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SUCCESSIVE DIFFERENTIAL COEFFICIENTS FOR MHD VELOCITY SLIP BOUNDARY LAYER FLOW OVER A PLANE PLAQUE

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ABSTRACT

Effects of MHD and velocity slip on boundary layer flow over a plane plaque is investigated. Similarity transformations are employed to transform the governing partial differential equations into ordinary ones, which are then solved by successive differential coeffficients (SDC) via Leibnitz-Maclaurin's method validated by numerical experiments via Runge-Kutta-Fehlberg method. The basic physically important parameters entering the problem are the Magnetic (M) and the Velocity Slip or Fluid Sliding (L) parameters. The results clearly represented the characteristics of the MHD velocity slip effected Blasius problem. It is seen that the SDC and numerical solutions demonstrated excellent agreements. Comparisons with available results in literature showed high degree of agreements. Typically, the velocity increases steadily and asymptotes linearly at large distances from the plate to approach the free stream mean velocity. It is observed that increase in the magnetic parameter, increases the velocity, but eliminates the linear asymptocity at the far boundary. As the magnetic parameter increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow. Consequently, the magnetic parameter reduces the similarity variable boundary layer η, thereby increasing the boundary layer thickness $\delta(x)$. On the other hand, increase in the fluid sliding parameter contributes to velocity jumps at the origin. The inclusion of the magnetic and velocity slip parameters modify the famous Blasius (1908) problem of boundary layer flow over a flat plate.

Keywords:

Blasius problem; Boundary layer; MHD; Similarity variables; Successive differential coefficients; Velocity slip.

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1. Introduction

In fluid dynamics, the no-slip condition for viscous fluids states that at a solid boundary, the fluid will have zero velocity relative to the boundary, whereas the slip flow is the condition in which molecular shearing occurs at hypersonic speeds. The no-slip condition is inappropriate and inconsistent with all the physical characteristics of fluid flow. For example, when fluid flows in micro-fluidic devices such as micro-electromechanical system (MEMS), the no-slip condition at the solid-fluid interface becomes inapplicable. It is natural that a fluid slides on the surface of a dynamic or static solid embedded in a particulate fluid such as emulsions, suspensions, foams, polymer solutions, and hence the slip condition. To this end, the slip flow model describes more accurately the non-equilibrium region near the interface. Therefore, it becomes imperative and necessary to replace the no-slip boundary condition by the slip boundary condition. On the other hand, Magnetohydrodynamics (MHD), which is also known as “magneto fluid dynamics” or “hydromagnetics” is defined as the study of the dynamics of electrically conducting fluids, such as plasmas, liquid metals, and salt water or electrolytes. MHD flows has lots of applications such as in engineering problems, geophysics, astrophysics, medicine, cosmic fluid dynamics and aerodynamics and are elsewhere reported in literatures.

There are many related literatures that make use of the velocity slip condition under different flow scenarios. This is because problems that involve slip boundary conditions are useful models for flows through pipes in which chemical reactions occur at the walls, flows with laminar film condensation, and certain two phase flows. Motivated by these applications and more, several studies have been made to investigate the effect of velocity slip. Anderson [1] worked on slip flow past a stretching surface, while Fang and Lee [2] looked at a moving –wall boundary layer flow of a slightly rarefied gas free stream over a moving flat plate. Fang and Lee [3], on the other hand, investigated and provided exact solutions of an incompressible couette flow with porous walls for slightly rarefied gas.

It is known that the slip flow condition gives more accurate description of flows when it has to do with near non-equilibrium regions. As such the slip flow condition has been applied to different flow scenarios in biomedical engineering, when blood flows through an artery slip, and this is evident from experimental observations [4, 5].

The slip flows under different flow configurations have been studied in recent years. Stagnation slip flow and heat transfer on a moving plate was studied by Wang [6]. For consideration of partial slip, Wang [7] made a study of stagnation flow on a cylinder, giving exact solution of the Navier-Stokes equations. Wang [8] considered analysis of viscous flow due to a stretching sheet with surface slip and suction. Fang et al. [9] provided an exact solution for slip MHD viscous flow over a stretching sheet. For an extension and consideration of second order slip flow model, Fang et al. [10] investigated viscous flow over a shrinking sheet.

The pioneering work of Blasius [11] idealized flow problem of a viscous fluid past an infinitesimally thick, semi-infinite flat plate has been extended to include slip flow boundary condition by many other researchers. Martin and Boyd [12] considered momentum and heat transfer in a laminar boundary layer flow over a flat plate with slip boundary condition. Cao and Baker [13] presented local non-similar solutions to the boundary layer equations for mixed convection over a vertical isothermal plate, considering, velocity slip and thermal jump boundary conditions. Harris et al. [14] studied the mixed convection boundary layer stagnation point flow on a vertical surface in a porous medium with slip. Aziz [15], on the other hand, studied the boundary layer slip flow over a flat plate with constant heat flux condition at the surface, where the local similarity incorporated the slip boundary condition. Bhattacharyya et al. [16, 17, 18] discussed the MHD slip flow over a flat plate and the steady slip flow on a flat plate in porous medium.

The literatures of velocity slip condition for flow and heat transfer problems have quite been explored! Mebine [19], recently, investigated MHD dynamic boundary layer flow over a plane plaque by the use of von Karman integral method. It was stated that the von Karman integral analysis is a tool that paves the way to using the right similarity variables. This work is an extension of the work of Mebine [19] to investigate the similarity variables solution of the “MHD velocity slip boundary layer flow over a plane plaque” via “successive differential coefficients” complemented by “numerical experimentations.”

2. Mathematical Formulation

Consider a semi-finite plane plague situated on the *Ox* axis, having the edge at *O*, attached under a null angle. The flow is a plane steady two-dimensional laminar flow of a viscous incompressible and electrically conducting fluid of density ρ, near the leading edge of a flat plate, where *Oxy* is the plane of the flow such that (*x, y*) is the Cartesian coordinates of any point in the domain of the flow, where *x* – axis is along the plate and *y* – axis is normal to the plate. Figure 1 gives a description of the flow configuration of the hydromagnetic boundary layer flow with slip velocity over a plane plaque. The slip flow is possible at lower gas pressures, when the free mean path of the gas molecules, say λ approximates the characteristic dimension L_1 of the surface, where the fluid slides upon. That is, when $\lambda \approx L_1$, the flow seems to “slip” along the surface and $u \neq 0$ at $y = 0$ such that $u = L_1 \dfrac{\partial u}{\partial y}$. This situation is appropriately called a slip flow.

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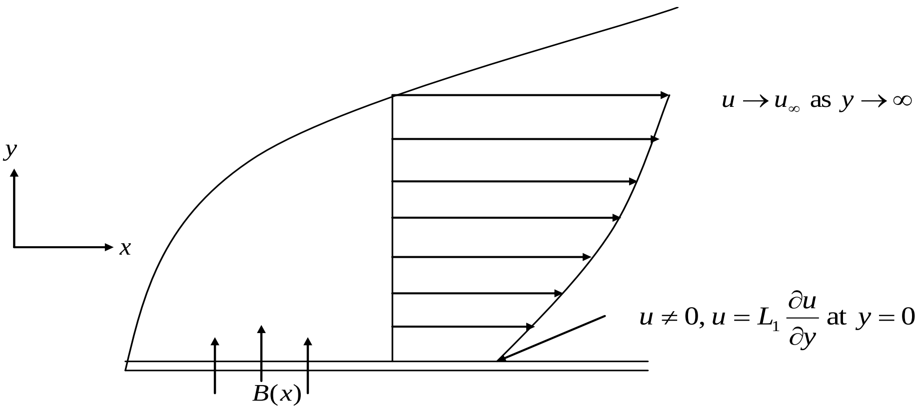

Figure 1: Flow Configuration of Hydromagnetic Boundary Layer

It is assumed that *u* and *v* are the respective velocity component in the *x* and *y* directions, and a constant magnetic field B_0 is applied normally to the plate. Following Mebine [19], within the boundary layer approximations, the continuity and MHD aided momentum equations are written in the usual notations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B(x)^2}{\rho}u, \tag{2.2}$$

with the associated boundary conditions:

$$y = 0: \quad u(x,0) = L_1\frac{\partial u}{\partial y}(x,0), v(x,0) = 0, \tag{2.3a}$$

$$y \to \infty: \quad u(x,\infty) = u_\infty, \tag{2.3b}$$

where ν is the kinematic viscosity, σ is the electrical conductivity of the fluid and $B(x)$ is the magnetic field intensity. The first boundary condition at $y = 0$ signifies the fact that the fluid in contact with the plaque, slides on its surface.

Mebine [19] investigated the problem of MHD dynamic boundary layer sliding flow over a plane plaque using von Karman integral boundary layer analysis method. It was stated that the integral boundary layer analysis is a key to validating similarity variables for numerical or analytical computations. Therefore, in order to determine the method of solution, first of all consider a stream function ψ such that the velocity components are defined by

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{2.4}$$

The use of equation (2.4) automatically satisfies the continuity equation (2.1) and renders the system of partial differential equations (2.1 – 2.2) to an ordinary differential equation. In this case a similarity solution is sorted such that

$$\psi = y\sqrt{\nu x u_\infty}\, f(\eta), \eta = y\sqrt{\frac{u_\infty}{\nu x}}. \tag{2.5}$$

With the introduction of the equations (2.4) and (2.5), equation (2.2) now becomes

$$f'''(\eta) + \frac{1}{2}f(\eta)f''(\eta) - Mf'(\eta) = 0, \tag{2.6}$$

where $M = \frac{\sigma B_0^2}{\rho u_\infty} \equiv$ Magnetic Parameter and the Magnetic Field Intensity, $B^2(x) = \frac{B_0^2}{x}$ for $x \neq 0$ or $x > 0$. The accompanying boundary conditions (2.3) are now written as

$$\eta = 0: f(0) = 0, f'(0) = Lf''(0), \quad (2.7a)$$

$$\eta \to \infty: f'(\infty) \to 1, \quad (2.7b)$$

where $L = L_1\sqrt{\frac{u_\infty}{\nu x}} \equiv$ Sliding Parameter.

3. The Blasius Problem and Benchmark results

For $M = 0$ respectively $L = 0$, the equation (2.6) together with the boundary conditions (2.7) reduce to the famous Blasius [11] problem:

$$f'''(\eta) + \frac{1}{2} f(\eta) f''(\eta) = 0 \quad (3.1)$$

subject to the boundary conditions:

$$\eta = 0: f(0) = 0, f'(0) = 0, \quad (3.2a)$$

$$\eta \to \infty: f'(\infty) \to 1. \quad (3.3b)$$

Boyd [20, 21] gave extensive expository studies of different solution considerations and properties of Blasius problem. Herein, a summary of power series, asymptotic approximations and Blasius integro-differential equation results, as discussed in Boyd [20, 21] are presented.

3.1 Power Series

The power series approximation is given by

$$f(\eta) \approx \frac{1}{2!}\alpha\eta^2 - \frac{1}{2}\frac{\alpha^2}{5!}\eta^5 + \frac{11}{4}\frac{\alpha^3}{8!}\eta^8 - \frac{375}{8}\frac{\alpha^4}{11!}\eta^{11} + \cdots, \quad (3.3)$$

where α is the second derivative of the function at the origin, which numerical value is given by $\alpha = 0.33205733621519630$. It is emphasized here that in order to overcome the limitation of a finite radius of convergence, the power series could be constructed, from either Pade approximants or an Euler-accelerated series, which both apparently converges for all positive real η [20, 22].

3.2 Asymptotic Approximations

For large positive η,

$$f(\eta) \sim B + \eta + \text{exponentially decaying terms}, \eta >> 1, \quad (3.4)$$

and

$$f''(\eta) = Q \exp\left\{-\frac{1}{4}\eta(\eta + 2B)\right\}, \quad (3.5)$$

where the definition and numerical value of the respective symbols are

$$B = \lim_{\eta\to\infty} \left(f(\eta) - \eta\right) = -1.720787657520503,$$

$$Q = \lim_{\eta\to\infty} \exp\left\{\frac{1}{4}\eta(\eta + 2B)\right\} f(\eta) = 0.233727621285063.$$

The linear polynomial $B + \eta$ is termed as the leading order asymptotic approximation for $f(\eta)$, and it is an exact solution to the differential equation for all η, but fails to satisfy the boundary conditions.

3.3 Blasius's Integro-differential Equation

Consider the first order linear ODE, which has an explicit solution :

$$v'(\eta)+\phi(\eta)v(\eta)=0\rightarrow v(\eta)=v(0)\exp\left(-\int_0^\eta \phi(\varsigma)d\varsigma\right). \quad (3.6)$$

This could be applied to the Blasius third order differential equation by identifying

$$v(\eta)=f''(\eta),\phi(\eta)=f(\eta) \quad (3.7)$$

such that

$$f''(\eta)=\alpha\exp(-F(\eta)), \quad (3.8)$$

where $F(\eta)\equiv\frac{1}{2}\int_0^\eta f(\varsigma)d\varsigma\approx\frac{1}{12}\alpha\eta^3-\frac{1}{2880}\alpha^2\eta^6+\frac{11}{2903040}\alpha^3\eta^9-\frac{5}{102187008}\alpha^4\eta^{12}+\cdots.$

This converts the Blasius third order differential equation into an integrodifferential equation of second order. It is noted here that insights and approximations could be extracted by substituting local approximations for $F(\eta)$. For example, as noted earlier, $f\sim B+\eta$ for some constant B as $\eta\rightarrow\infty$. This implies that $F(\eta)$ is asymptotically a quadratic polynomial and therefore $f''(\eta)\sim Q\exp\left\{-\frac{1}{4}\eta(\eta+2B)\right\}$ as $\eta\rightarrow\infty$, as stated before.

4. The MHD Velocity Slip effected Blasius Problem

The main objective of the problem is to investigate the solution of the equation (2.6) together with the boundary conditions (2.7) in order to explore the effects of M and L. The power series, asymptotic approximations and the Blasius integrodifferential equation results serve as building blocks to exploring the solution of the problem at hand (equations 2.6, 2.7). Therefore, the solution of the problem is tackled through "order reduction via numerical solution" and "successive differential coefficients via Leibnitz method [23]."

4.1 Order Reduction and Numerical Solution

The nonlinear ordinary differential equation (2.6) along with the boundary conditions (2.7) form a two point boundary value problem (BVP) by converting into an initial value problem (IVP). In this method it is necessary to choose a suitable finite guess value of $\eta\rightarrow\infty$, say η_∞. The solution process is repeated with various values until two successive values of the results differ only after desired digit(s), signifying the limit of the boundary along $\eta\rightarrow\infty$. The following first-order system is set such that

$$f'(\eta)=p(\eta),p'(\eta)=q(\eta), \quad (4.1)$$

then

$$q'(\eta)=-\frac{1}{2}f(\eta)q(\eta)+Mp(\eta), \quad (4.2)$$

with the boundary conditions

$$f(0)=0,p(0)=Lq(0). \quad (4.3)$$

To solve the first-order system (4.1 – 4.3) as an IVP the values for $q(0)$ i.e. $f''(0)$ is the most needed but no such value is given. The initial guess value for $f''(0)$ is chosen and applying IVP *Runge-Kutta-Fehlberg* method in **Maple** Software produces a solution accurate to a fifth order.

4.2 Successive Differential Coefficients via Leibnitz Method

For two differentiable and continuous functions, say u and v, which are functions of say x, Leibnitz concise formula for the nth differential coefficient of their product is:

$$D^n(uv)=\sum_{r=0}^{n}{}^nC_rD^{n-r}v\cdot D^ru, \quad (4.4)$$

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where ${}^{n}C_{r} = \dfrac{n!}{(n-r)!r!}, D = \dfrac{d}{dx}, D^{2} = \dfrac{d^{2}}{dx^{2}}, D^{3} = \dfrac{d^{3}}{dx^{3}}, \ldots, D^{n} = \dfrac{d^{n}}{dx^{n}}$.

The Leibnitz's formula is applied to the differential equations (2.6, 2.7a) in obtaining a recurrence relation between successive differential coefficients. This forms a step towards finding a power series solution of the problem at hand.

In the application of Leibnitz's formula, the solution of the problem is written in terms of Maclaurin series which is a special case of Taylor series, such that

$$f(\eta) = f(0) + \frac{\eta}{1!} f'(0) + \frac{\eta^{2}}{2!} f''(0) + \ldots = \sum_{r=0}^{n} \frac{\eta^{r}}{r!} f^{(r)}(0), \tag{4.5}$$

where $f(0) = f(\eta)\big|_{\eta=0}, f'(0) = \dfrac{df}{d\eta}\Big|_{\eta=0}, f''(0) = \dfrac{d^{2}f}{d\eta^{2}}\Big|_{\eta=0}, \cdots, f^{(r)}(0) = \dfrac{d^{r}f}{d\eta^{r}}\Big|_{\eta=0}$.

From the boundary conditions, it is observed that the value of $f''(0)$ is unknown. To apply the Leibnitz formula to finding the "Successive Differential Coefficients" (SDC) at $\eta = 0$, let $f''(0) = \alpha$, be an undetermined coefficient that would be computed with the help of condition (2.7b). From the physical point of view, $f''(0)$ is the quantity of interest, and it is the skin friction coefficient which represents the wall shear stress. The first eleven SDC of the application of equation (4.4) to the equation (2.6) with the conditions (2.7a) are stated as follows:

$$\begin{aligned}
& f'(0) = L\alpha, \\
& f''(0) = \alpha, \\
& f'''(0) = LM\alpha, \\
& f^{(iv)}(0) = -\frac{1}{2}L\alpha^{2} + M\alpha, \\
& f^{(v)}(0) = -\frac{1}{2}\alpha^{2} + LM^{2}\alpha - L^{2}M\alpha^{2}, \\
& f^{(vi)}(0) = \frac{3}{4}L^{2}\alpha^{3} - 4LM\alpha^{2} + M^{2}\alpha, \\
& f^{(vii)}(0) = \frac{11}{4}L\alpha^{3} - 4M\alpha^{2} + LM^{3}\alpha - 5L^{2}M^{2}\alpha^{2} + 2L^{3}M\alpha^{3}, \\
& f^{(viii)}(0) = \frac{11}{4}\alpha^{3} - \frac{39}{2}LM^{2}\alpha^{2} + 20L^{2}M\alpha^{3} + M^{3}\alpha - \frac{15}{8}L^{3}\alpha^{4}, \\
& f^{(ix)}(0) = -\frac{129}{8}L^{2}\alpha^{4} - \frac{39}{2}M^{2}\alpha^{2} + \frac{243}{4}LM\alpha^{3} - 21L^{2}M^{3}\alpha^{2} + 30L^{3}M^{2}\alpha^{3} + LM^{4}\alpha - 6L^{4}M\alpha^{4}, \\
& f^{(x)}(0) = -\frac{375}{8}L\alpha^{4} + \frac{105}{16}L^{4}\alpha^{5} + \frac{243}{4}M\alpha^{3} + M^{4}\alpha + \frac{1077}{4}L^{2}M^{2}\alpha^{3} - \frac{989}{8}L^{3}M\alpha^{4} - 83LM^{3}\alpha^{2}, \\
& f^{(xi)}(0) = -\frac{375}{8}\alpha^{4} - 83M^{3}\alpha^{2} + \frac{1761}{16}L^{3}\alpha^{5} + LM^{5}\alpha + 24L^{5}M\alpha^{5} + 798LM^{2}\alpha^{3} - \frac{6399}{8}L^{2}M\alpha^{4} \\
& \qquad -85L^{2}M^{4}\alpha^{2} + 330L^{3}M^{3}\alpha^{3} - 218L^{4}M^{2}\alpha^{4}.
\end{aligned} \tag{4.6}$$

For want of more accuracy, as many SDC as possible may be computed, but with much more difficulty in the computations. Of course, with the aid of Symbolic Computation Softwares such as Maple, Mathematica and Matlab, as many terms as possible and as desired could be computed! It is important to note that the Taylor series and the Maclaurin series only represent the function $f(\eta)$ in their intervals of convergence as earlier stated.

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5. Analyses of Results

The basic physically important parameters entering the problem are the Magnetic (M) and the Velocity Slip or Fluid Sliding (L) parameters. For $M = 0$ respectively $L = 0$ gives the Blasius problem, and it is clearly seen that the power series result (3.3) and the SDC result (4.6) are exactly the same.

To discuss the effects of the parameters, tabular and graphical representations are made using the values $M = 0.0, 0.2, 0.5, 0.8, 1.2$ and $L = 0.0, 0.2, 0.5$, respectively. The value of $\eta_\infty = 5.1$ as $\eta \to \infty$ is used for the numerical and SDC computations. The SDC results employed sixty-eight (68) terms. In order to demonstrate the linear asymptocity of the Blasius velocity $f'(\eta)$, the value of $\eta_\infty = 8$ as $\eta \to \infty$ is used for the graphical representations.

Table 1 gives the computations of $f''(0)$ for the numerical (NUM) and SDC results, respectively. It is observed that increase in magnetic parameter has a retarding effect on the coefficient of skin friction $f''(0)$, while increase in the sliding parameter enhances its reduction. From the table it is seen that the NUM results agree excellently well with the results obtained using the SDC. The result corresponding to $M = 0.0, L = 0.0$ implies Blasius (1908) problem. For a boundary layer flow of a Newtonian fluid over a flat plate, Blasius obtained an exact solution that gives $f''(0) = 0.332$ [24], which agrees excellently well with the NUM result of $f''(0) = 0.335$, giving an error of 1% approximately. The SDC result, on the other hand, gives $f''(0) = 0.331$, and when compared to the Blasius exact solution gives an error of 0.3% approximately. The SDC results could be improved via Pade approximants or Euler-accelerated series, as discussed ealier! It is also equally observed that the values of $f''(0)$ for $M = 0.0$ of the SDC results for $L = 0.0, 0.2$ and $L = 0.5$, respectively, are the only results at slight variance to some degree with the NUM results. For $0.2 \le M \le 1.2$, the accuracy of the NUM and SDC results is from order-six and above. It must be emphasized here that the numerical experimentations performed in this work used very few discretizations in constrast to Boyd [20], who utilized $50,000$ grid points by using order-four Runge-Kutta time matching to integrate the Blasius problem to achieve $f''(0) = 0.33205733621519630$. This implies that appropriate discretizations would achieve more excellent agreements for the NUM and SDC results.

	$L = 0.0$ $f''(0)$		$L = 0.2$ $f''(0)$		$L = 0.5$ $f''(0)$	
M	**NUM**	**SDC**	**NUM**	**SDC**	**NUM**	**SDC**
0.0	0.33545428	0.33075751	0.33215128	0.32329739	0.32114700	0.60449562
0.2	0.14508270	0.14508251	0.13651217	0.13651218	0.12497960	0.12497961
0.5	0.05141947	0.05141947	0.04546723	0.04546723	0.03872380	0.03872380
0.8	0.02290654	0.02290654	0.01950172	0.01950172	0.01594432	0.01594433
1.2	0.00945992	0.00945991	0.00776987	0.00776987	0.00612754	0.00612753

Table 1: Comparison of methods of Numerical and Successive Differential Coefficients for the skin friction coefficient $f''(0)$ for various of M and L.

From the Table 1, it is clearly seen that $f''(0)$, the curvature (i.e. skin friction coefficient) decreases as M increses, signifying that M has retarding effect analogous to a flow against a positive gradient. Physically, M increases the sliding of the fluid on the plane plaque, thereby reducing the similarity variable boundary layer with the velocity slip L positively contributing to velocity jumps at the orgin, much more pronounced for $M = 0$ (see Figures 2(b) and (c)). Table 2 demonstrates the velocity jumps at the origin for $0.2 \le M \le 1.2$ aided by the velocity slip for $L = 0.2$ and $L = 0.5$. It is clearly seen that increase in the magnetic parameter reduces the velocity jumps.

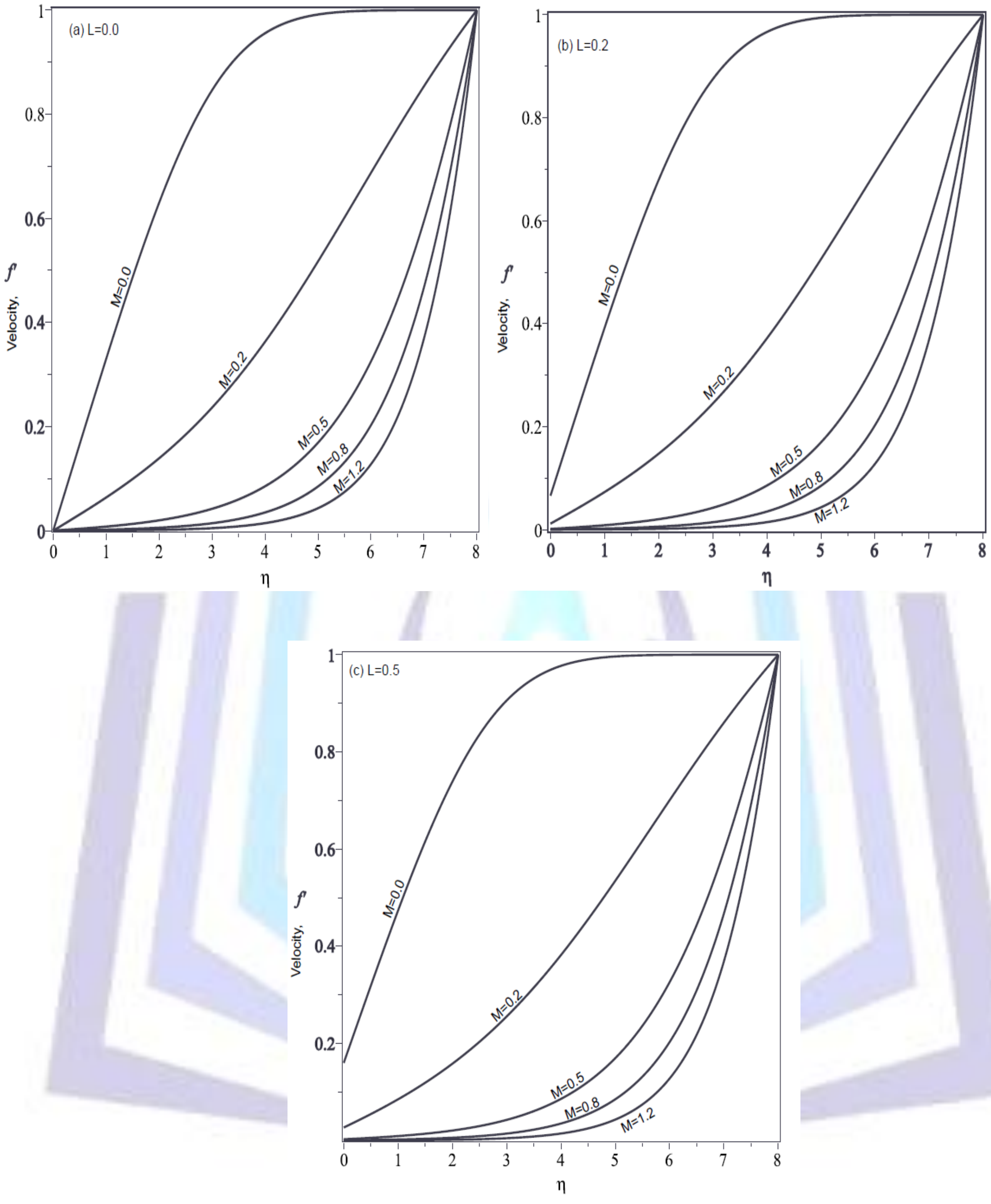

Figure 2: Velocity profiles for various values of M with L

Figure 2 display plots of the velocity $f'(\eta)$ versus η for various values of M and L. From the profiles, one can observe that for $M = 0.0$ with $0.0 \leq L \leq 0.5$, the velocity steadily increases from 0 and at about $\eta = 5.0$, the velocity asymptotes linearly to the free stream mean velocity. Physically, the Blasius velocity $f'(\eta)$ is constant at large distances from the plate, and this implies that $f(\eta)$ should asymptote to a linear function of η far from the plate. Near the surface of the plate at $\eta = 0$, the stream function $f(\eta)$ curves away from the straight line to satisfy the boundary conditions at $\eta = 0$, creating a region of rapid variation called a "boundary layer." For $M > 0.0$, the velocity increases, but the linear asymptocity is no longer retained, rather it sags towards the far boundary. As M increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow. Consequently, the similarity variable boundary layer η is reduced by increasing M . Physically, increasing magnetic parameter increases the boundary layer thickness $\delta(x)=\frac{y}{\eta}=\sqrt{\frac{\nu x}{u_\infty}}$ by equation (2.5). On the other hand, it is also observed that increase in the fluid sliding parameter contributes to the velocity jumps at $\eta=0$ (see Figure 2 (b) and (c) and Table 2). In terms of the velocity jumps at $\eta=0$, the NUM and SDC results excellently agree for all values of M for $L=0.0$, and for values of $0.2\leq M\leq 1.2$, the NUM and SDC results once again excellently agree for $L=0.2$ and $L=0.5$ to an order-six and above as reported before. Figure 2 (b) and (c) depicts the velocity jumps.

	$L=0.0$ $f'(0)$		$L=0.2$ $f'(0)$		$L=0.5$ $f'(0)$	
M	**NUM**	**SDC**	**NUM**	**SDC**	**NUM**	**SDC**
0.0	0.00000000	0.00000000	0.06643026	0.05595515	0.16057335	0.30224781
0.2	0.00000000	0.00000000	0.02730243	0.02420706	0.06248980	0.06248981
0.5	0.00000000	0.00000000	0.00909345	0.00698586	0.01936190	0.01936190
0.8	0.00000000	0.00000000	0.00390034	0.00274657	0.00797216	0.00797216
1.2	0.00000000	0.00000000	0.00155397	0.00100502	0.00306064	0.00306377

Table 2: Comparison of methods of Numerical and Successive Differential Coefficients for the velocity $f'(0)$ (i.e. at the origin) for various values of M and L.

7. Concluding Remarks

The purpose of this study was the exploration and exploitation of successive differential coefficients versus numerical experimentations for the investigation of MHD velocity slip effected famous Blasius problem of boundary layer flow over a flat plate. It is observed that the inclusion of the MHD and fluid sliding factor to the Blasius problem presented considerable mathematical interests for the intended investigations. From the results, the following main conclusions are made:

1) That the results represent the characteristics of the problem.
2) The velocity increases steadily and asymptotes linearly at large distances from the plate to approach the free stream mean velocity.
3) That increase in the magnetic parameter increases the velocity, but eliminates the linear asymptocity at the far boundary. As M increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow.
4) That increase in the fluid sliding parameter contributes to velocity jumps at the origin.
5) That the velocity jumps at the origin gives the significant difference of the present problem to that of the Blasius.
6) That the numerical and SDC results demonstrated excellent agreements.
7) That the inclusion of the parameters M and L modify the Blasius problem of boundary layer flow over a flat plate.
8) That the SDC results could be improved via Pade approximants or Euler-accelerated series.

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