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Kenmotsu pseudo-Riemannian Manifolds

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ABSTRACT

In this paper, we study some properties of Kenmotsu manifolds with pseudo-Riemannian metrics. We prove that this kind of manifolds satisfying certain conditions such as $R(X,Y)\cdot R=0$, $R(X,Y)\cdot S=0$ are Einstein manifolds. Also some sufficient conditions for a Kenmotsu pseudo-Riemannian metric manifold to be Einstein are obtained.

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1 INTRODUCTION

Almost Kenmotsu manifolds were first stated and studied by K. Kenmotsu [6] in 1972, then G. Dileo and A. Pastore [5] studied many properties of almost Kenmotsu manifolds.

Obversely a normal almost Kenmotsu manifold is a Kenmotsu manifold. Recently [3], [4] investigated Kenmotsu manifolds and got many properties for Kenmotsu manifolds. However, the metrics they used above are all Riemannian.

We have already noticed that G. Galvaruso and D. Perrone [1] studied contact manifolds with pseudo-Riemannian metrics in 2010, J. Liu and X. Liu [10] investigated almost Kenmotsu manifolds with pseudo-Riemannian metrics. In this paper, we focus on Kenmotsu manifolds with pseudo-Riemannian metrics. We study Kenmotsu manifolds with pseudo-Riemannian metrics, we prove that this kind of manifolds satisfying certain conditions such as $R(X,Y)\cdot R=0$, $R(X,Y)\cdot S=0$ are Einstein manifolds. Also some sufficient conditions for a Kenmotsu pseudo-Riemannian metric manifold to be Einstein are obtained.

This paper is organized in the following way. In section 2, some preliminaries are given, in section 3, we studied Kenmotsu pseudo-Riemannian metric manifolds satisfying certain conditions. Some sufficient conditions for a Kenmotsu pseudo-Riemannian metric manifold to be Einstein is obtained in section 4.

2 PRELIMINARIES

Let M^n be an n -dimensional (where $n=2m+1$) smooth manifold, an almost contact structure on M^n is a triplet (φ,ξ,η) , where φ is a (1,1)-tensor, ξ a global vector field and η a 1-form, such that

$$\text{(i)}\ \varphi(\xi)=0, \quad \eta\circ\varphi=0, \quad \text{(ii)}\ \eta(\xi)=1, \quad \varphi^2=-I+\eta\otimes\xi \tag{2.1}$$

and φ has rank $n-1$.

If a pseudo-Riemannian metric g on M^n satisfies

$$g(\varphi X,\varphi Y)=g(X,Y)-\varepsilon\eta(X)\eta(Y), \tag{2.2}$$

where $\varepsilon=\pm1$, then g is said to be compatible with the almost contact structure (φ,ξ,η) , (M^n,φ,ξ,η,g) is said to be an almost contact pseudo-Riemannian metric manifold.

It is easy to know by (2.1) and (2.2) that $\eta(X)=\varepsilon g(\xi,X)$ for any compatible metric. In particular, $g(\xi,\xi)=\varepsilon$ and so , the characteristic vector field ξ is either space-like or time-like, but cannot be light-like. Remark that $g(\varphi X,Y)=-g(X,\varphi Y)$.

Definition 2.1 Let (M^n,φ,ξ,η,g) be an almost contact pseudo-Riemannian metric manifold, if it satisfies

$$d\eta=0, \quad d\phi=2\eta\wedge\phi,$$

where we put $\phi(X,Y)=g(X,\varphi Y)$, then (M^n,φ,ξ,η,g) is called an almost Kenmotsu pseudo-Riemannian metric manifold.

If an almost Kenmotsu pseudo-Riemannian metric manifold (M^n,φ,ξ,η,g) is normal, it is said to be Kenmotsu pseudo-Riemannian metric manifold. J. Liu and X. Liu [10] proved that an almost Kenmotsu pseudo-Riemannian metric manifold (M^n,φ,ξ,η,g) is Kenmotsu if and only if

$$(\nabla_X\varphi)Y=-\eta(X)\varphi Y-\varepsilon g(X,\varphi Y)\xi. \tag{2.3}$$

Taking $Y=\xi$ in (2.3), we get $\varphi\nabla_X\xi=\varphi X$, applying φ on both sides of this equation, we can get

$$\nabla_X\xi=X-\eta(X)\xi. \tag{2.4}$$

Let (M^n,φ,ξ,η,g) be a Kenmotsu pseudo-Riemannian metric manifold, using the conditions above, we can calculate some basic formulas about this kind of manifolds.

We denote by R its curvature tensor of M , $R(X,Y)=[\nabla_X,\nabla_Y]-\nabla_{[X,Y]}$, S is the Ricci tensor, $S(X,Y)=g(QX,Y)$, where Q is the Ricci operator.

$$R(X,Y)\xi=\nabla_X\nabla_Y\xi-\nabla_Y\nabla_X\xi-\nabla_{[X,Y]}\xi$$

$$=\nabla_X(Y-\eta(Y)\xi)-\nabla_Y(X-\eta(X)\xi)+\eta([X,Y])\xi-[X,Y]$$

$$=\eta(X)Y-\eta(Y)X\ . \qquad (2.5)$$

$$S(X,Y)=\sum_{i=1}^{n}\varepsilon_i g(R(e_i,X)\xi,e_i)$$

$$=\sum_{i=1}^{n}\varepsilon_i g(\eta(e_i)X-\eta(X)e_i,e_i)$$

$$=\sum_{i=1}^{n}\varepsilon\varepsilon_i g(e_i,\xi)g(X,e_i)-\sum_{i=1}^{n}\varepsilon\varepsilon_i g(X,\xi)g(e_i,e_i)$$

$$=\varepsilon g(X,\xi)-n\varepsilon g(X,\xi)$$

$$=-(n-1)\eta(X), \qquad (2.6)$$

where $\{e_1,\cdots,e_n\}$ is a local pseudo-orthonormal basis, and $\varepsilon_i=g(e_i,e_i)=\pm1$ for all indices $i=1,2...n$.

$$(\nabla_X\eta)Y=\nabla_X\eta(Y)-\eta(\nabla_X Y)$$

$$=\varepsilon g(X,Y)-\eta(X)\eta(Y)\ . \qquad (2.7)$$

Using the symmetries of the curvature and (2.5), we can easily get

$$R(X,\xi)Y=\varepsilon g(X,Y)\xi-\eta(Y)X\ . \qquad (2.8)$$

$$\eta(R(X,Y)V)=\varepsilon\eta(Y)g(X,V)-\varepsilon\eta(X)g(Y,V)\ . \qquad (2.9)$$

Taking $Y=\xi$ in (2.8), we get

$$R(X,\xi)\xi=\eta(X)\xi-X\ . \qquad (2.10)$$

Using (2.10), we get

$$K(X,\xi)=-\frac{R(X,\xi,X,\xi)}{g(X,X)g(\xi,\xi)-g(X,\xi)g(X,\xi)}$$

$$=\frac{g(R(X,\xi)\xi,X)}{\varepsilon g(X,X)-\eta^2(X)}$$

$$=\frac{g(\eta(X)\xi-X,X)}{\varepsilon g(X,X)-\eta^2(X)}$$

$$=\frac{\varepsilon\eta^2(X)-g(X,X)}{\varepsilon g(X,X)-\eta^2(X)}$$

$$=-\varepsilon\ ,$$

where K denotes sectional curvature.

Definition 2.2 Let (M^n,φ,ξ,η,g) be a Kenmotsu pseudo-Riemannian metric manifold, if its Ricci tensor S is of the form

$$S(X,Y)=ag(X,Y)+\varepsilon b\eta(X)\eta(Y), \qquad (2.11)$$

for any vector fields X and Y, where a and b are functions on M^n, then we call M^n an η-Einstein manifold. If $b=0$, then η-Einstein manifolds become Einstein manifolds.

Theorem 2.1 Let (M^n,φ,ξ,η,g) be a Kenmotsu pseudo-Riemannian metric manifold, if (M^n,φ,ξ,η,g) is an η-Einstein manifold, that is to say $S(X,Y)=ag(X,Y)+\varepsilon b\eta(X)\eta(Y)$, then we have

$$a+b=-\varepsilon(n-1)\ . \qquad (2.12)$$

Proof. Taking $Y=\xi$ in (2.11) , using (2.6) , then

$$-(n-1)\eta(X)=\varepsilon a\eta(X)+\varepsilon b\eta(X),$$

we get

$$a+b=-\varepsilon(n-1).$$

□

Again taking $X=Y=e_i$ in (2.11) and then taking summation over the index i , we get

$$r=na+b, \tag{2.13}$$

where r denotes the scalar curvature.

Using (2.12) and (2.13), one has

$$a=\frac{r}{n-1}+\varepsilon, \quad b=\frac{r}{1-n}-n\varepsilon.$$

Theorem 2.2 For a Kenmotsu pseudo-Riemannian metric manifold (M^n,φ,ξ,η,g) , we have

$$div\xi=n-1.$$

Proof. Consider a local pseudo-orthonormal basis $\{e_1,\cdots,e_n\}$ on M^n , using (2.4), we have

$$\begin{aligned} div\xi &= tr\nabla\xi \\ &= \sum_{i=1}^{n}\varepsilon_i g(\nabla_{e_i}\xi,e_i) \\ &= \sum_{i=1}^{n}\varepsilon_i g(e_i-\eta(e_i)\xi,e_i) \\ &= \sum_{i=1}^{n}\varepsilon_i g(e_i,e_i)-\sum_{i=1}^{n}g(\xi,\xi) \\ &= n-1. \end{aligned}$$

□

3 KENMOTSU PSEUDO-RIEMANNIAN METRIC MANIFOLDS SATISFYING CERTAIN CONDITIONS

In this section, we investigate Kenmotsu pseudo-Riemannian metric manifolds (M^n,φ,ξ,η,g) satisfying the following conditions:

(i) M is Ricci semi-symmetric, that is to say, $R(X,Y)\cdot S=0$,

(ii) M is semi-symmetric, that is to say, $R(X,Y)\cdot R=0$,

(iii) M is locally symmetric, that is to say $\nabla R=0$,

(iv) $R(X,\xi)\cdot P=0$, where P is projective curvature.

Let's consider a Kenmotsu pseudo-Riemannian metric manifold M^n ($n=2m+1$) which satisfies the condition

$$R(X,Y)\cdot S=0. \tag{3.1}$$

From (3.1), we have

$$R(X,Y)S(U,V)-S(R(X,Y)U,V)-S(U,R(X,Y)V)=0. \tag{3.2}$$

It is easy to see $R(X,Y)S(U,V)=0$, so (3.2) becomes

$$S(R(X,Y)U,V)+S(U,R(X,Y)V)=0. \tag{3.3}$$

Putting $U=\xi$ in (3.3) and using (2.5), (2.6) and (2.9), we get

$$\eta(X)S(Y,V)-\eta(Y)S(X,V)-(n-1)\left[\eta(Y)g(X,V)-\eta(X)g(Y,V)\right]=0. \tag{3.4}$$

Then taking $Y=\xi$ in (3.4), considering (2.1) and (2.6), we have

$$S(X,V)=-\varepsilon(n-1)g(X,V). \tag{3.5}$$

Hence by (3.5), we have the following theorem:

Theorem 3.1 A Ricci semi-symmetric Kenmotsu pseudo-Riemannian metric manifold is an Einstein one.

Corollary 3.1 The scalar curvature of a Ricci semi-symmetric Kenmotsu pseudo-Riemannian metric manifold is $-\varepsilon n(n-1)$.

Now suppose a Kenmotsu pseudo-Riemannian metric manifold M^n ($n=2m+1$) satisfies the condition

$$R(X,Y)\cdot R=0. \tag{3.6}$$

By the definition of (3.6),we have

$$R(X,Y)R(Z,V)W-R(R(X,Y)Z,V)W-R(Z,R(X,Y)V)W-R(Z,V)R(X,Y)W=0. \tag{3.7}$$

Putting $V=\xi$ in (3.7) , then using (2.5) and (2.8) , it becomes

$$\varepsilon g(Z,W)R(X,Y)\xi-\eta(X)R(Z,Y)W+\eta(Y)R(Z,X)W+\eta(R(\xi,Y)W)Z=0. \tag{3.8}$$

Again we take $X=\xi$ in (3.8), using (2.1) and (2.8),

$$-\varepsilon g(Z,W)(\eta(Y)\xi-Y)-R(Z,Y)W+\eta(Y)(\varepsilon g(Z,W)\xi-\eta(W)Z)-\eta(\varepsilon g(Y,W)\xi-\eta(W)Y)Z=0. \tag{3.9}$$

That is to say,

$$R(Z,Y)W=\varepsilon(g(Z,W)Y-g(Y,W)Z). \tag{3.10}$$

Consider a local pseudo-orthonormal basis $\{e_1,\cdots,e_n\}$ on M^n, contracting (3.10) with respect to Z, we have

$$\begin{aligned} S(Y,W)&=\sum_{i=1}^{n}\varepsilon_i g(R(e_i,Y)W,e_i)\\ &=\sum_{i=1}^{n}\varepsilon\varepsilon_i g(g(e_i,W)Y-g(Y,W)e_i,e_i)\\ &=\varepsilon(g(Y,W)-ng(Y,W))\\ &=\varepsilon(1-n)g(Y,W). \end{aligned}$$

Then we can easily get the scalar curvature $r=\varepsilon n(1-n)$.

Hence we get the following theorem:

Theorem 3.2 A semi-symmetric Kenmotsu pseudo-Riemannian metric manifold is an Einstein one and its scalar curvature is $r=\varepsilon n(1-n)$.

Now we consider the condition

$$(\nabla_Z R)(X,Y)W=0. \tag{3.11}$$

From the definition of $(\nabla_Z R)(X,Y)W$, we get

$$\nabla_z(R(X,Y)W)-R(\nabla_Z X,Y)W-R(X,\nabla_Z Y)W-R(X,Y)\nabla_Z W=0. \tag{3.12}$$

Putting $W=\xi$, using (2.4) and (2.6), (3.12) becomes

$$\nabla_Z(\eta(X)Y-\eta(Y)X)-\eta(\nabla_Z X)Y+\eta(Y)\nabla_Z X-\eta(X)\nabla_Z Y+\eta(\nabla_Z Y)X-R(X,Y)(Z-\eta(Z)\xi)=0. \tag{3.13}$$

By a straightforward calculation through (3.13) , we get

$$R(X,Y)Z=\varepsilon\left(g(X,Z)Y-g(Y,Z)X\right). \tag{3.14}$$

Using the method of Theorem (3.2) , we can get

$$S(Y,Z)=\varepsilon(1-n)g(Y,Z). \tag{3.15}$$

Then we get the following theorem:

Theorem 3.3 A locally symmetric Kenmotsu pseudo-Riemannian metric manifold is an Einstein one and its scalar curvature is $r=\varepsilon n(1-n)$.

The projective curvature tensor is given by

$$P(X,Y)Z=R(X,Y)Z-\frac{1}{n-1}\{S(Y,Z)X-S(X,Z)Y\}. \tag{3.16}$$

If a Kenmotsu pseudo-Riemannian metric manifold satisfies the condition

$$R(X,\xi)\cdot P=0, \tag{3.17}$$

we have

$$R(X,\xi)P(U,V)W-P(R(X,\xi)U,V)W-P(U,R(X,\xi)V)W-P(U,V)R(X,\xi)W=0. \tag{3.18}$$

Considering (3.16), taking $V=\xi$, using (2.5), (2.6), (2.8) and (2.1), by a long but straightforward calculation, we get

$$-\varepsilon g(U,W)V+\varepsilon\eta(U)g(X,W)\xi+\frac{1}{n-1}\eta(U)S(X,W)\xi+R(U,X)W$$

$$-\frac{1}{n-1}S(X,W)U+\varepsilon\eta(W)g(X,U)\xi+\frac{1}{n-1}\eta(W)S(U,X)\xi=0. \tag{3.19}$$

Then taking $W=\xi$, we get

$$S(U,X)=-\varepsilon(n-1)g(U,X). \tag{3.20}$$

Thus we have the following theorem:

Theorem 3.4 A Kenmotsu pseudo-Riemannian metric manifold satisfying the condition $R(X,\xi)\cdot P=0$ is an Einstein one and its scalar curvature is $r=\varepsilon n(1-n)$.

4 OTHER CONDITIONS FOR A KENMOTSU PSEUDO-RIEMANNIAN MRTRIC MANIFOLD TO BE EINSTEIN

G.P. Pokhariyal and R.S. Mishra [2] defined M-projective curvature tensor field W^* on Riemannian manifold

$$W^*(X,Y)Z=R(X,Y)Z-\frac{1}{2(n-1)}\left[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY\right]. \tag{4.1}$$

Theorem 4.1 A Kenmotsu pseudo-Riemannian metric manifold is Einstein if it is M-projectively flat.

Proof. In view of $W^*=0$, then we have

$$R(X,Y)Z=\frac{1}{2(n-1)}\left[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY\right]. \tag{4.2}$$

Putting $Z=\xi$, using (2.5) , (2.6) and $\eta(X)=\varepsilon g(X,\xi)$, (4.2) becomes

$$\eta(X)Y-\eta(Y)X=\frac{1}{2(n-1)}\left[-(n-1)\eta(Y)X+(n-1)\eta(X)Y+\varepsilon\eta(Y)QX-\varepsilon\eta(X)QY\right]. \tag{4.3}$$

Concerning $S(X,Y)=g(QX,Y)$, using (2.6), we get

$$Q\xi=-\varepsilon(n-1)\xi. \tag{4.4}$$

Taking $Y=\xi$ in (4.3), using (4.4), we have

$$QX = -\varepsilon(n-1)X \, , \tag{4.5}$$

which proves it is an Einstein manifold. □

Corollary 4.1 If a Kenmotsu pseudo-Riemannian metric manifold is M-projectively flat, the scalar curvature is $r = \varepsilon n(1-n)$.

The con-circular curvature tensor, con-harmonic curvature tensor and conformal curvature tensor are given by

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}\left[g(Y,Z)X - g(X,Z)Y\right], \tag{4.6}$$

$$L(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\left[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\right], \tag{4.7}$$

$$V(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\left[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\right]$$

$$+\frac{r}{(n-1)(n-2)}\left[g(Y,Z)X - g(X,Z)Y\right]. \tag{4.8}$$

Theorem 4.2 A Kenmotsu pseudo-Riemannian metric manifold is Einstein if its M-projective curvature tensor W^* and concircular curvature tensor C are linearly dependent.

Proof. Let $W^*(X,Y)Z = kC(X,Y)Z$, where k is a non-zero constant. By (4.1) and (4.6), we have

$$(1-k)R(X,Y)Z - \frac{1}{2(n-1)}\left[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\right]$$

$$+\frac{kr}{n(n-1)}\left[g(Y,Z)X - g(X,Z)Y\right] = 0. \tag{4.9}$$

Contracting (4.9) with respect to X , we get

$$\sum_{i=1}^{n}\varepsilon_i(1-k)g(R(e_i,Y)Z,e_i) - \frac{1}{2(n-1)}\sum_{i=1}^{n}\left[S(Y,Z)\varepsilon_i g(e_i,e_i) - S(e_i,Z)\varepsilon_i g(Y,e_i) + g(Y,Z)\varepsilon_i g(Qe_i,e_i) - g(e_i,Z)\varepsilon_i g(QY,e_i)\right]$$

$$+\frac{kr}{n(n-1)}\sum_{i=1}^{n}\left[g(Y,Z)\varepsilon_i g(e_i,e_i) - g(e_i,Z)\varepsilon_i g(Y,e_i)\right] = 0. \tag{4.10}$$

From (4.10) we get

$$\left[1-k-\frac{n-2}{2(n-1)}\right]S(Y,Z) = \left[\frac{r}{2(n-1)} - \frac{kr}{n}\right]g(Y,Z). \tag{4.11}$$

Then we have $S(Y,Z) = \frac{r}{n}g(Y,Z)$, which proves the theorem. □

Theorem 4.3 A Kenmotsu pseudo-Riemannian metric manifold is Einstein if its M-projective curvature tensor W^* and conharmonic curvature tensor L are linearly dependent.

Proof. Let $W^*(X,Y)Z = \alpha L(X,Y)Z$, where α being any non-zero constant. Using (4.1) and (4.7), we have

$$(1-\alpha)R(X,Y)Z = \left[\frac{1}{2(n-1)} - \alpha\frac{1}{n-2}\right]\left[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\right]. \tag{4.12}$$

Contracting (4.12) with respect to X like (4.10), we get

$$\frac{n}{2(n-1)}S(Y,Z) = \left[\frac{r}{2(n-1)} - \frac{\alpha r}{n-2}\right]g(Y,Z), \tag{4.13}$$

which implies $S(Y,Z) = \frac{(n-2)r - 2\alpha r(n-1)}{n(n-2)}g(Y,Z)$, so it is Einstein. □

Theorem 4.4 A Kenmotsu pseudo-Riemannian metric manifold is Einstein if its M-projective curvature tensor W^* and conformal curvature tensor V are linearly dependent.

Proof. Let $W^*(X,Y)Z=\alpha V(X,Y)Z$, where α being any non-zero constant. Using (4.1) and (4.8), we have

$$(1-\alpha)R(X,Y)Z-\left(\frac{1}{2(n-1)}-\frac{\alpha}{n-2}\right)\left[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY\right]$$
$$-\frac{\alpha r}{(n-1)(n-2)}\left[g(Y,Z)X-g(X,Z)Y\right]=0. \tag{4.14}$$

Contracting (4.12) with respect to X, we get

$$(1-\alpha)S(Y,Z)-\left(\frac{1}{2(n-1)}-\frac{\alpha}{n-2}\right)\left[nS(Y,Z)-S(Y,Z)+rg(Y,Z)-S(Y,Z)\right]$$
$$-\frac{\alpha r}{(n-1)(n-2)}\left[ng(Y,Z)-g(Y,Z)\right]=0.$$

Then we have $S(Y,Z)=\frac{r}{n}g(Y,Z)$, which completes the proof. □

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