# Inventory Model with Time-Dependent Holding cost under Inflation when Seller Credits to Order Quantity 

Rajendra Sharma ${ }^{1}$, Jasvinder Kaur ${ }^{2}$<br>Department Of Mathematics, Graphic Era University Dehradun (UK), India<br>Email: rs.2103@yahoo.in, jasvinddn@gmail.com


#### Abstract

In this study an inventory model is developed under which the seller provides the retailer a permissible delay in payments, if the retailer orders a large quantity. In this paper we establish an inventory model for non deteriorating items and time dependent holding cost under inflation when seller offers permissible delay to the retailer, if the order quantity is greater than or equal to a predetermined quantity. We then obtain optimal solution for finding optimal order quantity, optimal replenishment time and optimal total relevant cost. Finally, numerical example is given to illustrate the theoretical results and made sensitive analysis of various parameters on the optimal solution.


Keywords: Inventory; inflation; order quantity; time dependent holding cost.


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## I. Introduction

In inventory management, one of the important problems is how to maintain and control the inventories of deteriorating items. Deterioration refers to spoilage of material with time. Food items, radioactive materials, chemicals, green vegetables and blood are few examples of such items. In classical inventory models it is considered that the demand rate is either constant or time dependent but independent of the stock status. In recent years, mathematical ideas have been used in different areas in real life problems, particularly inventory. One of the most important concerns of the managements is the decide when and how much to order or to manufacture so that the total cost associated with the inventory system should be minimum.

Selling Price plays an important role in inventory system. Burwell et al. [1] developed economic lot size model for dependent demand rate under quantity and freight discount. An inventory model with price and time dependent demand is developed by mandal et al. [2] and you [3]. Chang et al. [4] developed an EOQ model when supplier offers trade credit to the buyer if the order quantity is greate than or equal to a pre-determined quantity. Chang and Huang [5] presented a model optimal ordering policy under conditions of allowable shortages and permissible delay in payments.

In most of the models, holding cost is taken as constant. But in real life and from marketing point of view holding cost may not be constant. Various function describing holding cost were considered by several researchers like Muhdlemann, A.P. and valtis spamopoulos [6], weiss [7], and Goh [8]. Patra et al. [9] developed a generalized EOQ model for deteriorating items where deterioration rate and holding cost are expressed as linearly increasing functions of time and demand rate is a function of selling price. Alferes [10] developed inventory model with stock level dependent demand rate and variable holding cost. In model [10] the holding cost is an increasing step function of the time spent in storage. In model [10] two types of time dependent holding cost increase functions are considered ie. Retroactive increase and incremental increase.

In most of the business transaction, the supplier will offer the trade terms mixing cash discount and trade credit to the customer. The concept of inflation and time value of money was employed by wee and law [11] into a model when the demand is price dependent and storage is allowed. In modal [11] a production environment with a finite replenishment rate was considered. A note on EOQ model under cash discount and payment delay was discussed by huang [12]. Jaggi et al [13] developed a model optimal order policy for deteriorating items with inflations induced demand .In model [13] the demand rate is assumed to be a function of inflation. An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting was developed by Hou [14]. In this paper Hou [14] discussed an inventory model for deterioration items with stock dependent consumption rate and shortage under inflation and time- discounting over a finite planning horizon. Hou and Lin [15] developed a cash flow oriented EOQ model with deteriorating items under permissible delay in payments. In this study Hou and Lin applied the discounted cash flows approach for problem analysis chung [16] presented the discounted cash flows (DCF) approach for the analysis of optimal inventory policy in the presence of the trade credit. Jsggi and Aggarwal [17] extended chung [16] to develop an inventory model for obtaining the optimal order quantity of deteriorating items in the presence of trade credit using DCF approach. Jaggi et al [18] presented retailer's optimal ordering policy under two stage trade credit financing. In paper [18] an inventory model under two levels of trade credit policy by assuming that demand is a function of credit period offered by the retailer to the customer using discounted cash flow (DCF) approach is developed.

This study develops a deterministic inventory model for non deteriorating items and time dependent holding cost. Four different cases have been discussed. The effect of inflation is also discussed. In addition optimal solution is given for cycle time, total costs and order quantity.

The rest of the paper is organized as follows: in section 2, assumptions and notations are given. Sections 3, deals with development of mathematical model. In section 4, theoretical results are given followed by numerical example in section 5. In section 6, sensitivity analysis is given. Finally, conclusion and future research directives are given in the last section.

## II. Proposed Assumptions \& Notations

## 1. Assumptions

1.1 The demand is known and is constant.
1.2 The inflation rate is a constant
1.3 Replenishment is instantaneous
1.4 Shortages are not allowed
1.5 Holding cost is time dependent i.e. $h=h(t)=h t$
1.6 If $Q<Q_{d}$ then the payment for the items received must be made immediately
1.7 If $Q \geq Q_{d}$ then the delay in payments up to $M$ is permitted. During the trade credit period the account is not settled and generated sales revenue is deposited in an interest bearing account. At the end of credit period, the customer pays off all units ordered, and starts paying for the interest charges on the items in stocks.

## 2. NOTATIONS:

2.1 H : length of planning horizon and $\mathrm{H}=\mathrm{nT}$, where n is an integer for the number of replenishments to be made during period H and T is an interval of time between replenishments.
2.2 h : holding cost per unit time i.e. $\mathrm{h}(\mathrm{t})=\mathrm{ht}$
$2.3 I(t)$ : Inventory level at any time $t, 0 \leq t \leq T$.
$2.4 r$ : constant rate of inflation, $0<r<1$
2.5 D : the demand rate per unit time.
2.6 $\mathrm{P}(\mathrm{t})=\mathrm{pe}^{\mathrm{tt}}$ : the selling price per unit time, p is the initial selling price at $\mathrm{t}=0$.
2.7 $\mathrm{S}(\mathrm{t})=\mathrm{se}^{\mathrm{rt}}$ : the ordering cost per order at time t , s is the initial ordering cost at $\mathrm{t}=0$.
2.8 $C(t)=c e^{r t}$ : the purchasing cost at time $t, c$ is the initial purchase price at $t=0, c<p$.
$2.9 \mathrm{I}_{\mathrm{c}}$ : interest charged / \$ / year by the supplier per order.
2.10 ld : the interest earned / \$/year.
2.11 Q : order quantity.
$2.12 \mathrm{Q}_{\mathrm{d}}$ : minimum order quantity for which the delay in payments is allowed.
2.13 T : the replenishment time interval.
$2.14 \mathrm{~T}_{d}$ : the time interval that $Q_{d}$ units are depleted to zero due to demand only.
$2.15 \mathrm{Z}(\mathrm{t})$ : the total relevant cost over $(\mathrm{O}, \mathrm{H})$.
Note that the total relevant cost consists of (i) cost of placing order, (ii) cost of purchasing, (iii) cost of carrying inventory excluding interest charges, (iv) cost of interest charges for unsold items at $t=0$ or after credit period $M$ and (v) interest earned from sales revenue during the credit period.

## III. Mathematical Formulation and Equations

The level of inventory $I(t)$ gradually decreases mainly to meet demands only. Thus, the rate of charge of inventory with respect to time can be described by the following differential equations:
$\frac{\mathrm{di}(\mathrm{t})}{\mathrm{dt}}=-\mathrm{D}, \quad 0 \leq \mathrm{t} \leq \mathrm{T}$
The solution of $(1)$, with boundary condition $\mathrm{I}(\mathrm{t})=0$ is
$\mathrm{I}(\mathrm{t})=\mathrm{D}(\mathrm{T}-\mathrm{t}), \quad 0 \leq \mathrm{t} \leq \mathrm{T}$
And the order quantity is
$\mathrm{Q}=\mathrm{I}(0)=\mathrm{DT}$
From the above equation (3) we can find the time interval in which $Q_{d}$ units are depleted to zero due to demand only

$$
\begin{equation*}
\mathrm{T}_{\mathrm{d}}=\frac{\mathrm{Q}_{\mathrm{d}}}{\mathrm{D}} \tag{4}
\end{equation*}
$$

Hence it is easy to see that the inequality
$\mathrm{Q}<\mathrm{Q}_{\mathrm{d}}$ iff $\mathrm{T}<\mathrm{T}_{\mathrm{d}}$
Again the length of time intervals are all the same, hence we have
$\mathrm{I}(\mathrm{KT}+\mathrm{t})=\mathrm{D}(\mathrm{T}-\mathrm{t}), \quad 0 \leq \mathrm{k} \leq \mathrm{n}-1, \quad 0 \leq \mathrm{t} \leq \mathrm{T}$
For total relevant cost in $(0, H)$, we need following elements
(i) cost of placing order

$$
\begin{equation*}
S(0)+S(T)+S(2 T)+\ldots \ldots \ldots \ldots \ldots \ldots+S\{(n-1) T\}=S\left(\frac{e^{r H}-1}{e^{r T}-1}\right) \tag{6}
\end{equation*}
$$

(ii) cost of purchasing

$$
\begin{equation*}
Q[C(0)+C(T)+C(2 T)+\ldots \ldots \ldots \ldots \ldots+C\{(n-1) T\}]=C D T\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right) \tag{7}
\end{equation*}
$$

(iii) cost of carrying inventory
$\sum_{K=0}^{n-1} C(K, T) \int_{O}^{T} h(t) \cdot I(K T+t) d t=\operatorname{chD} \frac{\mathrm{T}^{3}}{6}\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right)$
(iv) Regarding interest charged and earned, we have the following four possible cases based on the values of $\mathrm{T}, \mathrm{M}$ and $\mathrm{T}_{\mathrm{d}}$

Case I, $0<T<\mathrm{T}_{\mathrm{d}}$

Since $T<T_{d}$ (i.e. $Q<Q_{d}$ ). In this case the interest charges for all unsold items start at the initial time, we obtain the interest payable in $(0, \mathrm{H})$ as
$I_{C} \sum_{k=0}^{n+1} C(K, T) \int_{0}^{T} I(K T+t) d t=\frac{\mathrm{T}^{2}}{2} I_{C} c D\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right)$
$\therefore Z_{1}(T)=$ total cost in $(0, \mathrm{H})$

$$
\begin{equation*}
Z_{1}(T)=\left[s+c D T+c h D \frac{\mathrm{~T}^{3}}{6}+I_{C} c D \frac{\mathrm{~T}^{2}}{2}\right]\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right) \tag{10}
\end{equation*}
$$

Case II, $\mathrm{T}_{\mathrm{d}} \leq T<M$
In this case there is a permissible delay $M$ which is longer than $T$. As a result there is no interest charged, but the interest earned in $(0, H)$ is
$I_{d} \sum_{k=0}^{n-1} P(K, T)\left[\int_{0}^{T} D t d t+D T(M-T)\right]=I_{d} P D\left(\mathrm{TM}-\frac{\mathrm{T}^{2}}{2}\right)\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right)$
$\therefore Z_{2}(T)=$ total relevant cost in $(0, H)$

$$
\begin{equation*}
Z_{2}(T)=\left[s+c D T+c h D \frac{\mathrm{~T}^{3}}{6}-I_{d} P D\left(\mathrm{TM}-\frac{\mathrm{T}^{2}}{2}\right)\right]\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right) \tag{12}
\end{equation*}
$$

Case III, $\quad \mathrm{T}_{\mathrm{d}} \leq M \leq T$
In this case, $T$ is longer than or equal to both $T_{d}$ and $M$ then delay in payment is permitted and the total relevant cost includes both the interest charged and the interest earned. The interest payable in $(0, H)$ is
$I_{c} \sum_{k=0}^{n-1} C(K, T) \int_{M}^{T} I(K T+t) d t=\frac{1}{2} c D(T-M)^{2}\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right)$
The interest earned in $(0, \mathrm{H})$ is

$$
\begin{equation*}
I_{d} \sum_{k=0}^{n-1} P(K, T) \int_{0}^{M} D t d t=I_{d} p D \frac{\mathrm{M}^{2}}{2}\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right) \tag{14}
\end{equation*}
$$

$\therefore Z_{3}(T)=$ total relevant cost in $(0, H)$

$$
\begin{equation*}
Z_{3}(T)=\left[s+c D T+c h D \frac{\mathrm{~T}^{3}}{6}+\frac{1}{2} \mathrm{I}_{\mathrm{c}} \mathrm{cD}(T-M)^{2}+I_{d} P D \frac{\mathrm{M}^{2}}{2}\right]\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right) \tag{15}
\end{equation*}
$$

Case IV, $\quad \mathrm{M} \leq \mathrm{T}_{\mathrm{d}} \leq T$
In this case, the replenishment time interval T is also greater than or equal to both $\mathrm{T}_{\mathrm{d}}$ and M . Hence case IV is similar to case III. Thus total relevant cost in $(0, \mathrm{H})$ is

$$
\begin{equation*}
Z_{4}(T)=\left[s+c D T+\operatorname{ch} D \frac{\mathrm{~T}^{3}}{6}+\frac{1}{2} \mathrm{I}_{\mathrm{c}} \mathrm{cD}(T-M)^{2}+I_{d} P D \frac{\mathrm{M}^{2}}{2}\right]\left(\frac{\mathrm{e}^{\mathrm{rH}}-1}{\mathrm{e}^{\mathrm{rT}}-1}\right) \tag{16}
\end{equation*}
$$

## IV. Theoretical Results

Since inflation rate $r$ is very small. Using truncated taylor's series expansion for the exponential terms, we get the modified (approximated) values of $Z_{i}(T), i=1,2,3 \& 4$ as follows

$$
\begin{align*}
& Z_{1}(T) \approx \frac{1}{r}\left[\frac{S}{T}+c D+c h D \frac{T^{2}}{6}+I_{c} c D \frac{T}{2}\right]\left(e^{r H}-1\right), \quad\left(\because \mathrm{e}^{\mathrm{rT}} \approx 1+\mathrm{Rt}\right.  \tag{17}\\
& Z_{2}(T) \approx \frac{1}{r}\left[\frac{S}{T}+c D+c h D \frac{T^{2}}{6}-I_{d} p D\left(\mathrm{M}-\frac{\mathrm{T}}{2}\right)\right]\left(e^{r H}-1\right)  \tag{18}\\
& \mathrm{Z}_{3}(\mathrm{~T}) \approx \frac{1}{r}\left[\frac{S}{T}+c D+\operatorname{ch} D \frac{T^{2}}{6}+\frac{1}{2} I_{c} c D\left(\mathrm{~T}+\frac{\mathrm{M}^{2}}{\mathrm{~T}}-2 \mathrm{M}\right)+\frac{\mathrm{I}_{\mathrm{d}} \mathrm{PD}{ }^{2}}{2 \mathrm{~T}}\right]\left(\mathrm{e}^{\mathrm{rH}}-1\right)  \tag{19}\\
& \mathrm{Z}_{4}(\mathrm{~T}) \approx \frac{1}{r}\left[\frac{S}{T}+c D+\operatorname{ch} D \frac{T^{2}}{6}+\frac{1}{2} I_{c} c D\left(\mathrm{~T}+\frac{\mathrm{M}^{2}}{\mathrm{~T}}-2 \mathrm{M}\right)+\frac{\mathrm{I}_{\mathrm{d}} \mathrm{PDM}}{2 \mathrm{~T}}\right]\left(\mathrm{e}^{\mathrm{rH}}-1\right) \tag{20}
\end{align*}
$$

The optimal solutions are obtained by taking the first and second order derivatives of $Z_{i}(T), i=1,2,3 \& 4$ with respect to $T$, we obtain

$$
\begin{align*}
& \frac{d Z_{1}(T)}{d T}=\frac{1}{r}\left[-\frac{S}{T^{2}}+\frac{c h D T}{3}+\frac{I_{c} c D}{2}\right]\left(e^{r H}-1\right)  \tag{21}\\
& \frac{d Z_{2}(T)}{d T}=\frac{1}{r}\left[-\frac{S}{T^{2}}+\frac{c h D T}{3}+\frac{I_{d} p D}{2}\right]\left(e^{r H}-1\right) \tag{22}
\end{align*}
$$

$$
\begin{align*}
\frac{d Z_{3}(T)}{d T} & =\frac{1}{\mathrm{r}}\left[-\frac{S}{T^{2}}+\frac{c h D T}{3}+\frac{I_{c} c D}{2}\left(1-\frac{\mathrm{M}^{2}}{\mathrm{~T}^{2}}\right)-\frac{\mathrm{I}_{\mathrm{d}} \mathrm{PDM}^{2}}{2 \mathrm{~T}^{2}}\right]\left(e^{r H}-1\right)  \tag{23}\\
\frac{d^{2} Z_{1}(T)}{d T^{2}} & =\frac{1}{r}\left[\frac{2 S}{T^{3}}+\frac{c h D}{3}\right]\left(e^{r H}-1\right)>0  \tag{24}\\
\frac{d^{2} Z_{2}(T)}{d T^{2}} & =\frac{1}{r}\left[\frac{2 S}{T^{3}}+\frac{c h D}{3}\right]\left(e^{r H}-1\right)>0  \tag{25}\\
\frac{d^{2} Z_{3}(T)}{d T^{2}} & =\frac{1}{\mathrm{r}}\left[\frac{2 S}{T^{3}}+\frac{c h D}{3}+\frac{\mathrm{I}_{\mathrm{C}} \mathrm{CDM}^{2}}{\mathrm{~T}^{3}}+\frac{\mathrm{I}_{\mathrm{d}} \mathrm{PDM}^{2}}{\mathrm{~T}^{3}}\right]\left(e^{r H}-1\right)>0 \tag{26}
\end{align*}
$$

For optimal (minimum) solution, put $\frac{d Z_{i}(T)}{d T}=0, i=1,2,3,4$, we obtain
From (21) $\quad \frac{d Z_{1}(T)}{d T}=0$

$$
\begin{equation*}
2 c h D T^{3}+3 I_{c} c D T^{2}-6 s=0 \tag{27}
\end{equation*}
$$

From (22) $\frac{d Z_{2}(T)}{d T}=0$

$$
\begin{equation*}
2 c h D T^{3}+3 I_{d} p D T^{2}-6 s=0 \tag{28}
\end{equation*}
$$

From (23) $\frac{d Z_{2}(T)}{d T}=0$
$2 c h D T^{3}+3 I_{c} c D T^{2}-\left(6 \mathrm{~S}+3 \mathrm{CI}_{c} D M^{2}+3 \mathrm{I}_{\mathrm{d}} \mathrm{PDM}^{2}\right)=0$

## V. Examples and Tables

## 1.NUMERICAL EXAMPLES:

Case I, $0<T<\mathrm{T}_{\mathrm{d}}$
Example 1. Let $\mathrm{s}=\$ 150 /$ order, $\mathrm{c}=\$ 25 /$ units, $\mathrm{h}=\$ 2 / \mathrm{unit} /$ year, $I_{c}=0.10 / \$ /$ year, $\mathrm{D}=500$ unit/year, $\mathrm{p}=\$ 30$ per unit, $\mathrm{r}=0.05$ per unit, $I_{d}=0.05 / \$$ year, $h=1$ year, Substituting these values in (27) and (3) and (10) we obtain
$1000 T^{3}+75 T^{2}-18=0$
Solving (30) we get
$\mathrm{T}=T_{1}^{*}=0.239308525$ year
$\therefore$ optimal order quantity
$\mathrm{Q}=Q_{1}^{*}\left(T_{1}^{*}\right)=D T_{1}^{*}=500 \times 0.239308525=119.6542625$ units
If $Q_{d}=120$ units, then $T_{d}=\frac{Q_{d}}{D}=\frac{120}{500}=0.24$ year
Which verifies that, when $Q_{1}^{*}<Q_{d}$ then $T_{1}^{*}<T_{d}$, which proves case 1
Also $Z_{1}^{*}\left(T_{1}^{*}\right)=\$ 13858.56996$
Case II, $\mathrm{T}_{\mathrm{d}} \leq T<M$
Example 2. let $D=100$ units, $\theta=0.02, c=\$ 30 /$ units, $p=\$ 40$ per unit, $h=\$ 2 /$ unit/year, $l_{d}=0.05 / \$ /$ year, $H=1$ year, $s=\$$ $50 /$ order, $I_{c}=0.08 / \$ /$ year, $r=0.05$ per unit, $M=110$ days, Substituting these values in (28) and (3) and (12) we get
$2000 T^{3}+100 T^{2}-50=0$
Solving (31) we get
$\mathrm{T}=T_{2}^{*}=0.276649$ year
$\therefore$ optimal order quantity
$\mathrm{Q}=Q_{2}^{*}\left(T_{2}^{*}\right)=D T_{1}^{*}=27.6649$ unit
If $Q_{d}=25$ units, then $T_{d}=\frac{Q_{d}}{D}=\frac{25}{100}=0.25$ year
which proves case 2
Also $Z_{2}^{*}\left(T_{2}^{*}\right)=\$ 1159.8975$
Case III, $\mathrm{T}_{\mathrm{d}} \leq M \leq T$

Example 3. let $D=100$ units, $c=\$ 10 / u n i t s, p=\$ 20$ per unit, $h=\$ 2 /$ unit/year, $I_{d}=0.05 / \$ /$ year, $H=1$ year, $s=\$ 100 / o r d e r$, $I_{c}=0.10 / \$ /$ year, M=90days, Substituting these values in (29) and (3) and (15) we get the values
$4000 \mathrm{~T}^{3}+300 \mathrm{~T}^{2}-627.3596538=0$
Solving (32) we get
$\mathrm{T}=T_{3}^{*}=0.518004$ year
And corresponding optimal order quantity
$\mathrm{Q}=Q_{3}^{*}\left(T_{3}^{*}\right)=51.8004$ units
If $Q_{d}=50$ units, then $T_{d}=\frac{Q_{d}}{D}=\frac{50}{500}=0.5$ year
which proves case 3
Also $Z_{3}^{*}\left(T_{3}^{*}\right)=\$ 1328.854169$

## 2. SENSITIVITY ANALYSIS:

Sensitivity analysis has been performed by considering various values of the parameters like unit ordering cost (s), unit purchasing cost (c), holding cost (h) and credit period (M), the corresponding values obtained with respect to the changes in above parameters are replenishment cycle time ( $T$ ), economic order quantity $Q$ and total relevant $\operatorname{cost} Z(T)$ by taking into consideration the following different cases.
i. When $0<T<\mathrm{T}_{\mathrm{d}} \quad$ [tables 1 (a), 1 (b), 1 (c)]
ii. When $\mathrm{T}_{\mathrm{d}} \leq T<M$ [tables 2(a),2(b),2(c)]
iii. When $\mathrm{T}_{\mathrm{d}} \leq M \leq T$ [tables 3(a),3(b),3(c),3(d)]

Table 1. (case 1: When $0<T<\mathbf{T}_{\mathbf{d}}$ )
Table 1(a): Sensitivity analysis on ' $s$ '

| S | $T_{1}^{*}$ | $Q_{1}^{*}\left(T_{1}^{*}\right)$ | $Z_{1}^{*}\left(T_{1}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 160 | .244963 | 122.4815 | 13900.91739 |
| 170 | .250388 | 125.194 | 13942.31781 |
| 180 | .255606 | 127.803 | 13982.84756 |
| 190 | .260635 | 130.3175 | 14022.57294 |
| 200 | .265492 | 132.746 | 14061.55197 |

Table 1(b): Sensitivity analysis on ' $c$ '

| C | $T_{1}^{*}$ | $Q_{1}^{*}\left(T_{1}^{*}\right)$ | $Z_{1}^{*}\left(T_{1}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 26 | .235932 | 117.966 | 14387.02082 |
| 27 | .232725 | 116.3625 | 14915.12194 |
| 28 | .229673 | 114.8365 | 15442.89397 |
| 29 | .226764 | 113.382 | 15970.35566 |
| 30 | .223986 | 111.993 | 16497.52411 |

Table 1(c): Sensitivity analysis on ' $h$ '

| H | $T_{1}^{*}$ | $Q_{1}^{*}\left(T_{1}^{*}\right)$ | $Z_{1}^{*}\left(T_{1}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 2.1 | .236101 | 118.0505 | 13870.63944 |
| 2.2 | .233066 | 116.533 | 13882.39414 |
| 2.3 | .230186 | 115.093 | 13893.85441 |
| 2.4 | .227449 | 113.7425 | 13905.03856 |


| 2.5 | .224843 | 112.4215 | 13915.9631 |
| :--- | :--- | :--- | :--- |

Table 2. (case 2: When $\mathrm{T}_{\mathrm{d}} \leq \boldsymbol{T}<\boldsymbol{M}$ ):
Table 2(a): Sensitivity analysis on ' $s$ '

| S | $T_{2}^{*}$ | $Q_{2}^{*}\left(T_{2}^{*}\right)$ | $Z_{2}^{*}\left(T_{2}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 50 | 0.276649 | 27.6649 | 3317.874602 |
| 55 | 0.286060 | 28.6060 | 3336.09602 |
| 60 | 0.294919 | 29.4919 | 3353.74254 |
| 65 | 0.303299 | 30.3299 | 3370.884971 |
| 70 | 0.311260 | 31.1260 | 3387.569617 |
| 75 | 0.318852 | 31.8852 | 3403.842535 |

Table 2(b): Sensitivity analysis on ' $c$ '

| C | $T_{2}^{*}$ | $Q_{2}^{*}\left(T_{2}^{*}\right)$ | $Z_{2}^{*}\left(T_{2}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 30 | 0.276649 | 27.6649 | 3317.874601 |
| 31 | 0.273960 | 27.3960 | 3423.007294 |
| 32 | 0.271388 | 27.1388 | 3528.090614 |
| 33 | 0.268888 | 26.8888 | 3633.126891 |
| 34 | 0.266515 | 26.6515 | 3738.11829 |
| 35 | 0.264192 | 26.4192 | 3843.066807 |

Table 2(c): Sensitivity analysis on ' $h$ '

| H | $T_{2}^{*}$ | $Q_{2}^{*}\left(T_{2}^{*}\right)$ | $Z_{2}^{*}\left(T_{2}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 2.0 | 0.276649 | 27.6649 | 3317.874602 |
| 2.1 | 0.272655 | 27.2655 | 3321.741688 |
| 2.2 | 0.268888 | 26.8888 | 3325.500375 |
| 2.3 | 0.265326 | 26.5326 | 3329.157921 |
| 2.4 | 0.261949 | 26.1949 | 3332.721156 |
| 2.5 | 0.258742 | 25.8742 | 3336.195986 |

Table 3. (case 3: When $\mathrm{T}_{\mathrm{d}} \leq M \leq T$ )
Table 3(a): Sensitivity analysis on ' $s$ '

| S | $T_{3}^{*}$ | $Q_{3}^{*}\left(T_{3}^{*}\right)$ | $Z_{3}^{*}\left(T_{3}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 100 | 0.518004 | 51.8004 | 1328.854169 |
| 110 | 0.534491 | 53.4491 | 1347.91904 |
| 120 | 0.550058 | 55.0058 | 1366.797242 |
| 130 | 0.564823 | 56.4823 | 1385.191432 |
| 140 | 0.578388 | 57.8388 | 1403.122159 |
| 150 | 0.592316 | 59.2316 | 1420.632087 |

Table 3(b): Sensitivity analysis on ' $c$ '

| C | $T_{3}^{*}$ | $Q_{3}^{*}\left(T_{3}^{*}\right)$ | $Z_{3}^{*}\left(T_{3}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 10 | 0.518004 | 51.8004 | 1328.404704 |
| 11 | 0.501594 | 50.1594 | 1440.521103 |
| 12 | 0.487090 | 48.7090 | 1551.395947 |
| 13 | 0.474141 | 47.4141 | 1663.066349 |
| 14 | 0.462479 | 46.2479 | 1773.64007 |
| 15 | 0.451900 | 45.1900 | 1883.821707 |

Table 3(c): Sensitivity analysis on ' $h$ '

| H | $T_{3}^{*}$ | $Q_{3}^{*}\left(T_{3}^{*}\right)$ | $Z_{3}^{*}\left(T_{3}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 2.0 | 0.518004 | 51.8004 | 1328.404705 |
| 2.1 | 0.510367 | 51.0367 | 1328.922617 |
| 2.2 | 0.503170 | 50.3170 | 1337.311092 |
| 2.3 | 0.496369 | 49.6369 | 1341.579272 |
| 2.4 | 0.489929 | 48.9929 | 1345.735161 |
| 3.5 | 0.483817 | 48.3817 | 1349.785971 |

Table 3(d): Sensitivity analysis on 'M'

| M | $T_{3}^{*}$ | $Q_{3}^{*}\left(T_{3}^{*}\right)$ | $Z_{3}^{*}\left(T_{3}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 90 | 0.518004 | 51.8004 | 1328.854169 |
| 100 | 0.520417 | 52.0417 | 1328.411936 |
| 110 | 0.523059 | 52.3059 | 1328.739832 |
| 120 | 0.525930 | 52.5930 | 1329.266953 |
| 130 | 0.529004 | 52.9004 | 1330.105132 |
| 140 | 0.532293 | 53.2293 | 1331.212023 |
| 150 | 0.535783 | 53.5783 | 1332.582313 |

## VI. Conclusion

Analysis of the Results shown in tables 1 to 3 :

1. It is observed from the computational results shown in table 1 (a) that for higher values of ordering cost ' $s$ ', the corresponding values of replenishment cycle time $T_{1}^{*}$, order quantity $Q_{1}^{*}\left(T_{1}^{*}\right)$ and total relevant cost $Z_{1}^{*}\left(T_{1}^{*}\right)$ also go higher.
2. The computational results shown in table 1(b) indicate that with the increasing of unit purchasing cost ' $c$ ', the corresponding values of replenishment cycle time ( $T_{1}^{*}$ ) order quantity $Q_{1}^{*}\left(T_{1}^{*}\right)$ are decreasing while the total relevant cost $Z_{1}^{*}\left(T_{1}^{*}\right)$ is increasing with the increasing values of unit purchasing cost ' c '.
3. The computational results shown in table 1(c) indicate that the higher values of holding cost ' $h$ ' imply lower values of replenishment cycle time $T_{1}^{*}$ and order quantity $Q_{1}^{*}\left(T_{1}^{*}\right)$ but higher values of total relevant cost $Z_{1}^{*}\left(T_{1}^{*}\right)$, the tendency of these results is the same as those shown in table 1(b).

4 The computational results obtained in table 2(a) indicate that ordering cost ' $s$ ' is directly proportional to the replenishment cycle time ( $T_{2}^{*}$ ), economic order quantity $Q_{2}^{*}\left(T_{2}^{*}\right)$ and total relevant cost $Z_{2}^{*}\left(T_{2}^{*}\right)$ i.e. an increase in ' $s$ ' implies the proportional increase in $T_{2}^{*}, Q_{2}^{*}\left(T_{2}^{*}\right)$ and $Z_{2}^{*}\left(T_{2}^{*}\right)$.

5 The computational results obtained in table 2(b) indicate that purchasing cost ' $c$ ' is inversely proportional to replenishment cycle time $\left(T_{2}^{*}\right)$ and economic order quantity $Q_{2}^{*}\left(T_{2}^{*}\right)$ and directly proportional to the total relevant $\operatorname{cost} Z_{2}^{*}\left(T_{2}^{*}\right)$ i.e. an increase in ' $c$ ' shows proportional decrease in $T_{2}^{*}$ and $Q_{2}^{*}\left(T_{2}^{*}\right)$ while as increase in $Z_{2}^{*}\left(T_{2}^{*}\right)$
6 The computational results obtained in table 2(c) indicate that higher values of holding cost ' h ' are associated with the lower values of the replenishment cycle time $T_{2}^{*}$ and economic order quantity $Q_{2}^{*}\left(T_{2}^{*}\right)$ and higher values of total relevant cost $Z_{2}^{*}\left(T_{2}^{*}\right)$.

7 The computational results obtained in table 3(a) indicate that unit ordering cost 's' is directly proportional to all the three values i.e. replenishment cycle time $T_{3}^{*}$ and economic order quantity $Q_{3}^{*}\left(T_{3}^{*}\right)$ and total relevant $\operatorname{cost} Z_{3}^{*}\left(T_{3}^{*}\right)$.
8 The computational results obtained 1 n table 3 (b) show that the value of replenishment cycle time $T_{3}^{*}$ and economic order quantity $Q_{3}^{*}\left(T_{3}^{*}\right)$ decrease with the increasing of unit purchasing cost ' $c$ ' while total relevant cost $Z_{3}^{*}\left(T_{3}^{*}\right)$ increase with the increasing values of unit purchasing cost ' $c$ '.
9 The computational results obtained in table 3(c) indicate that higher values of holding cost ' $h$ ' imply the lower values of the replenishment cycle time $T_{3}^{*}$ and economic order quantity $Q_{3}^{*}\left(T_{3}^{*}\right)$ and higher values of total relevant $\operatorname{cost} Z_{3}^{*}\left(T_{3}^{*}\right)$.

10 The computational results obtained In table 3(d) indicate that higher values of credit period ' $M$ ' are associated with higher values of replenishment cycle time $T_{3}^{*}$, economic order quantity $Q_{3}^{*}\left(T_{3}^{*}\right)$ and total relevant cost $Z_{3}^{*}\left(T_{3}^{*}\right)$.

## VII. Proposed model

The proposed model can be extended in many more ways such as, we can consider the demand rate in quadratic time dependent form. We can also consider the demand as a function of quantity or selling price. Further the shortages may also be taken in to account to generalize the model thus this paper can be useful developed as a wholesaler and retailer system model.


Case 1. $0<T<T_{d}$

## Inventory level



Case 2. $\mathrm{T}_{\mathrm{d}} \leq \mathrm{T}<\mathrm{M}$

## Inventory level


$0 T_{d} M T T_{d(1)} M_{(1)} 2 T \quad T_{d(n-2)} M_{(n-2)}(n-1) T \quad n T=H$
Case 3. $T_{d} \leq M \leq T$


Case 4. $M \leq T_{d} \leq T$
Fig. 1 Four possible inventory systems.

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