\pi, (F^A)_\pi, A\right) \to \left((G_B)^\pi, (G^B)_\pi, B\right)$ is the homomorphism of intuitionistic fuzzy soft modules? Since

$$(F_\pi^A)(x + imd) = \inf_{z\in \mathrm{Im}\, d} F^A(x+z) \geq \inf_{z\in \mathrm{Im}\, d} G_B(f(x+z)) = \inf_{z\in \mathrm{Im}\, d} G_B(f(x) + f(z))$$

$$= \inf_{y=f(z)} G^B(f(x)+y) \geq \inf_{y\in \mathrm{Im}\, d} (f(x)+y) = (G^B)_\pi \left(\varprojlim^{(1)} \underline{f}(x + \mathrm{Im}\, d)\right),$$

$\varprojlim^{(1)}$ is a factor.

We investigate another property of $\varprojlim^{(1)}$ factor, let us introduce the category of chain complexes of intuitionistic fuzzy soft modules ([5]).

Let $\{(F_n, A)\}_{n\in Z}$ be intuitionistic fuzzy soft modules over $\{M_n\}_{n\in Z}$ and let for $\forall n \in Z$,

$$(\partial_n, 1_A) : (F_n, A) \to (F_{n-1}, A)$$

be homomorphism of intuitionistic fuzzy soft modules.

Definition 4.4. If for all $a \in A$ $\left\{\left(M_n, F_{na}, F_n^a\right), \partial_n : \left(M_n, F_{na}, F_n^a\right) \to \left(M_{n-1}, F_{n-1a}, F_{n-1}^a\right)\right\}$ is chain complex of intuitionistic fuzzy soft modules, then the following sequence is said to be a chain complex of intuitionistic fuzzy soft modules

$$(F, A) = \left\{(F_n, A), (\partial_n, 1_A) : (F_n, A) \to (F_{n-1}, A)\right\}.$$

Let $(F, A) = \left\{(F_n, A), (\partial_n, 1_A)\right\}$ be a chain complex of intuitionistic fuzzy soft modules. Then for each $a \in A$ we obtain the fuzzy homology module

$$H_n(F, \alpha) = \ker \partial_n \setminus \mathrm{Im}\partial_{n+1}$$

for the fuzzy chain complex

$$\{(M_n, F_n(a)), \partial_n : (M_n, F_n(a)) \to (M_{n-1}, F_{n-1}(a))\}.$$

Thus, for all $a \in A$ the fuzzy module $H_n(F.a)$ is a quotient module in $\{(M_n, F_{na})\}$. If there exist a one to one and covered connection with every fuzzy submodule of fuzzy quotient module of (M_n, F_{na}) and fuzzy submodule of we can think the intuitionistic fuzzy module $H_n(F, a)$ as a fuzzy submodule of (M_n, F_{na}). Thus,

$$H_n(F,-) : A \to FSM(M_n)$$

is an intuitionistic fuzzy soft module?

Definition 4.5. Intuitionistic fuzzy soft module $(H_n(F,-), A)$ is said to be $n-$dimensional fuzzy soft homology module of chain complex of intuitionistic fuzzy soft modules

$$(F, A) = \{(F_n, A), \quad (\partial_n, 1_A)\}.$$

Definition 4.6. Let $\{(F_n, A), \quad (\partial_n, 1_A)\}$ and $\{(G_n, A), \quad (\partial'_n, 1_B)\}$ be chain complexes of intuitionistic fuzzy soft modules over $\{M_n\}_{n \in Z}$ and $\{N_n\}_{n \in Z}$ respectively and let $\{f_n : M_n \to N_n\}$ is homomorphism of modules, $g : A \to B$ is a mapping of sets. If for all $a \in A$, $f_n : (M_n, F_n^a) \to (N_n, G_n^{g(a)})$ is a fuzzy homomorphism of intuitionistic fuzzy modules and the condition $\partial'_n \circ f_n = f'_{n-1} \circ \partial_n$ is satisfied, then

$$(f_n, g) : (F_n, A) \to (G_n, A)$$

is said to be morphism of chain complexes of intuitionistic fuzzy soft modules.

Definition 4.7. Let $(\{\varphi_n\}, g)$, $(\{\psi_n\}, g) : \{(F_n, A), \partial_n\} \to \{(G_n, B), \partial'_n\}$ be morphism of chain complexes of intuitionistic fuzzy soft modules and let

$$D = (\{D_n\}, g) : \{(F_n, A), \partial_n\} \to \{(G_{n+1}, B), \partial'_{n+1}\}$$

be a family of homomorphisms of intuitionistic fuzzy soft modules. If the condition $\varphi_n - \psi_n = D_{n-1}\partial_n + \partial'_{n+1}D_n$ is satisfied then the family of homomorphism of intuitionistic fuzzy soft modules $D = (\{D_n\}, g)$ is said to be chain homotopy morfism $(\{\varphi_n\}, g)$, $(\{\psi_n\}, g)$ is said to be chain homotopy mappings and denoted by $(\{\varphi_n\}, g) \sim (\{\psi_n\}, g)$.

The following theorem can be easily proved.

Theorem 4.8. The chain homotopy relation is an equivalence relation and homology (cohomology) modules are invariant with respect to this relation.

Let

$$\left(\{(F_\alpha \; A\}_{\alpha \in \Lambda}, \{(p_\alpha^{\alpha'}, 1_A) : (F_{\alpha'}, A) \to (F_\alpha, A)\}_{\alpha \pi \alpha'}\right)$$

be an inverse system of intuitionistic fuzzy soft modules.

Let us consider the following cochain complex of fuzzy soft modules

$$\overline{0} \to (\Pi F_\alpha, A) \overset{\overline{d}}{\to} (\Pi F_\alpha, A) \to \overline{0}.$$

Cohomology modules of this complex are $\ker \overline{d}$ and $co\ker \overline{d}$.

Lemma 4.9. $\underset{\leftarrow}{\lim}(F_\alpha, A) = \ker \overline{d}$ and $\underset{\leftarrow}{\lim}^{(1)}(F_\alpha, A) = co\ker \overline{d}$ **.**

Proof. The proof of lemma is trivial.

We accept natural numbers set which is index set of inverse system.

Theorem 5.11. Let the sequence

$$(F_1, A) \overset{p_1^2}{\leftarrow} (F_2, A) \overset{p_2^3}{\leftarrow} \ldots$$

be inverse sequence of intuitionistic fuzzy soft modules. For each infinite subsequence of this sequence, $\underset{\leftarrow}{\lim}^{(1)}$ dose not change.

Proof. Let $S = \{i, j, k, \ldots\}$ be infinite subsequence of natural numbers N. From Lemma 1, $\underset{\leftarrow}{\lim}^{(1)}$ is defined by the following homomorphism of fuzzy soft modules as appropriate subsequence

$$\overline{d}' : \left(\prod_{s \in S} F_s, A \right) \to \left(\prod_{s \in S} F_s, A \right).$$

We may define

$$f_0, f_1 : \prod_{s \in S} M_s \to \prod_{n \in N} M_n$$

homomorphisms of modules with this formula:

$$f_0(x_i, x_j, x_k, \ldots) = \left(p_1^i(x_i), p_2^i(x_i), \ldots, p_{i-1}^i(x_i), x_i, p_{i+1}^j(x_j), \ldots, p_{j-1}^j(x_j), x_j, \ldots \right)$$

$$f_1(x_i, x_j, x_k, \ldots) = (0, 0, \ldots, x_i, 0, \ldots, x_j, 0, \ldots, x_k, 0, \ldots).$$

Also, for each $a \in A$

$$\left(\underset{n \in N}{\wedge} F_{na} \right) \left(p_1^i(x_i), \ldots, p_{i-1}^i(x_i), x_i, p_{i+1}^j(x_j), \ldots, p_{j-1}^j(x_j), x_j, \ldots \right)$$

$$= F_{1a}(p_1^i(x_i)) \wedge \ldots \vee F_{i-1a}(p_{i-1}^i(x_i)) \wedge F_{ia}(x_i) \wedge$$

$$F_{i+1a}(p_{i+1}^j(x_j)) \wedge \ldots \wedge \mu_j(x_j) \wedge \ldots$$

$$\geq [F_{ia}(x_i) \wedge \ldots \wedge F_{ia}(x_i) \wedge F_{ia}(x_i)] \wedge [F_{ja}(x_j) \wedge \ldots \wedge F_{ja}(x_j)] \wedge \ldots$$

$$= F_{ia}(x_i) \wedge F_{ja}(x_j) \wedge \ldots = \underset{s \in S}{\wedge} F_{sa}(x_s)$$

$$\underset{n\in N}{\vee} F_n^a \left(p_1^i(x_i),\ldots,p_{i-1}^i(x_i),x_i,p_{i+1}^j(x_j),\ldots,p_{j-1}^j(x_j),x_j,\ldots\right)$$

$$=F_1^a(p_1^i(x_i))\vee\ldots\vee F_{i-1}^a(p_{i-1}^i(x_i))\vee F_i^a(x_i)\vee\ldots$$

$$F_{i+1}^a(p_{i+1}^j(x_j))\vee\ldots\vee\mu_j(x_j)\vee\ldots$$

$$\leq\left[F_i^a(x_i)\vee\ldots\vee F_i^a(x_i)\vee F_i^a(x_i)\right]\vee\left[F_j^a(x_j)\vee\ldots\vee F_j^a(x_j)\right]\vee\ldots$$

$$=F_i^a(x_i)\vee F_j^a(x_j)\vee\ldots=\underset{s\in S}{\vee} F_s^a(x_s)$$

and

$$\left(\underset{n\in N}{\wedge} F_{na}\right)\left(0,\,0,\ldots,x_i,x_i,0,\ldots,x_j,0,\ldots\right)$$

$$=F_{1a}(0)\wedge\ldots\wedge F_{ia}(x_i)\wedge F_{i+1a}(0)\wedge\ldots\wedge F_{ja}(x_j)\wedge\ldots$$

$$=F_{ia}(x_i)\wedge F_{ja}(x_i)\wedge\ldots=\underset{s\in S}{\wedge} F_{sa}(x_s),$$

$$\left(\underset{n\in N}{\vee} F_n^a\right)\left(0,\,0,\ldots,x_i,x_i,0,\ldots,x_j,0,\ldots\right)$$

$$=F_1^a(0)\vee\ldots\vee F_i^a(x_i)\vee F_{i+1}^a(0)\vee\ldots\vee F_j^a(x_j)\vee\ldots$$

$$=F_i^a(x_i)\vee F_j^a(x_i)\vee\ldots=\underset{s\in S}{\vee} F_s^a(x_s).$$

Then $\bar{f}_0,\,\bar{f}_1:\left(\underset{s\in S}{\Pi} F_s,A\right)\to\left(\underset{n\in N}{\Pi} F_n,A\right)$ are homomorphisms of intuitionistic fuzzy soft modules. It is clear that the following diagram is commutative:

$$\begin{array}{ccc}\left(\underset{s\in S}{\Pi} F_s,A\right) & \to & \left(\underset{n\in N}{\Pi} F_n,A\right)\\ \bar{d}'\downarrow & & \downarrow\bar{d}\\ \left(\underset{s\in S}{\Pi} F_s,A\right) & \to & \left(\underset{n\in N}{\Pi} F_n,A\right)\end{array}$$

i.e. $\left\{\bar{f}_0,\,\bar{f}_1\right\}$ are morphisms of cochin complexes. Now, let us define

$$g_0,\,g_1:\underset{n\in N}{\Pi} M_n\to\underset{s\in S}{\Pi} M_s$$

homomorphisms with this formula:

$$g_0(x_1,x_2,x_3,\ldots)=(x_i,x_j,x_k,\ldots)$$

$$g_1(x_1,x_2,x_3,\ldots)=\begin{pmatrix} x_i+p_i^{i+1}(x_{i+1})+\ldots+p_i^{j-1}(x_{j-1}),x_j \\ +p_j^{j+1}(x_{j+1})+\ldots+p_j^{k-1}(x_{k-1}),\ldots \end{pmatrix}.$$

For

$$\left(\underset{s\in S}{\wedge}F_{sa}\right)\left(x_i,x_j,x_k,\ldots\right)=F_{ia}(x_i)\wedge F_{ja}(x_j)\wedge\ldots\geq\underset{n\in N}{\wedge}F_{na}(x_n)$$

and

$$\left(\underset{s\in S}{\wedge}F_{sa}\right)\left(x_i+p_i^{i+1}(x_{i+1})+\ldots+p_i^{j-1}(x_{j-1}),\ x_j+\ldots+p_j^{k-1}(x_{k-1}),\ldots\right)$$

$$=F_{ia}\left(x_i+p_i^{i+1}(x_{i+1})+\ldots+p_i^{j-1}(x_{j-1})\right)\wedge F_{ja}\left(x_j+\ldots+p_j^{k-1}(x_{k-1})\right)\wedge\ldots$$

$$\geq\min\left\{F_{ia}(x_i),F_{ia}\left(p_i^{i+1}(x_{i+1})\right),\ldots,F_{ia}\left(p_j^{j-1}(x_{j-1})\right)\right\}\wedge$$

$$\min\left\{F_{ja}(x_j),\ldots,F_{ja}\left(p_j^{k-1}(x_{k-1})\right)\right\}\wedge\ldots$$

$$\geq\min\left\{F_{ia}(x_i),F_{i+1a}(x_{i+1}),\ldots,F_{j-1a}(x_{j-1})\right\}\wedge\ldots$$

$$\min\left\{F_{ja}(x_j),\ldots,F_{j+1a}(x_{j+1}),\ldots,F_{k-1a}(x_{k-1})\right\}\wedge\ldots$$

$$=\underset{m\in S}{\wedge}F_{ma}(x_m)\geq\underset{n\in N}{\wedge}F_{na}(x_n),$$

$$\left(\underset{s\in S}{\vee}F_s^a\right)\left(x_i,x_j,x_k,\ldots\right)=F_i^a(x_i)\vee F_j^a(x_j)\vee\ldots\leq\underset{n\in N}{\vee}F_n^a(x_n)$$

and

$$\left(\underset{s\in S}{\vee}F_s^a\right)\left(x_i+p_i^{i+1}(x_{i+1})+\ldots+p_i^{j-1}(x_{j-1}),\ x_j+\ldots+p_j^{k-1}(x_{k-1}),\ldots\right)$$

$$=F_i^a\left(x_i+p_i^{i+1}(x_{i+1})+\ldots+p_i^{j-1}(x_{j-1})\right)\vee F_j^a\left(x_j+\ldots+p_j^{k-1}(x_{k-1})\right)\vee\ldots$$

$$\leq\max\left\{F_i^a(x_i),F_i^a\left(p_i^{i+1}(x_{i+1})\right),\ldots,F_i^a{}_{ia}\left(p_j^{j-1}(x_{j-1})\right)\right\}\vee$$

$$\max\left\{F_j^a(x_j),\ldots,F_j^a\left(p_j^{k-1}(x_{k-1})\right)\right\}\vee\ldots$$

$$\leq\max\left\{F_i^a(x_i),F_{i+1}^a(x_{i+1}),\ldots,F_{j-1}^a(x_{j-1})\right\}\vee\ldots$$

$$\vee\max\left\{F_j^a(x_j),\ldots,F_{j+1}^a(x_{j+1}),\ldots,F_{k-1}^a(x_{k-1})\right\}\vee\ldots=\underset{m\in S}{\vee}F_m^a(x_m),$$

thus, $\bar{g}_0, \bar{g}_1 : \left(\prod_{n\in N} F_n, A\right) \to \left(\prod_{s\in S} F_s, A\right)$ are homomorphisms of fuzzy soft modules and $\bar{d}' \circ \bar{g}_0 = \bar{g}_1 \circ \bar{d}$ are satisfied, i.e., $\{\bar{g}_0, \bar{g}_1\}$ are homomorphisms of cochin complexes. It is clear that

$$\bar{g}_0 \circ \bar{f}_0 = \bar{g}_1 \circ \bar{f}_1 = \bar{1}_{\left(\prod_{s\in S} F_s, A\right)}.$$

Hence, we give

$$D : \prod_{n\in N} M_n \to \prod_{n\in N} M_n$$

homomorphism of modules with this formula:

$$D(x_1, x_2, x_3, \ldots) = \left(x_i + p_1^2(x_2) + \ldots + p_1^{i-1}(x_{i-1}), x_2 + p_2^3(x_3) + \ldots + p_2^{i-1}(x_{i-1}), \ldots\right.$$
$$\left. x_{i-1}, 0,\ x_{i+1} + p_{i+1}^{i+2}(x_{i+2}) + \ldots + p_{i+1}^{j-1}(x_{j-1}), x_{i+2} + \ldots + p_{i+2}^{j-1}(x_{j-1}), 0, \ldots\right).$$

For,

$$\left(\bigwedge_{n\in N} F_{na}\right)\left(x_1 + p_1^2(x_2) + \ldots + p_1^{i-1}(x_{i-1}), x_2 + p_2^3(x_3) + \ldots + p_2^{i-1}(x_{i-1}), \ldots, x_{i-1}, 0, \ldots\right)$$

$$= F_{1a}(x_1 + p_1^2(x_2) + \ldots + p_1^{i-1}(x_{i-1})) \wedge F_{2a}(x_2 + p_2^3(x_3) + \ldots + p_2^{i-1}(x_{i-1})) \wedge \ldots$$

$$\wedge F_{i-1a}(x_{i-1}) \wedge F_{ia}(0) \wedge F_{i+1a}\left(x_{i+1} + p_{i+1}^{i+2}(x_{i+2}) + \ldots + p_{i+1}^{j-1}(x_{j-1})\right) \wedge \ldots$$

$$\geq \min\left\{F_{1a}(x_1), F_{1a}\left(p_1^2(x_2)\right), \ldots, F_{1a}\left(p_1^{i-1}(x_{i-1})\right)\right\} \wedge$$

$$\min\left\{F_{2a}(x_2), F_{2a}\left(p_2^3(x_3)\right), \ldots, F_{2a}\left(p_2^{i-1}(x_{i-1})\right)\right\} \wedge F_{i-1a}(x_{i-1}) \wedge 1 \wedge$$

$$\min\left\{F_{i+1a}(x_{i+1}), F_{i+1a}\left(p_{i+1}^{i+2}(x_{i+2})\right), \ldots, F_{i+1a}\left(p_{i+1}^{j-1}(x_{j-1})\right)\right\} \wedge \ldots$$

$$\geq \min\left\{F_{1a}(x_1), F_{2a}(x_2), \ldots, F_{i-1a}(x_{i-1})\right\} \wedge$$

$$\min\left\{F_{2a}(x_2), F_{3a}(x_3), \ldots, F_{i-1a}(x_{i-1})\right\} \wedge F_{i-1a}(x_{i-1}) \wedge F_{i+1a}(x_{i+1}) \wedge \ldots$$

$$= \bigwedge_{k=1}^{i-1} F_{ka}(x_k) \wedge \bigwedge_{k=2}^{i-1} F_{ka}(x_k) \wedge \ldots = \bigwedge_{n\in N} F_{na}(x_n),$$

$$\left(\bigvee_{n\in N} F_n^a\right)\left(x_1 + p_1^2(x_2) + \ldots + p_1^{i-1}(x_{i-1}), x_2 + p_2^3(x_3) + \ldots + p_2^{i-1}(x_{i-1}), \ldots, x_{i-1}, 0, \ldots\right)$$

$$= F_1^a(x_1 + p_1^2(x_2) + \ldots + p_1^{i-1}(x_{i-1})) \vee F_2^a(x_2 + p_2^3(x_3) + \ldots + p_2^{i-1}(x_{i-1})) \vee \ldots$$

$$\vee F_{i-1}^a(x_{i-1}) \vee F_i^a(0) \vee F_{i+1}^a\left(x_{i+1} + p_{i+1}^{i+2}(x_{i+2}) + \ldots + p_{i+1}^{j-1}(x_{j-1})\right) \vee \ldots$$

$$\leq \max\left\{F_1^a(x_1), F_1^a\left(p_1^2(x_2)\right), \ldots, F_1^a\left(p_1^{i-1}(x_{i-1})\right)\right\} \vee$$

$$\max\left\{F_2^a(x_2), F_2^a\left(p_2^3(x_3)\right),..., F_2^a\left(p_2^{i-1}(x_{i-1})\right)\right\} \vee F_{i-1}^a(x_{i-1}) \vee 0 \vee$$

$$\max\left\{F_{i+1}^a(x_{i+1}), F_{i+1}^a\left(p_{i+1}^{i+2}(x_{i+2})\right),..., F_{i+1}^a\left(p_{i+1}^{j-1}(x_{j-1})\right)\right\} \vee ...$$

$$\leq \max\left\{F_1^a(x_1), F_2^a(x_2),..., F_{i-1}^a(x_{i-1})\right\} \vee$$

$$\max\left\{F_2^a(x_2), F_3^a(x_3),..., F_{i-1}^a(x_{i-1})\right\} \vee F_{i-1}^a(x_{i-1}) \vee F_{i-1}(x_{i-1}) \vee F_{i+1}(x_{i+1}) \vee ...$$

$$= \bigvee_{k=1}^{i-1} F_k^a(x_k) \vee \bigvee_{k=2}^{i-1} F_k^a(x_k) \vee ... = \bigvee_{n\in N} F_n^a(x_n).$$

$\overline{D}:\left(\prod_{n\in N} F_n, A\right) \to \left(\prod_{n\in N} F_n, A\right)$ is a homomorphism of fuzzy soft modules? By using simplicity of calculation, it is shown that $\overline{D}$ is a chain homotopy between $\bar{f}_0 \circ \bar{g}_0$ and $\bar{f}_1 \circ \bar{g}_1$ homomorphisms. Then the following cohomology modules of cochin complexes

$$0 \to \left(\prod_{n\in N} F_n, A\right) \xrightarrow{\bar{d}} \left(\prod_{n\in N} F_n, A\right) \to 0$$

$$0 \to \left(\prod_{s\in S} F_s, A\right) \xrightarrow{\bar{d}} \left(\prod_{s\in S} F_s, A\right) \to 0$$

are fuzzy soft isomorphic. Since $\underset{\leftarrow}{\lim}$ is first cohomology module, the theorem is proved. Since $\underleftarrow{\lim}(F_n, A) = \ker \bar{d}$ and $p_n^{n+1}(x_{n+1}) = x_n$ is satisfied for each $\{x_n\} \in \underleftarrow{\lim} M_n$,

$$F_{na}(x_n) = F_{na}\left(p_n^{n+1}(x_{n+1})\right) \geq F_{n+1a}(x_{n+1})$$

$$F_n^a(x_n) = F_n^a\left(p_n^{n+1}(x_{n+1})\right) \leq F_{n+1}^a(x_{n+1})$$

i.e., for each $\{x_n\} \in \ker \bar{d}$, $\{F_{na}(x_n)\}$ is decreasing sequence.

Theorem 4.11. For all $\{x_n''\} \in \ker \bar{d}$, if $\lim_{n\to\infty} F_{na}''(x_n'') = 0$ or $\lim_{n\to\infty} F_n''^a(x_1'') = 1$ and the following diagram is short exact sequence of inverse of inverse system of fuzzy soft modules

$$\begin{array}{ccccccccc} & & \mathrm{M} & & \mathrm{M} & & \mathrm{M} & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & (F_2', A) & \to & (F_2, A) & \to & (F_2'', A) & \to & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & (F_1', A) & \to & (F_1, A) & \to & (F_1'', A) & \to & 0 \end{array}$$

then the sequence

$$0 \to \underleftarrow{\lim}(F_n', a) \to \underleftarrow{\lim}(F_n, a) \to \underleftarrow{\lim}(F_n'', a) \to$$
$$\underleftarrow{\lim}^{(1)}(F_n', a) \to \underleftarrow{\lim}^{(1)}(F_n, a) \to \underleftarrow{\lim}^{(1)}(F_n'', a) \to 0$$

is exact

Proof. For inverse system of fuzzy soft modules $\{(F_n, A)\}_{n\in N}$,

$$C = 0 \xrightarrow{\bar{0}} \left(\prod_{n\in N} F_n, A\right) \xrightarrow{\bar{d}} \left(\prod_{n\in N} F_n, A\right) \xrightarrow{\bar{0}} 0 \xrightarrow{\bar{0}} \ldots$$

is a cochin complexes of fuzzy soft modules?

$$H^0(C) = \underleftarrow{\lim}(F_n, a), \quad H^1(C) = \underleftarrow{\lim}^{(1)}(F_n, a), \quad H^k(C) = 0, \quad k \geq 2 \tag{4.1}$$

are fuzzy soft cohomology modules of this complexes. Similarly, for the inverse system of fuzzy soft modules $\{(F'_n, A)\}$ and $\{(F''_n, A)\}$, we can constitute the following intuitionistic fuzzy cochin complex

$$C' = 0 \xrightarrow{\bar{0}} \left(\prod_{n\in N} F'_n, A\right) \xrightarrow{\bar{d}'} \left(\prod_{n\in N} F'_n, A\right) \xrightarrow{\bar{0}} 0 \xrightarrow{\bar{0}} \ldots$$

$$C'' = 0 \xrightarrow{\bar{0}} \left(\prod_{n\in N} F''_n, A\right) \xrightarrow{\bar{d}''} \left(\prod_{n\in N} F''_n, A\right) \xrightarrow{\bar{0}} 0 \xrightarrow{\bar{0}} \ldots .$$

It is clear that fuzzy cohomology modules of this complexes is the form in (4.1). From the condition of this theorem, the following sequence

$$0 \to C' \to C \to C'' \to 0$$

is short exact sequence of cochin complexes of fuzzy soft modules. But generally, the following sequence of cohomology modules of this sequence

$$\begin{aligned} 0 &\to H^0(C') \to H^0(C) \to H^0(C'') \xrightarrow{\bar{\partial}} H^1(C') \\ &\to H^1(C) \to H^1(C'') \to H^2(C') \to \Lambda \end{aligned}$$

is not exact, because $\bar{\partial}$ is usually not homomorphism of fuzzy soft modules. Since $H^0(C'') = \ker d''$ and $\lim_{n\to\infty} F''_{na}(x''_n) = 0$, grade function F'' of fuzzy soft module $\left(H^0(C''), \mu'', \lambda''\right)$ is equal to grade function $\bar{0}$. Thus $\bar{\partial}$ is homomorphism of fuzzy soft modules. Therefore, the sequence

$$\begin{aligned} 0 &\to H^0(C') \to H^0(C) \to H^0(C'') \xrightarrow{\bar{\partial}} H^1(C') \\ &\to H^1(C) \to H^1(C'') \to H^2(C') \to \Lambda \end{aligned}$$

is exact. By using the 5.1, we obtain the following exact sequence of intuitionistic fuzzy modules

$$\begin{aligned} 0 &\to \underleftarrow{\lim}(M'_n, \mu'_n, \lambda'_n) \to \underleftarrow{\lim}(F_n, a) \to \underleftarrow{\lim}(F''_n, a) \\ &\to \underleftarrow{\lim}^{(1)}(F'_n, a) \to \underleftarrow{\lim}^{(1)}(F_n, a) \to \underleftarrow{\lim}^{(1)}(F''_n, a) \to 0. \end{aligned}$$

References

1. U. Acar, F. Koyuncu, and B. Tanay, Soft sets and soft rings, Comput. Math. Appl. 59 (2010) 3458-3463.

2. H. Aktaş and N. Çagman, Soft sets and soft group, Inform. Sci. 177 (2007) 2726-2735.

3. K.T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Studies on Fuzziness and Soft Computing, 35, (1999) Physica-Verlag, Heidelberg.

4. S.A. Bayramov, Fuzzy and fuzzy soft structures in algebras, Lambert Academic Publishing, 2012.

5. S. Bayramov, C. Gunduz, M. Ibrahim Yazar, Inverse system of fuzzy soft modules. Annals of Fuzzy Mathematics and Informatics 4 (2012) 349-363.

6. S. Eilenberg and N. Steenrood, Foundations of algebraic topology, Princeton, 1952.

7. F. Feng, Y.B. Jun and X. Zhao, Soft semirings, Compute. Math. Appl. 56 (2008) 2621-2628.

8. M. Ghadiri and B.Davvaz, Direct system and direct limit of modules, Iran. J. Sci. Technol. Trans. A Sci 128(A2) (2004) 267-275.

9. C. Gunduz (Aras) and S. Bayramov, Fuzzy soft modules, Int. Math. Forum 6(11) (2011) 517-527.

10. C. Gunduz (Aras) and S. Bayramov, Inverse and direct system in category of fuzzy modules, Fuzzy Sets, Rough Sets and Multivalued Operations and Applications 2(1) (2011) 11-25.

11. C. Gunduz (Aras), S.A. Bayramov, Intuitionistic fuzzy soft modules, Computers and Mathematics with Application, 62 (2011) 2480-2486.

12. C. Gunduz (Aras) and S. Bayramov, Inverse and direct system in category of fuzzy modules, Fuzzy Sets, Rough Sets and Multivalued Operations and Applications 2(1) (2011) 11-25.

13. L. Jin-liang, Y. Rui-xia and Y. Bing-xue, Fuzzy soft sets and fuzzy soft groups, Chinese Control and Decision Conference (2008) 2626-2629.

14. S.R. Lopez-Permouth and D.S. Malik, on categories of fuzzy modules, Inform. Sci. 52 (1990) 211-220.

15. P.K. Maji, A.R. Roy and R. Bismas, an application of soft sets in a decision-making problem, Compute. Math. Appl. 44 (2002) 1077-1083.

16. P.K. Maji, A.R. Roy and R. Bismas, an application of soft sets in a decision-making problem, Compute. Math. Appl. 44 (2002) 1077-1083.

17. D. Molodtsov, Soft set theory-first results, Compute. Math. Appl. 37 (1999) 19-31.

18. A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512-517.

19. A.R. Roy and P.K. Maji, A fuzzy soft set theoretic approach to decision making problems, J. Compute. Appl. Math. 203 (2007) 412-418.

20. Qiu-Mei Sun, Zi-Liong Zhang and Jing Liu, Soft sets and soft modules, Lecture Notes in Compute. Sci. 5009 (2008) 403-409.