ISSN 2347-192

Short Communication

A note on "The Ideal Generated by Codense Sets and the Banach Localization Property"

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Abstract

In this note we show by producing counter examples that some results which appeared in the articles by Jankovic´ and Hamlett [3] are incorrect.

Mathematics Subject Classification 2000: 54A05, 54A10.

Keywords: Codense sets and boundary sets.

Council for Innovative Research

Peer Review Research Publishing System

Journal: Journal of Advances in Mathematics

Vol 8, No 2

editor@cirjam.org

www.jam.cirworld.com, www.cirworld.com

1. Introduction

The notion of ideal topological spaces was studied by Kuratowski [5] and Vaidyanathaswamy [8]. Applications to various fields were further investigated by Jankovic´ and Hamlett [4]; Dontchev et al. [2]; Mukherjee et al. [6]; Arenas et al.[1]; Nasef and Mahmoud [7], etc.

In this note, we show that there are Mathematical errors of the claimed results of the paper of Jankovic´ and Hamlett [3].

2. Preliminaries and Some Examples

Throughout the present note, (X, τ) will denote a topological space with no separation properties assumed. For a subset A of a topological space (X, τ), $Cl(A)$, $Int(A)$ and $Bd(A)$ will denote the closure, the interior and the boundaries of A in (X, τ), respectively.

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies;

(i) $A \in I$ and $B \subseteq A$ implies $B \in I$, (hereditry),

(ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$, (finite additivity).

It is interesting to note that if I is a proper ideal (i.e. $X \notin I$), then the collection of the complements of the members of I form a filter on X. This is why sometimes ideals are called dual filters.

The following collections form important ideals [3, 4, 8] on a topological space (X, τ):

1. $\{\emptyset\}$ or $I_{\{\emptyset\}}$: the trivial ideal.
2. $\mathcal{P}(X)$: the improper ideal.
3. $\mathcal{F}$: (I_f) the ideal of all finite sets.
4. $\mathcal{C}$: (I_c) the ideal of all countable subsets of X.
5. $\mathcal{CD}$: (I_{cd}) the ideal of all closed and discrete sets in (X, τ).

   i.e. $I_{cd}= \{A \subseteq X: A^d= \emptyset\}$ where A^d is the derived set of A.
6. $\mathcal{N}$: (I_n) the ideal of all nowhere dense sets.

   i.e. $I_n= \{A \subseteq X: Int(Cl(A))= \emptyset\}$.
7. $\mathcal{M}$: (I_m) the ideal of all first category (= meager) sets.

   ($A \in I_m$ iff A is a countable union of nowhere dense sets)

   i.e. $I_m= \{A \subseteq X: A= \cup\{B_i , i \in \mathbb{N}; Int(Cl(B_i))= \emptyset\}$.
8. $\mathcal{S}$: (I_s) the ideal of all scattered sets (here X must be T_0) or τ_D-space.
9. $\mathcal{L}$: (I_{L0}) the ideal of all Lebesgue null sets (here X stands for the real line).
10. $\mathcal{PB}$: the ideal of all parabounded subsets of (X, τ).
11. $\mathcal{CDF}$: the ideal of all closed df- sets, where a subset A of a topological space (X, τ) is called discretely finite (= df- set) if for each $x \in A$, there exists an open set U containing x such that $U \cap A$ is finite.
12. I_K: the ideal of relatively compact sets in (X, τ).

    (A is relatively compact iff $Cl(A)$ is compact).
13. <A>: $I(A)$ the principal ideal generated by any subset A of (X, τ).

    i. e. $I(A)= \mathcal{P}(A)= \{B \subseteq X: B \subseteq A\}$.

Observation 2.1.

In [3] D. Jankovic´ and T.R. Hamlett showed that the collection $\mathcal{C}= \{A\subseteq X: \mathrm{Int}(A)= \emptyset \}$ of all codense subsets in a topological space (X, τ) formed an ideal on X and denoted by $I(\mathcal{C})$.

The following example shows that, in general the family of all codense subsets in a topological space (X, τ) need not be an ideal on X.

Example 2.1. Let $X= \{a, b, c\}$ with a topology $\tau =\{X, \emptyset, \{b,c\}\}$. Then $I(\mathcal{C})= \{A\subseteq X: \mathrm{Int}(A)= \emptyset \}= \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$ which is not an ideal, since $\{b\}, \{c\}\in I(\mathcal{C})$ while $\{b, c\}$ does not belongs to $I(\mathcal{C})$.

Observation 2.2.

Jankovic´ and Hamlett [3] showed that the collection $\mathcal{B}= \{\mathrm{Bd}(A): A\subseteq X\}$ of all boundaries of subsets of the space (X, τ) form an ideal and denoted by $I(\mathcal{B})$.

The following example shows that, the family of all boundaries of subsets of the space (X, τ) is not an ideal on X in general.

Example 2.2. Let X be as in Example 2.1. with a topology $\tau= \{X, \emptyset, \{a\}, \{a, b\}\}$. Then one can deduce that $I(\mathcal{B})= \{ \emptyset, \{c\}, \{b, c\}\}$ which is not an ideal, since $\{b\}\subseteq \{b, c\}\in I(\mathcal{B})$ while $\{b\}$ does not belongs to $I(\mathcal{B})$.

Observation 2.3.

Jankovic´ and Hamlett [4] claimed that $I\cup\{X\}$ is a topology on X, where I is an ideal on X.

The following example shows that, $I\cup\{X\}$ is not a topology on X in general.

Example 2.3. Let $X= \mathbb{R}$ be the set of all real numbers with an ideal $I= \{A \subseteq \mathbb{N}$: A is finite subset of the natural numbers $\mathbb{N}\}$.

It is clear that $\{n\}\in I$ for each $n\in \mathbb{N}$, but $\cup\{\{n\}: n\in \mathbb{N}\}= \mathbb{N} \notin I\cup\{X\}$. Hence, $I\cup\{X\}$ is not a topology on X.

This note, $I \cup\{X\}$ is a topology on X, is hold where X is finite set.

3. Conclusions

The aim of this note is to point out that one assertion (The collections of all codense sets and the collections of all boundary sets in any topological space) in a previous paper by Jankovic´ and Hamlett [D. Jankovic´ and T. R. Hamlett, The ideal generated by codense sets and the Banach localization property, Coll. Math. Soc. Janos Bolyai, 55, Topology, Pecs, Hungary, (1989), 349- 358] not represent ideal by a counterexamples. And so we show that, the note $I\cup\{X\}$ is not a topology on X in general which was introduced by Jankovic´ and Hamlett [D. Jankovic´ and T. R. Hamlett, New Topologies from old via ideals. Amer. Math. Monthly 97(1990), 295- 310].

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