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ON INTUITIONISTIC FUZZY IDEALS BITOPOLOGICAL SPACES

Mohammed Jassim Tuaimah[1] , Mohammed Jassim Mohammed[2]

[1] Department of Mathematics, College of Education for Pure Science , Thi-Qar University , Thi-Qar , Iraq

[2] Department of Mathematics, College of Education for Pure Science , Thi-Qar University , Thi-Qar , Iraq

ABSTRACT

In this paper we introduce the notion of intuitionistic fuzzy ideals in intuitionistic fuzzy bitopological spaces and we prove some results about it .

Keywords

Intuitionistic fuzzy bitopological space ; intuitionistic fuzzy ideal and intuitionistic fuzzy local function .

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1.INTRODUCTION :

The concept of " fuzzy sets " was first introduced by Zadeh [7] in 1965 . Atanassov [1] in 1983 , initiated the study of an " intuitionistic fuzzy sets" and defined the " interior and closure " over the set of intuitionistic fuzzy sets . In 1995, Coker and Demirci [3] , constructed the basic concept so called " intuitionistic fuzzy points " and related objects as " quasi-coincidence ". Kandil , Nouh and El-Sheikh [5] in 1995 , initiated the study of " fuzzybitopological spaces ". In 1997 , Coker [4] defined the concept of " intuitionistic fuzzy topology ". The concept of " fuzzy ideals in fuzzy bitopological space and fuzzy pairwise local function " were first introduced by AbdEl_Monsef , Kozae , Salama and Elagmy [2] in 2012 . The notions of " intuitionistic fuzzy ideals and intuitionistic fuzzy local functions " studied by Salama and Alblowi [6] in 2012 .

In this paper we introduce the notion of an " intuitionistic fuzzy bitopological spaces " and study the concept of an " intuitionistic fuzzy ideals , intuitionistic fuzzy local functions in intuitionistic fuzzy ideal bitopological spaces " and prove some results about them .

2.PRELIMINARIES :

Definition. 2.1. [7] :

Let X be a non-empty set and $I = [0,1]$ be the closed interval of the real numbers . A fuzzy subset μ of X is defined to be a membership function $\mu : X \to I$, such that $\mu(x) \in I$ for every $x \in X$. The set of all fuzzy subsets of X denoted by I^X.

Definition. 2.2. [1] :

An intuitionistic fuzzy set (IFS , for short) A is an object have the form : $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle ; x \in X\}$, where the functions $\mu_A : X \to I$, $\nu_A : X \to I$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively , and $0 \le \mu_A(x) + \nu_A(x) \le 1$, for each $x \in X$. The set of all intuitionistic fuzzy sets in X denoted by $IFS(X)$.

Definition. 2.3. [4] :

$0_\sim = \langle x, 0,1 \rangle$, $1_\sim = \langle x, 1,0 \rangle$ are the intuitionistic fuzzy sets corresponding to empty set and the entire universe respectively .

Definition. 2.4. [3] :

Let X be a non-empty set . An intuitionistic fuzzy point (IFP , for short) denoted by $x_{(\alpha,\beta)}$ is an intuitionistic fuzzy set have the form : $x_{(\alpha,\beta)}(y) = \begin{cases} \langle x, \alpha, \beta \rangle & ; \quad x = y \\ \langle x, 0,1 \rangle & ; \quad x \neq y \end{cases}$, where $x \in X$ is a fixed point , and $\alpha , \beta \in [0,1]$ satisfy $\alpha + \beta \le 1$. The set of all $IFPs$ denoted by $IFP(X)$. If $A \in IFS(X)$, we say that $x_{(\alpha,\beta)} \in A$ if and only if $\alpha \le \mu_A(x)$ and $\beta \ge \nu_A(x)$, for each $x \in X$.

Definition. 2.5. [3] :

Let $A = \langle x, \mu_A(x), \nu_A(x) \rangle$, $B = \langle x, \mu_B(x), \nu_B(x) \rangle$ be two $IFSs$ in X and $x_{(\alpha,\beta)} \in IFP(X)$, then :

(1) A is said to be quasi-coincident with B (written AqB) if there exists an element $x \in X$, such that $\mu_A(x) + \mu_B(x) > 1$ and $\nu_A(x) + \nu_B(x) < 1$. Otherwise A is not quasi-coincident with B and denoted by $A\overline{q}B$.

(2) We say that A is quasi- coincident with $x_{(\alpha,\beta)}$, denoted by $x_{(\alpha,\beta)}qA$ if $\mu_A(x) + \alpha > 1$ and $\nu_A(x) + \beta < 1$.

Definition.2.6. [4] :

An intuitionistic fuzzy topology (IFT , for short) on a non-empty set X is a family τ of an intuitionistic fuzzy sets in X such that :

(i) $0_\sim$, $1_\sim$ $\in \tau$,

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$, for any arbitrary family $\{G_i : i \in \Gamma\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space ($IFTS$, in short) .

Definition. 2.7. [6] :

A non-empty collection of intuitionistic fuzzy sets L of a set X is called intuitionistic fuzzy ideal on X (IFI , for short) such that :

(i) If $A \in L$ and $B \leq A \Rightarrow B \in L$ (heredity) ,

(ii) If $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ (finite additivity) .

If (X, τ) be an $IFTS$, then the triple (X, τ, L) is called an intuitionistic fuzzy ideal topological space ($IFITS$, for short) .

Definition. 2.8. [6] :

Let (X, τ, L) be an $IFITS$. If $A \in IFS(X)$, then the intuitionistic fuzzy local function $A^\star(L, \tau)$ ($A^\star$, for short) of A in (X, τ, L) is the union of all intuitionistic fuzzy points $x_{(\alpha,\beta)}$ such that :

$A^\star(L, \tau) = \vee\{ x_{(\alpha,\beta)} : A \wedge U \notin L , for\ every\ \ U \in N(x_{(\alpha,\beta)}, \tau) \}$, where $N(x_{(\alpha,\beta)}, \tau)$ is the set of all quasi-neighborhoods of an IFP $x_{(\alpha,\beta)}$ in τ . The intuitionistic fuzzy closure operator of an IFS A is defined by $cl^\star(A) = A \vee A^\star$, and $\tau^\star(L)$ is an IFT finer than τ generated by $cl^\star(.)$ and defined as $\tau^\star(L) = \{ A : cl^\star(A^c) = A^c \}$.

Definition. 2.9. [5] :

A fuzzy bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ ($FBTS$, for short) is a non-empty set X equipped with two fuzzy topologies $\mathcal{T}_1$ and $\mathcal{T}_2$.

Definition. 2.10. [2] :

A fuzzy set μ in a $FBTS\,(X, \mathcal{T}_1, \mathcal{T}_2)$ is called pairwise quasi-neighborhood of a fuzzy point x_t if and only if there exists a $\mathcal{T}_i$ –fuzzy open set ν , $i \in \{1,2\}$ such that $x_t\, q\, \nu \leq \mu$. The set of all pairwise quasi-neighborhoods of x_t in $\mathcal{T}_i$ denoted by $PN(x_t, \mathcal{T}_i)$.

Definition. 2.11. [2] :

Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a $FBTS$ and $\mathcal{L}$ be a fuzzy ideal on X , then $(X, \mathcal{T}_1, \mathcal{T}_2, \mathcal{L})$ is called fuzzy ideal bitopological space ($FIBTS$) . The pairwise fuzzy local function $P\mu^\star(\mathcal{L}, \mathcal{T}_i)$ of a fuzzy set μ is the union of all fuzzy points x_t such that if $\rho \in PN(x_t, \mathcal{T}_i)$, $i \in \{1,2\}$ and $\gamma \in \mathcal{L}$, then there is at least one $r \in X$ for which $\rho(r) + \mu(r) - 1 > \gamma(r)$. The pairwise fuzzy closure operator of a fuzzy set μ is defined by $\mathcal{T}_i - cl^\star(\mu) = \mu \vee P\mu^\star(\mathcal{L}, \mathcal{T}_i)$ and $\mathcal{T}_i^\star(\mathcal{L})$ is the fuzzy bitopology finer than $\mathcal{T}_i$ generated by $\mathcal{T}_i - cl^\star(.)$ and defined as follows : $\mathcal{T}_i^\star(\mathcal{L}) = \{ \mu : \mathcal{T}_i - cl^\star(\mu^c) = \mu^c \}$.

3.SOME RESULTS OF INTUITIONISTIC FUZZY IDEALS BITOPOLOGICAL SPACES

Definition. 3.1 :

Let τ_1 and τ_2 be two intuitionistic fuzzy topologies on a non-empty set X . The triple (X, τ_1, τ_2) is called an "intuitionistic fuzzy bitopological space" ($IFBTS$, for short) , every member of τ_i is called τ_i –intuitionistic fuzzy open set $(\tau_i - IFOS)$, $i \in \{1,2\}$ and the complement of $\tau_i - IFOS$ is τ_i –intuitionistic fuzzy closed set $(\tau_i - IFCS)$, $i \in \{1,2\}$.

Example. 3.2 :

Let $X = \{e, d\}$ and $A, B \in IFS(X)$ such that : $A = \langle x, (0.3,0.1), (0.5,0.6)\rangle$, $B = \langle x, (0.2,0.4), (0.7,0.3)\rangle$.

Let $\tau_1 = \{0_\sim , 1_\sim, A\}$ and $\tau_2 = \{0_\sim , 1_\sim, B\}$ be two $IFTs$ on X . Then (X, τ_1, τ_2) is $IFBTS$.

Definition. 3.3 :

Let (X, τ_1, τ_2) be an $IFBTS$, $A \in IFS(X)$ and $x_{(\alpha,\beta)} \in IFP(X)$. Then A is said to be quasi-neighborhood of $x_{(\alpha,\beta)}$ if there exists a $\tau_i - IFOS$ B , $i \in \{1,2\}$ such that $x_{(\alpha,\beta)} qB \le A$. The set of all quasi-neighborhoods of $x_{(\alpha,\beta)}$ in (X, τ_1, τ_2) is denoted by $N(x_{(\alpha,\beta)}, \tau_i)$, $i \in \{1,2\}$.

Definition. 3.4 :

An intuitionistic fuzzy bitopological space (X, τ_1, τ_2) with an intuitionistic fuzzy ideal L on X is called " intuitionistic fuzzy ideal bitopological space " (X, τ_1, τ_2, L) and denoted by $IFIBTS$.

Example. 3.5 :

Let $X = \{e\}$ and A , $B \in IFS(X)$ such that : $A = \langle x, 0.3, 0.5\rangle$, $B = \langle x, 0.2, 0.4\rangle$. Let (X, τ_1, τ_2) be an $IFBTS$, where $\tau_1 = \{0_\sim , 1_\sim, A\}$ and $\tau_2 = \{0_\sim , 1_\sim, B\}$. If $L = \{0_\sim , A, C : C \in IFS(X) \text{ and } C \le A\}$ be an IFI on X . Then (X, τ_1, τ_2, L) is $IFIBTS$.

Definition. 3.6 :

Let (X, τ_1, τ_2, L) be an $IFIBTS$ and $A \in IFS(X)$. Then the " intuitionistic fuzzy local function " of A in (X, τ_1, τ_2, L) denoted by $A^*(L, \tau_i)$, $i \in \{1,2\}$ and defined as follows : $A^*(L, \tau_i) = \vee\{x_{(\alpha,\beta)} : A \wedge U \notin L, \text{for every } U \in N(x_{(\alpha,\beta)}, \tau_i)\}$.

Definition. 3.7 :

An " intuitionistic fuzzy local function " $A^*(L, \delta)$ having the form : $A^*(L, \delta) = \vee\{x_{(\alpha,\beta)} : A \wedge U \notin L, \text{for every } U \in N(x_{(\alpha,\beta)}, \delta)\}$, where $\delta = \tau_1 \vee \tau_2$ is an intuitionistic fuzzy topology generated by τ_1, τ_2 and $N(x_{(\alpha,\beta)}, \delta)$ is the set of all quasi-neighborhoods of $x_{(\alpha,\beta)}$ in δ .

Definition. 3.8 :

Let (X, τ_1, τ_2) be an $IFBTS$ and $A \in IFS(X)$. Then " intuitionistic fuzzy interior " and " intuitionistic fuzzy closure " of A with respect to τ_i , $i \in \{1,2\}$ are defined by : $\tau_i - int(A) = \vee\{G : G \text{ is a } \tau_i - IFOS, G \le A\}$, $\tau_i - cl(A) = \wedge\{K : K \text{ is a } \tau_i - IFCS , A \le K\}$.

Remark. 3.9 :

We denote $\delta - int(A)$, $\delta - cl(A)$ the interior and closure of A respectively , with respect to $\delta = \tau_1 \vee \tau_2$.

Proposition. 3.10 :

Let (X, τ_1, τ_2) be an $IFBTS$ and $A \in IFS(X)$. Then we have :

(i) $\tau_i - int(A) \le A$, $i \in \{1,2\}$.

(ii) $\tau_i - int(A)$ is a largest $\tau_i - IFOS$ contains in A .

(iii) A is a $\tau_i - IFOS$ if and only if $\tau_i - int(A) = A$.

(iv) $\tau_i - int(\tau_i - int(A)) = \tau_i - int(A)$.

(v) $A \leq \tau_i - cl(A)$, $i \in \{1,2\}$.

(vi) $\tau_i - cl(A)$ is a smallest $\tau_i - IFCS$ contains A .

(vii) A is a $\tau_i - IFCS$ if and only if $\tau_i - cl(A) = A$.

$(viii)$ $\tau_i - cl(\tau_i - cl(A)) = \tau_i - cl(A)$.

(ix) $[\, \tau_i - int(A)\,]^c = \tau_i - cl(A^c)$, $i \in \{1,2\}$.

(x) $[\, \tau_i - cl(A)\,]^c = \tau_i - int(A^c)$, $i \in \{1,2\}$.

Proof : Clearly . ■

Definition. 3.11 :

We define " $\star$ −intuitionistic fuzzy closure operator " for intuitionistic fuzzy bitopology $\tau_i^\star(L)$ as follows : $\tau_i - cl^\star(A) = A \vee A^\star(L, \tau_i)$ for every $A \in IFS(X)$. Also , $\tau_i^\star(L)$ is called an intuitionistic fuzzy bitopology generated by $\tau_i - cl^\star(A)$ and defined as : $\tau_i^\star(L) = \{\, A : \tau_i - cl^\star(A^c) = A^c \ ,\ i \in \{1,2\} \}$.

Note : $\tau_i^\star(L)$ finer than intuitionistic fuzzy bitopology τ_i , (i.e. $\tau_i \leq \tau_i^\star(L)$) .

Remark. 3.12 :

(i) If $L = \{0_\sim\} \Rightarrow A^\star(L, \tau_i) = \tau_i - cl(A)$, for any $A \in IFS(X)$

$\Rightarrow \tau_i - cl^\star(A) = A \vee A^\star(L, \tau_i) = A \vee \tau_i - cl(A) = \tau_i - cl(A) \Rightarrow \tau_i^\star(\{0_\sim\}) = \tau_i$, $i \in \{1,2\}$.

(ii) If $L = IFS(X) \Rightarrow A^\star(L, \tau_i) = 0_\sim$, for any $A \in IFS(X) \Rightarrow \tau_i - cl^\star(A) = A \vee A^\star(L, \tau_i) = A \vee 0_\sim = A$

$\Rightarrow \tau_i^\star(L)$ is the intuitionistic fuzzy discrete bitopology on X .

Theorem. 3.13 :

Let (X, τ_1, τ_2) be an $IFBTS$ and L , J be two $IFIs$ on X . Then for any $A, B \in IFS(X)$, we have :

(i) If $A \leq B \Rightarrow A^\star(L, \tau_i) \leq B^\star(L, \tau_i)$, $i \in \{1,2\}$.

(ii) If $L \leq J \Rightarrow A^\star(J, \tau_i) \leq A^\star(L, \tau_i)$, $i \in \{1,2\}$.

(iii) $A^\star(L, \tau_i) = \tau_i - cl(A^\star) \leq \tau_i - cl(A)$, $i \in \{1,2\}$.

(iv) $A^{\star\star}(L, \tau_i) \leq A^\star(L, \tau_i)$, $i \in \{1,2\}$.

(v) $(A \vee B)^\star(L, \tau_i) = A^\star(L, \tau_i) \vee B^\star(L, \tau_i)$, $i \in \{1,2\}$.

(vi) $(A \wedge B)^\star(L, \tau_i) = A^\star(L, \tau_i) \wedge B^\star(L, \tau_i)$, $i \in \{1,2\}$.

(vii) If $B \in L \Rightarrow (A \vee B)^\star(L, \tau_i) = A^\star(L, \tau_i)$, $i \in \{1,2\}$.

Proof :

$(\boldsymbol{i})$ Let $x_{(\alpha,\beta)} \in A^\star(L, \tau_i) \Rightarrow A \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

Since $A \leq B \Rightarrow B \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in B^\star(L, \tau_i)$. Therefore $A^\star(L, \tau_i) \leq B^\star(L, \tau_i)$.

$(\boldsymbol{ii})$ Let $x_{(\alpha,\beta)} \in A^\star(J, \tau_i) \Rightarrow A \wedge U \notin J$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

Since $L \leq J \Rightarrow A \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in A^\star(L, \tau_i)$. Thus $A^\star(J, \tau_i) \leq A^\star(L, \tau_i)$.

$(\boldsymbol{iii})$ Since $\{0_\sim\} \leq L$, for every IFI on $X \Rightarrow$ from (ii) and (Remark. 3.12:) , we get :

$A^\star(L, \tau_i) \leq A^\star(\{0_\sim\}, \tau_i) = \tau_i - cl(A)$, for every $A \in IFS(X) \Rightarrow A^\star(L, \tau_i) \leq \tau_i - cl(A)$.

Now , we will prove that : $A^\star(L, \tau_i) = \tau_i - cl(A^\star)$.

$(\Leftarrow)$ Suppose that $x_{1(\alpha,\beta)} \in \tau_i - cl(A^\star) \Rightarrow A^\star \wedge U \neq 0_\sim$, for every $U \in N\left(x_{1(\alpha,\beta)}, \tau_i\right)$

$\Rightarrow$ there exists $x_{2(\alpha,\beta)} \in A^\star \wedge U$, such that $A \wedge V \notin L$, for every $V \in N\left(x_{2(\alpha,\beta)}, \tau_i\right)$

Since $U \wedge V \in N\left(x_{2(\alpha,\beta)}, \tau_i\right) \Rightarrow A \wedge (U \wedge V) \notin L \Rightarrow A \wedge U \notin L$, for every $U \in N\left(x_{1(\alpha,\beta)}, \tau_i\right)$

$\Rightarrow\ x_{1(\alpha,\beta)} \in A^{\star}(L, \tau_i)$. Hence $\tau_i - cl(A^{\star}) \le A^{\star}(L, \tau_i) \ldots\ldots (1)$

$(\Rightarrow)$ From (Proposition. 3.10:(v)) , we get : $A^{\star}(L, \tau_i) \le \tau_i - cl(A^{\star}) \ldots\ldots (2)$

$\Rightarrow$ From (1) and (2) , we have : $A^{\star}(L, \tau_i) = \tau_i - cl(A^{\star})$.

$(\boldsymbol{iv})$ From (iii) , we get : $A^{\star\star}(L, \tau_i) = \tau_i - cl(A^{\star\star}) \le \tau_i - cl(A^{\star}) = A^{\star}(L, \tau_i) \Rightarrow A^{\star\star}(L, \tau_i) \le A^{\star}(L, \tau_i)$.

$(\boldsymbol{v})$ $(\Rightarrow)$ Suppose that $x_{(\alpha,\beta)} \in (A \vee B)^{\star}(L, \tau_i) \Rightarrow (A \vee B) \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

$\Rightarrow (A \wedge U) \vee (B \wedge U) \notin L \Rightarrow A \wedge U \notin L$ and $B \wedge U \in L$ or $A \wedge U \in L$ and $B \wedge U \notin L$

$\Rightarrow A \wedge U \notin L$ or $B \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

$\Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L, \tau_i)$ or $x_{(\alpha,\beta)} \in B^{\star}(L, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L, \tau_i) \vee B^{\star}(L, \tau_i) \Rightarrow (A \vee B)^{\star}(L, \tau_i) \le A^{\star}(L, \tau_i) \vee B^{\star}(L, \tau_i) \ldots\ldots (1)$

$(\Leftarrow)$ Since $A \le A \vee B$ and $B \le A \vee B \Rightarrow$ From (i), we get : $A^{\star}(L, \tau_i) \le (A \vee B)^{\star}(L, \tau_i)$ and $B^{\star}(L, \tau_i) \le (A \vee B)^{\star}(L, \tau_i)$

$\Rightarrow A^{\star}(L, \tau_i) \vee B^{\star}(L, \tau_i) \le (A \vee B)^{\star}(L, \tau_i) \ldots\ldots (2)$

$\Rightarrow$ From (1) and (2) , we get : $(A \vee B)^{\star}(L, \tau_i) = A^{\star}(L, \tau_i) \vee B^{\star}(L, \tau_i)$.

$(\boldsymbol{vi})$ $(\Rightarrow)$ Since $A \wedge B \le A$ and $A \wedge B \le B \Rightarrow$ from (i), we get :$(A \wedge B)^{\star}(L, \tau_i) \le A^{\star}(L, \tau_i)$ and $(A \wedge B)^{\star}(L, \tau_i) \le B^{\star}(L, \tau_i)$

$\Rightarrow (A \wedge B)^{\star}(L, \tau_i) \le A^{\star}(L, \tau_i) \wedge B^{\star}(L, \tau_i) \ldots\ldots (1)$

$(\Leftarrow)$ Suppose that $x_{(\alpha,\beta)} \in A^{\star}(L, \tau_i) \wedge B^{\star}(L, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L, \tau_i)$ and $x_{(\alpha,\beta)} \in B^{\star}(L, \tau_i)$

$\Rightarrow A \wedge U \notin L$ and $B \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i) \Rightarrow (A \wedge U) \wedge (B \wedge U) \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

$\Rightarrow (A \wedge B) \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in (A \wedge B)^{\star}(L, \tau_i)$

$\Rightarrow A^{\star}(L, \tau_i) \wedge B^{\star}(L, \tau_i) \le (A \wedge B)^{\star}(L, \tau_i) \ldots\ldots (2)$

$\Rightarrow$ From (1) and (2) , we get : $(A \wedge B)^{\star}(L, \tau_i) = A^{\star}(L, \tau_i) \wedge B^{\star}(L, \tau_i)$

$(\boldsymbol{vii})$ $(\Rightarrow)$ Suppose that $x_{(\alpha,\beta)} \in (A \vee B)^{\star}(L, \tau_i) \Rightarrow$ from (v), we have : $x_{(\alpha,\beta)} \in A^{\star}(L, \tau_i) \vee B^{\star}(L, \tau_i)$

$\Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L, \tau_i)$ or $x_{(\alpha,\beta)} \in B^{\star}(L, \tau_i)$

If $x_{(\alpha,\beta)} \in A^{\star}(L, \tau_i) \Rightarrow (A \vee B)^{\star}(L, \tau_i) \le A^{\star}(L, \tau_i) \ldots\ldots (*_1)$

If $x_{(\alpha,\beta)} \in B^{\star}(L, \tau_i) \Rightarrow B \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

Since $B \in L$, and $B \wedge U \notin L \Rightarrow U \notin L \Rightarrow A \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L, \tau_i)$

$\Rightarrow (A \vee B)^{\star}(L, \tau_i) \le A^{\star}(L, \tau_i) \ldots\ldots (*_2)$

$(\Leftarrow)$ Since $A \le A \vee B \Rightarrow$ from (i) , we have : $A^{\star}(L, \tau_i) \le (A \vee B)^{\star}(L, \tau_i) \ldots\ldots (**)$

$\Rightarrow$ From $(*_1)$, $(*_2)$ and $(**)$, we get : $(A \vee B)^{\star}(L, \tau_i) = A^{\star}(L, \tau_i)$. ■

Theorem. 3.14 :

Let (X, τ_1, τ_2, L) be an $IFIBTS$, $A, B \in IFS(X)$ and $\delta = \tau_1 \vee \tau_2$ be an intuitionistic fuzzy topology generated by τ_1, τ_2 . Then we have :

(i) $A^{\star}(L, \delta) \le A^{\star}(L, \tau_i)$, $i \in \{1,2\}$.

(ii) If $A \le B \Rightarrow A^{\star}(L, \delta) \le B^{\star}(L, \tau_i)$, $i \in \{1,2\}$.

(iii) $A^{\star}(L, \delta) \le \delta - cl(A) \le \tau_i - cl(A)$, $i \in \{1,2\}$.

(iv) $A^{\star\star}(L, \delta) \le A^{\star}(L, \tau_i)$, $i \in \{1,2\}$.

Proof :

$(\boldsymbol{i})$ Let $x_{(\alpha,\beta)} \notin A^{\star}(L, \tau_i) \Rightarrow$ there exists $U \in N(x_{(\alpha,\beta)}, \tau_i)$ such that $A \wedge U \in L$

Since $\tau_i \le \delta = \tau_1 \vee \tau_2 \Rightarrow N(x_{(\alpha,\beta)}, \tau_i) \le N(x_{(\alpha,\beta)}, \delta)$

$\Rightarrow U \in N(x_{(\alpha,\beta)}, \delta)$ and $A \wedge U \in L \Rightarrow x_{(\alpha,\beta)} \notin A^{\star}(L, \delta) \Rightarrow A^{\star}(L, \delta) \le A^{\star}(L, \tau_i)$.

$(\boldsymbol{ii})$ Let $x_{(\alpha,\beta)} \in A^{\star}(L, \delta) \Rightarrow A \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \delta)$

Since $A \le B \Rightarrow B \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)}, \delta) \Rightarrow x_{(\alpha,\beta)} \in B^{\star}(L, \delta) \Rightarrow A^{\star}(L, \delta) \le B^{\star}(L, \delta)$

$\Rightarrow$ from (i), we get : $B^{\star}(L,\delta) \leq B^{\star}(L,\tau_i)$, $i \in \{1,2\}$. Therefore $A^{\star}(L,\delta) \leq B^{\star}(L,\tau_i)$, $i \in \{1,2\}$.

$(\boldsymbol{iii})$ From (Theorem. 3.13:(iii)) , we have : $A^{\star}(L,\delta) \leq \delta - cl(A)$

Since $\tau_i \leq \delta = \tau_1 \vee \tau_2 \Rightarrow \delta - cl(A) \leq \tau_i - cl(A)$. Hence $A^{\star}(L,\delta) \leq \delta - cl(A) \leq \tau_i - cl(A)$

$(\boldsymbol{iv})$ From (Theorem. 3.13:(iv)) , we get : $A^{\star\star}(L,\delta) \leq A^{\star}(L,\delta)$

$\Rightarrow$ from (i) , we have : $A^{\star}(L,\delta) \leq A^{\star}(L,\tau_i)$, $i \in \{1,2\}$. Thus $A^{\star\star}(L,\delta) \leq A^{\star}(L,\tau_i)$, $i \in \{1,2\}$. ■

Lemma. 3.15 :

Let (X,τ_1,τ_2,L) be an $IFIBTS$, $A \in IFS(X)$. Then we have : $\tau_i - cl^{\star}(A) \leq \tau_i - cl(A)$, $i \in \{1,2\}$.

Proof :

From (Theorem.3.13: (iii)) , we get : $A^{\star}(L,\tau_i) \leq \tau_i - cl(A) \Rightarrow A \vee A^{\star}(L,\tau_i) \leq A \vee \tau_i - cl(A)$

Since $A \leq \tau_i - cl(A) \Rightarrow A \vee \tau_i - cl(A) = \tau_i - cl(A) \Rightarrow A \vee A^{\star}(L,\tau_i) \leq \tau_i - cl(A)$

Since $\tau_i - cl^{\star}(A) = A \vee A^{\star}(L,\tau_i) \Rightarrow \tau_i - cl^{\star}(A) \leq \tau_i - cl(A)$. ■

Theorem. 3.16 :

Let (X,τ_1,τ_2,L) be an $IFIBTS$, $A \in IFS(X)$ and $\tau_1 \leq \tau_2$. Then :

(i) $A^{\star}(L,\tau_2) \leq A^{\star}(L,\tau_1)$.

(ii) $\tau_1{}^{\star}(L) \leq \tau_2{}^{\star}(L)$.

(iii) $A^{\star}(L,\tau_1{}^{\star}(L)) \leq A^{\star}(L,\tau_2{}^{\star}(L))$.

Proof :

$(\boldsymbol{i})$ Suppose that $x_{(\alpha,\beta)} \notin A^{\star}(L,\tau_1) \Rightarrow$ there exists $U \in N(x_{(\alpha,\beta)},\tau_1)$ such that $A \wedge U \in L$.

Since $\tau_1 \leq \tau_2 \Rightarrow U \in N(x_{(\alpha,\beta)},\tau_2)$

Now , $U \in N(x_{(\alpha,\beta)},\tau_2)$ and $A \wedge U \in L \Rightarrow x_{(\alpha,\beta)} \notin A^{\star}(L,\tau_2)$. Therefore $A^{\star}(L,\tau_2) \leq A^{\star}(L,\tau_1)$.

$(\boldsymbol{ii})$ Let $A \in \tau_1{}^{\star}(L) \Rightarrow$ from (Definition. 3.11:) , we get : $\tau_1 - cl^{\star}(A^c) = A^c \Rightarrow A^c \vee (A^c)^{\star}(L,\tau_1) = A^c$

Now , $\tau_2 - cl^{\star}(A^c) = A^c \vee (A^c)^{\star}(L,\tau_2)$

$\Rightarrow$ From (i) , we have : $A^c \vee (A^c)^{\star}(L,\tau_2) \leq A^c \vee (A^c)^{\star}(L,\tau_1) = A^c \Rightarrow \tau_2 - cl^{\star}(A^c) \leq A^c$ ... ... (1)

Since $A^c \leq \tau_2 - cl^{\star}(A^c)$ ... ... (2)

$\Rightarrow$ From (1) and (2) , we get : $\tau_2 - cl^{\star}(A^c) = A^c \Rightarrow A \in \tau_2{}^{\star}(L)$. Thus $\tau_1{}^{\star}(L) \leq \tau_2{}^{\star}(L)$.

$(\boldsymbol{iii})$Suppose that $x_{(\alpha,\beta)} \in A^{\star}(L,\tau_1{}^{\star}(L)) \Rightarrow A \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)},\tau_1{}^{\star}(L))$

$\Rightarrow$ From (ii) , we have : $U \in N(x_{(\alpha,\beta)},\tau_2{}^{\star}(L)) \Rightarrow A \wedge U \notin L$, for every $U \in N(x_{(\alpha,\beta)},\tau_2{}^{\star}(L)) \Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L,\tau_2{}^{\star}(L))$

$\Rightarrow A^{\star}(L,\tau_1{}^{\star}(L)) \leq A^{\star}(L,\tau_2{}^{\star}(L))$. ■

Theorem. 3.17 :

Let L_1 and L_2 be two $IFIs$ on $IFBTS\,(X,\tau_1,\tau_2)$. Then we have :

(i) If $L_1 \leq L_2 \Rightarrow \tau_i{}^{\star}(L_1) \leq \tau_i{}^{\star}(L_2)$, $i \in \{1,2\}$.

(ii) $A^{\star}(L_1 \wedge L_2,\tau_i) = A^{\star}(L_1,\tau_i) \vee A^{\star}(L_2,\tau_i)$, $i \in \{1,2\}$.

(iii) $A^{\star}(L_1 \vee L_2,\tau_i) = A^{\star}(L_1,\tau_i) \wedge A^{\star}(L_2,\tau_i)$, $i \in \{1,2\}$.

(iv) $\tau_i{}^{\star}(L_1 \wedge L_2) = \tau_i{}^{\star}(L_1) \wedge \tau_i{}^{\star}(L_2)$, $i \in \{1,2\}$.

(v) $\tau_i{}^{\star}(L_1 \vee L_2) = \tau_i{}^{\star}(L_1) \vee \tau_i{}^{\star}(L_2)$, $i \in \{1,2\}$.

Proof :

$(\boldsymbol{i})$ Suppose that $A \in \tau_i{}^{\star}(L_1) \Rightarrow$ from (Definition. 3.11:) , we have : $\tau_i - cl^{\star}(A^c) = A^c$

$\Rightarrow A^c \vee (A^c)^{\star}(L_1, \tau_i) = A^c \Rightarrow (A^c)^{\star}(L_1, \tau_i) \leq A^c$

Since $L_1 \leq L_2 \Rightarrow$ from (Theorem. 3.13: (ii)) , we get : $(A^c)^{\star}(L_2, \tau_i) \leq (A^c)^{\star}(L_1, \tau_i)$

$\Rightarrow (A^c)^{\star}(L_2, \tau_i) \leq A^c \Rightarrow A^c \vee (A^c)^{\star}(L_2, \tau_i) = A^c \Rightarrow A \in \tau_i{}^{\star}(L_2)$. Thus $\tau_i{}^{\star}(L_1) \leq \tau_i{}^{\star}(L_2)$.

$(\boldsymbol{ii})(\Rightarrow)$ Let $x_{(\alpha,\beta)} \in A^{\star}(L_1 \wedge L_2, \tau_i) \Rightarrow A \wedge U \notin (L_1 \wedge L_2)$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

$\Rightarrow A \wedge U \notin L_1$ or $A \wedge U \notin L_2$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

$\Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L_1, \tau_i)$ or $x_{(\alpha,\beta)} \in A^{\star}(L_2, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L_1, \tau_i) \vee A^{\star}(L_2, \tau_i)$

$A^{\star}(L_1 \wedge L_2, \tau_i) \leq A^{\star}(L_1, \tau_i) \vee A^{\star}(L_2, \tau_i) \dots\dots (1)$

$(\Leftarrow)$ Since $L_1 \wedge L_2 \leq L_1$ and $L_1 \wedge L_2 \leq L_2 \Rightarrow$ From(Theorem. 3.13: (ii)) , we have :

$A^{\star}(L_1, \tau_i) \leq A^{\star}(L_1 \wedge L_2, \tau_i)$ and $A^{\star}(L_2, \tau_i) \leq A^{\star}(L_1 \wedge L_2, \tau_i) \Rightarrow A^{\star}(L_1, \tau_i) \vee A^{\star}(L_2, \tau_i) \leq A^{\star}(L_1 \wedge L_2, \tau_i) \dots\dots (2)$

$\Rightarrow$ From (1) and (2) $\Rightarrow A^{\star}(L_1 \wedge L_2, \tau_i) = A^{\star}(L_1, \tau_i) \vee A^{\star}(L_2, \tau_i)$.

$(\boldsymbol{iii})(\Rightarrow)$ Since $L_1 \leq L_1 \vee L_2$ and $L_2 \leq L_1 \vee L_2$

$\Rightarrow$ From (Theorem. 3.13: (ii)) , we have : $A^{\star}(L_1 \vee L_2, \tau_i) \leq A^{\star}(L_1, \tau_i)$ and $A^{\star}(L_1 \vee L_2, \tau_i) \leq A^{\star}(L_2, \tau_i)$.

$\Rightarrow A^{\star}(L_1 \vee L_2, \tau_i) \leq A^{\star}(L_1, \tau_i) \wedge A^{\star}(L_2, \tau_i) \dots\dots (1)$

$(\Leftarrow)$ Suppose that $x_{(\alpha,\beta)} \in A^{\star}(L_1, \tau_i) \wedge A^{\star}(L_2, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L_1, \tau_i)$ and $x_{(\alpha,\beta)} \in A^{\star}(L_2, \tau_i)$

$\Rightarrow A \wedge U \notin L_1$ and $A \wedge U \notin L_2$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i)$

$\Rightarrow A \wedge U \notin (L_1 \vee L_2)$, for every $U \in N(x_{(\alpha,\beta)}, \tau_i) \Rightarrow x_{(\alpha,\beta)} \in A^{\star}(L_1 \vee L_2, \tau_i)$

$\Rightarrow A^{\star}(L_1, \tau_i) \wedge A^{\star}(L_2, \tau_i) \leq A^{\star}(L_1 \vee L_2, \tau_i) \dots\dots (2)$

$\Rightarrow$ From (1) and (2) $\Rightarrow A^{\star}(L_1 \vee L_2, \tau_i) = A^{\star}(L_1, \tau_i) \wedge A^{\star}(L_2, \tau_i)$.

$(\boldsymbol{iv})$ Since $L_1 \wedge L_2 \leq L_1$ and $L_1 \wedge L_2 \leq L_2 \Rightarrow$ from (i) , we get : $\tau_i{}^{\star}(L_1 \wedge L_2) \leq \tau_i{}^{\star}(L_1)$ and $\tau_i{}^{\star}(L_1 \wedge L_2) \leq \tau_i{}^{\star}(L_2)$

$\Rightarrow \tau_i{}^{\star}(L_1 \wedge L_2) \leq \tau_i{}^{\star}(L_1) \wedge \tau_i{}^{\star}(L_2) \dots\dots (1)$

$(\Leftarrow)$ Suppose that $A \in [\tau_i{}^{\star}(L_1) \wedge \tau_i{}^{\star}(L_2)] \Rightarrow A \in \tau_i{}^{\star}(L_1)$ and $A \in \tau_i{}^{\star}(L_2)$

$\Rightarrow$ From (Definition. 3.11:) , we get : $A^c \vee (A^c)^{\star}(L_1, \tau_i) = A^c$ and $A^c \vee (A^c)^{\star}(L_2, \tau_i) = A^c$

$\Rightarrow [A^c \vee (A^c)^{\star}(L_1, \tau_i)] \vee [A^c \vee (A^c)^{\star}(L_2, \tau_i)] = A^c$

$\Rightarrow A^c \vee [(A^c)^{\star}(L_1, \tau_i) \vee (A^c)^{\star}(L_2, \tau_i)] = A^c \Rightarrow$ from (ii) , we get : $A^c \vee (A^c)^{\star}(L_1 \wedge L_2, \tau_i) = A^c \Rightarrow A \in \tau_i{}^{\star}(L_1 \wedge L_2) \dots\dots (2)$

$\Rightarrow$ From (1) and (2) , we get : $\tau_i{}^{\star}(L_1 \wedge L_2) = \tau_i{}^{\star}(L_1) \wedge \tau_i{}^{\star}(L_2)$.

$(\boldsymbol{v})(\Rightarrow)$ Suppose that $A \in \tau_i{}^{\star}(L_1 \vee L_2) \Rightarrow$ From (Definition.3.11:) , we get : $A^c \vee (A^c)^{\star}(L_1 \vee L_2, \tau_i) = A^c$

$\Rightarrow$ From (iii) , we have : $A^c \vee [(A^c)^{\star}(L_1, \tau_i) \wedge (A^c)^{\star}(L_2, \tau_i)] = A^c$

$\Rightarrow [A^c \vee (A^c)^{\star}(L_1, \tau_i)] \wedge [A^c \vee (A^c)^{\star}(L_2, \tau_i)] = A^c \Rightarrow A^c \vee (A^c)^{\star}(L_1, \tau_i) = A^c$ and $A^c \vee (A^c)^{\star}(L_2, \tau_i) = A^c$

$\Rightarrow$ From (Definition. 3.11:) , we get : $A \in \tau_i{}^{\star}(L_1)$ and $A \in \tau_i{}^{\star}(L_2)$

$\Rightarrow A \in [\tau_i{}^{\star}(L_1) \vee \tau_i{}^{\star}(L_2)] \Rightarrow \tau_i{}^{\star}(L_1 \vee L_2) \leq \tau_i{}^{\star}(L_1) \vee \tau_i{}^{\star}(L_2) \dots\dots (1)$

$(\Leftarrow)$ Since $L_1 \leq L_1 \vee L_2$ and $L_2 \leq L_1 \vee L_2 \Rightarrow$ from (i) , we get : $\tau_i{}^{\star}(L_1) \leq \tau_i{}^{\star}(L_1 \vee L_2)$ and $\tau_i{}^{\star}(L_2) \leq \tau_i{}^{\star}(L_1 \vee L_2)$

$\Rightarrow \tau_i{}^{\star}(L_1) \vee \tau_i{}^{\star}(L_2) \leq \tau_i{}^{\star}(L_1 \vee L_2) \dots\dots (2) \Rightarrow$ from (1) and (2) , we have : $\tau_i{}^{\star}(L_1 \vee L_2) = \tau_i{}^{\star}(L_1) \vee \tau_i{}^{\star}(L_2)$. ∎

Theorem. 3.18 :

(i) $A^{\star}(L_1 \vee L_2, \tau_i) = A^{\star}(L_1, \tau_i{}^{\star}(L_1)) \wedge A^{\star}(L_2, \tau_i{}^{\star}(L_2))$, $i \in \{1,2\}$.

(ii) $A^{\star}(L_1 \wedge L_2, \tau_i) = A^{\star}(L_1, \tau_i{}^{\star}(L_1)) \vee A^{\star}(L_2, \tau_i{}^{\star}(L_2))$, $i \in \{1,2\}$.

(iii) $\tau_i{}^{\star}(L_1 \vee L_2) = \tau_i{}^{\star\star}(L_1) \wedge \tau_i{}^{\star\star}(L_2)$, $i \in \{1,2\}$.

(iv) $\tau_i{}^{\star}(L_1 \wedge L_2) = \tau_i{}^{\star\star}(L_1) \vee \tau_i{}^{\star\star}(L_2)$, $i \in \{1,2\}$.

Proof :

$(\boldsymbol{i})$ $(\Rightarrow)$ Let $x_{(\alpha,\beta)} \notin [A^\star(L_1, \tau_i{}^\star(L_1)) \wedge A^\star(L_2, \tau_i{}^\star(L_2))]$ $\Rightarrow$ $x_{(\alpha,\beta)} \notin A^\star(L_1, \tau_i{}^\star(L_1))$ or $x_{(\alpha,\beta)} \notin A^\star(L_2, \tau_i{}^\star(L_2))$

$\Rightarrow$ $A \wedge U_1 \in L_1$, for some $U_1 \in N\left(x_{(\alpha,\beta)}, \tau_i{}^\star(L_1)\right)$ or $A \wedge U_2 \in L_2$, for some $U_2 \in N\left(x_{(\alpha,\beta)}, \tau_i{}^\star(L_2)\right)$

$\Rightarrow$ $(A \wedge U_1) \vee (A \wedge U_2) \in (L_1 \vee L_2)$ $\Rightarrow$ $A \wedge (U_1 \vee U_2) \in (L_1 \vee L_2)$

Since $U_1 \in N\left(x_{(\alpha,\beta)}, \tau_i{}^\star(L_1)\right)$ and $\tau_i \leq \tau_i{}^\star(L_1)$ $\Rightarrow$ $U_1 \in N(x_{(\alpha,\beta)}, \tau_i)$

Since $U_2 \in N\left(x_{(\alpha,\beta)}, \tau_i{}^\star(L_2)\right)$ and $\tau_i \leq \tau_i{}^\star(L_2)$ $\Rightarrow$ $U_2 \in N(x_{(\alpha,\beta)}, \tau_i)$

$\Rightarrow$ $(U_1 \vee U_2) \in N(x_{(\alpha,\beta)}, \tau_i)$ $\Rightarrow$ $A \wedge (U_1 \vee U_2) \in (L_1 \vee L_2)$, for some $(U_1 \vee U_2) \in N(x_{(\alpha,\beta)}, \tau_i)$

$\Rightarrow$ $x_{(\alpha,\beta)} \notin A^\star(L_1 \vee L_2, \tau_i)$ $\Rightarrow$ $A^\star(L_1 \vee L_2, \tau_i) \leq A^\star(L_1, \tau_i{}^\star(L_1)) \wedge A^\star(L_2, \tau_i{}^\star(L_2)) \ldots\ldots (1)$

$(\Leftarrow)$ From (Definition.3.11:) , we have : $\tau_i \leq \tau_i{}^\star(L_1)$ and $\tau_i \leq \tau_i{}^\star(L_2)$

$\Rightarrow$ From (Theorem.3.16:(i)) , we get : $A^\star(L_1, \tau_i{}^\star(L_1)) \leq A^\star(L_1, \tau_i)$ and $A^\star(L_2, \tau_i{}^\star(L_2)) \leq A^\star(L_2, \tau_i)$

$\Rightarrow$ $A^\star(L_1, \tau_i{}^\star(L_1)) \wedge A^\star(L_2, \tau_i{}^\star(L_2)) \leq A^\star(L_1, \tau_i) \wedge A^\star(L_2, \tau_i)$

$\Rightarrow$ From (Theorem.3.17: (iii)) , we have : $A^\star(L_1, \tau_i{}^\star(L_1)) \wedge A^\star(L_2, \tau_i{}^\star(L_2)) \leq A^\star(L_1 \vee L_2, \tau_i) \ldots\ldots (2)$

$\Rightarrow$ From (1) and (2) , we get : $A^\star(L_1 \vee L_2, \tau_i) = A^\star(L_1, \tau_i{}^\star(L_1)) \wedge A^\star(L_2, \tau_i{}^\star(L_2))$.

$(\boldsymbol{ii})$ Similarly as $(\boldsymbol{i})$.

$(\boldsymbol{iii})$ By using $(\boldsymbol{i})$, we get : $\tau_i{}^\star(L_1 \vee L_2) = \tau_i{}^{\star\star}(L_1) \wedge \tau_i{}^{\star\star}(L_2)$.

$(\boldsymbol{iv})$ By using $(\boldsymbol{ii})$, we get : $\tau_i{}^\star(L_1 \wedge L_2) = \tau_i{}^{\star\star}(L_1) \vee \tau_i{}^{\star\star}(L_2)$. ■

Corollary. 3.19 :

Let (X, τ_1, τ_2, L) be an $IFIBTS$. Then we have :

(i) $A^\star(L, \tau_i) = A^\star(L, \tau_i{}^\star(L))$, $i \in \{1,2\}$.

(ii) $\tau_i{}^\star(L) = \tau_i{}^{\star\star}(L)$, $i \in \{1,2\}$.

Proof :

By taking $L_1 = L_2 = L$ in above Theorem , we have the required result . ■

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