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Moments of order statistics from nonidentically Distributed Lomax, exponential Lomax and exponential Pareto Variables

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Abstract

In this paper, the probability density function and the cumulative distribution function of the r^{th} order statistic arising from independent nonidentically distributed (INID) Lomax, exponential Lomax and exponential Pareto variables are presented. The moments of order statistics from INID Lomax, exponential lomax and exponential Pareto were derived using the technique established by Barakat and Abdelkader. Also, numerical examples are given.

Keywords

Moments of order statistics, nonidentically distributed order statistics, Lomax distribution, exponential Lomax distribution, exponential Pareto distribution..

1. Introduction

Three techniques have been established in literature to compute moments of order statistics of independent nonidentically distributed random variables. The first technique is created by Balakrishnan (1994). This technique requires a basic relation between to probability density function (pdf) and the cumulative distribution function (cdf) and referred to as differential equation technique (DET). Many authors used to this technique to derive the moments of INID order statistics for continuous distributions (See, Childs and Balakrishnan (2006) and Mohie Elidin et al (2007)).
Barakat and Abdelkader (2000) established the second technique to compute the moments of order statistics from nonidentically distributed weibull variables and referred to as (BAT). This technique that the cdf of the distribution can be written in the form $F(x) = 1 - \theta(x)$, which is satisfied in this distribution. Many authors used to BAT technique to compute the moments of INID order statistics for several continuous distributions (more details, see Abdelkader (2004a) Abdelkader (2004b), Jamjoom (2006), Abdelkader (2010), Jamjoom and Al-Saiary (2010) and Jamjoom and Al-Saiary (2013)).
The third technique was developed by Jamjoom and Al-Saiary (2011) which is the moment generating function technique it depends on (BAT) and referred to as (M.G.F. BAT). The third technique was used by Al-Saiary (2015) to compute the moments of INID order statistics for standard type II generalized logistic variables.
In this paper the pdf and the cdf of the r^{th} order statistic arising from INID Lomax, exponential Lomax and exponential Pareto distributions are given in section 2. In section 3, we drive the moments of the r^{th} order statistics of INID random variables arising from Lomax, exponential Lomax and exponential Pareto using (BAT). Finally, some conclusions are addressed in section 4.

2. Nonidentical order statistics from Lomax, exponential Lomax and exponential Pareto distributions

Let $X_1, X_2, \ldots, X_n$ be independent random variables having cumulative distribution functions (cdfs) $F_1(x), \ldots, F_n(x)$ and probability density functions (pdfs) $f_1(x), f_2(x), \ldots, f_n(x)$, respectively. Let $x_{1:n} \le \ldots \le x_{n:n}$ denote the order statistics obtained by arranging the n $X_i s$ in increasing order of magnitude. Then the pdf of the r^{th} order statistic $X_{r:n}$, $(1 \le r \le n)$, can be written as:

$$f_{r:n}(x) = \frac{1}{(r-1)!\ (n-r)!} \sum_P \prod_{a=1}^{r-1} F_{i_a}(x)\ f_{i_r}(x) \prod_{c=r+1}^{n} [1-F_{i_c}(x)] \qquad (1)$$

Where $\sum_P$ denotes the summation overall n! permutations $(i_1, i_2, \ldots, i_n)$ of $(1,2,\ldots,n)$ (See Bapat and Beg (1989)). Put the previous pdf of the r^{th} order statistic $X_{r:n}$ in the form of permanent as:

$$f_{r:n}(x) = \frac{1}{(r-1)!\ (n-r)!} \text{ Per} \left[\underbrace{F(x)}_{r-1} \quad \underbrace{F(x)}_{1} \quad \underbrace{(1-F(x))}_{n-r} \right]$$

$$= \frac{1}{(r-1)!\ (n-r)!} \begin{vmatrix} F_1(x) & F_2(x) & \dots & F_n(x) \\ \vdots & \vdots & & \vdots \\ F_1(x) & F_2(x) & \dots & F_n(x) \\ f_1(x) & f_2(x) & \dots & f_n(x) \\ 1-F_1(x) & 1-F_2(x) & \dots & 1-F_n(x) \\ \vdots & \vdots & & \vdots \\ 1-F_1(x) & 1-F_2(x) & \dots & 1-F_n(x) \end{vmatrix}^{+} \begin{matrix} \}(r-1)\ \text{rows} \\ 1\ \text{row} \\ \}(n-r)\ \text{rows} \end{matrix} \quad (2)$$

and the cdf of the r^th^ order statistic $X_{r:n}$ can be written as:

$$F_{r:n}(x) = \sum_{i=r}^{n} \sum_{P_j} \prod_{a=1}^{j} F_{i_a}(x) \prod_{a=j+1}^{n} [1-F_{i_a}(x)] \quad (3)$$

Where P_j is all permutations of $i_1, i_2, \dots, i_n$ for 1, 2, …, n which satisfy
$i_1 < i_2 < \dots < i_j$ and $i_{j+1} < i_{j+2} < \dots < i_n$ and it is conveniently expressed in terms of permanent as

$$F_{r:n}(x) = \sum_{i=r}^{n} \frac{1}{i!\ (n-i)!} \text{Per} \left[\underbrace{F(x)}_{i} \quad \underbrace{(1-F(x))}_{n-i} \right] \quad , -\infty < x < \infty \quad (4)$$

Where F(x) and [1 – F(x)] denote the column vectors $[F_1(x), F_2(x), \dots, F_n(x)]'$ and $[1 - F_1(x), 1 - F_2(x), \dots, 1 - F_n(x)]'$ respectively. So the $F_{r:n}(x)$ can be rewritten as

$$F_{r:n}(x) = \sum_{i=r}^{n} \frac{1}{i!\ (n-i)!} \text{Per} \begin{vmatrix} F_1(x) & 1-F_1(x) \\ F_2(x) & 1-F_2(x) \\ \vdots & \vdots \\ \underbrace{F_n(x)}_{i} & \underbrace{1-F_n(x)}_{n-i} \end{vmatrix}^{+} \quad , -\infty < x < \infty \quad (5)$$

The cdf of the smallest order statistic (r = 1) $X_{1:n}$ is given by:

$$F_{1:n}(x) = \sum_{i=1}^{n} \frac{1}{i!\ (n-i)!} \begin{vmatrix} F_1(x) & 1-F_1(x) \\ F_2(x) & 1-F_2(x) \\ \vdots & \vdots \\ \underbrace{F_n(x)}_{i} & \underbrace{1-F_n(x)}_{n-i} \end{vmatrix}^{+} \quad , -\infty < x < \infty$$

and the cdf of the largest order statistic (r = n) $X_{n:n}$ (x) is given by:

$$F_{n:n}(x) = \frac{1}{n!} \begin{vmatrix} F_1(x) \\ \\ \underbrace{F_n(x)}_{n} \end{vmatrix}^{+} \quad , -\infty < x < \infty$$

Note that permanent is a square matrix, which is defined similar to the determinants except that all elements in the expansion have a positive sign (Minc (1987) and Balakrishnan (1994)).

In this paper, we consider the case where the random variables X_i, i = 1, 2, …, n are independent and nonidentical having Lomax (L), exponential Lomax (EL) and exponential Pareto (EP) distributions with cdfs $F_L(x)$, $F_{EL}(x)$ and $F_{EP}(x)$ and pdfs $f_L(x)$, $f_{EL}(x)$ and f_{EP} (x) respectively (more details see, Lemonte and Corderio (2013), Abdel Al-Kadim and Boshi (2013) and El-Bassiouny et al (2015)).

Where

$$F_{Li}(x) = 1 - (1 + \beta x)^{-\alpha_i},\ f_{Li}(x) = \beta\alpha_i (1 + \beta x)^{-(\alpha_i+1)},\ x > 0,\ \alpha_i,\ \beta > 0 \qquad (6)$$

$$F_{ELi}(x) = 1 - e^{-\lambda_i(\frac{\beta}{x+\beta})^{-\alpha}},\ f_{ELi}(x) = \frac{\alpha\lambda_i}{\beta}(\frac{\beta}{x+\beta})^{-(\alpha+1)} e^{-\lambda_i(\frac{\beta}{x+\beta})^{-\alpha}},\ x > -\beta,\ \alpha,\ \beta,\ \lambda_i > 0 \qquad (7)$$

$$F_{EPi}(x) = 1 - e^{-\lambda_i(\frac{x}{p})^{\theta}},\ f_{EPi}(x) = \frac{\lambda_i\theta}{p}(\frac{x}{p})^{\theta-1} e^{-\lambda_i(\frac{x}{p})^{\theta}},\ x > 0,\ \theta,\ p,\ \lambda_i > 0 \qquad (8)$$

3. The moment of the r^th order statistics arising from INID Lomax, exponential lomax and exponential Pareto random variables

In this section, we drive the moments of order statistics from INID random variables arising from L, EL and EP. We need to following theorem which is established by Barakat and Abdelkader (2003).

Theorem 1: Let $X_1, X_2, \ldots, X_n$ be independent nonidentically distribution random variables, The k^{th} moment of order statistics $\mu_{r:n}^{(k)}$ for $1 \le r \le n$ and $k = 1, 2, \ldots$ is given by:

$$\mu_{r:n}^{(k)} = \sum_{j=n-r+1}^{n} (-1)^{j-(n-r+1)} \binom{j-1}{n-r} I_j(k) \qquad (9)$$

Where

$$I_j(k) = \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum k \int_0^{\infty} x^{k-1} \prod_{t=1}^{j} G_{i_t}(x)\, dx \quad ,\ j = 1,2,\ldots,n, \qquad (10)$$

$G_{i_t}(x) = 1 - F_{i_t}(x)$ with $(i_1, i_2, \ldots, i_n)$ are permutations of $(1, 2, \ldots, n)$ for which $i_1 < i_2 < \ldots < i_n$. The proof of this Theorem 1 see to Barakat and Abdelkader (2003).

3.1. Moments of order statistics from INID Lomax random variables

The following Theorem gives an explicit expression for $I_j(k)$ when $X_1, X_2, \ldots, X_n$ are INID Lomax random variables.

Theorem 2: for $1 \le r \le n$ and $k = 1, 2, \ldots$

$$I_j(k) = \frac{k}{\beta^{k-1}} \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum B\,(k, \sum_{t=1}^{j} \alpha_{i_t} - k) \qquad (11)$$

Where B (. , .) is the complete beta function

Proof: on applying Theorem 1 and using eq. (10), we get

$$I_j(k) = \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum k \int_0^{\infty} x^{k-1} \prod_{t=1}^{j} [1 - F_{Li}(x)]$$

$$I_j(k) = \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum k \int_0^{\infty} x^{k-1} \prod_{t=1}^{j} (1+\beta x)^{-\alpha_{i_t}}$$

$$= \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum k \int_0^{\infty} \frac{x^{k-1}}{(1+\beta x)^{\sum_{t=1}^{j} \alpha_{i_t}}}$$

$$= \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum \frac{k}{\beta^{k-1}} \int_0^{\infty} \frac{(\beta x)^{k-1}}{(1+\beta x)^{\sum_{t=1}^{j} \alpha_{i_t}}} dx \qquad (12)$$

By using the known relation $B(p,q) = \int_0^\infty \frac{x^{p-1}}{(1+x)^{p+q}}$, then

$$\int_0^\infty \frac{(\beta x)^{k-1}}{(1+\beta x)^{\sum_{t=1}^{j}\alpha_{i_t}}} dx = B\left[k, \sum_{t=1}^{j} \alpha_{i_t} - k\right] \tag{13}$$

By substituting eq. (13) in eq. (12), we get Theorem 2.

Result 1: Substituting eq. (11) in eq. (9), The k^{th} moments of the r^{th} order statistics from INID Lomax distribution can be written as:

$$\mu_{r:n}^{(k)} = \sum_{j=n-r+1}^{n} (-1)^{j-(n-r+1)} \binom{j-1}{n-r} \frac{k}{\beta^{k-1}} \sum_{1\le i_1<i_2<\ldots<i_j\le n} \ldots \sum B\left[k, \sum_{t=1}^{j} \alpha_{i_t} - k\right] \tag{14}$$

Also, we can obtain the k^{th} moment of the smallest order statistic $x_{1:n}$ and the largest order statistic $x_{n:n}$ from INID Lomax as follows:

$$\mu_{1:n}^{(k)} = \frac{k}{\beta^{k-1}} B\left[k , \sum_{i=1}^{n} \alpha_{i_t} - k\right] \tag{15}$$

and

$$\mu_{n:n}^{(k)} = \sum_{j=1}^{n} (-1)^{j-1} \frac{k}{\beta^{k-1}} \sum_{1\le i_1<i_2<\ldots<i_j\le n} \ldots \sum B\left[k, \sum_{t=1}^{j} \alpha_{i_t} - k\right] \tag{16}$$

Example (1)

Let n = 3, $X_1 \sim$ Lomax ($\beta = 2$, $\alpha_1 = 3$), $X_2 \sim$ Lomax ($\beta = 2$, $\alpha_2 = 3$) and $X_3 \sim$ Lomax ($\beta = 2$, $\alpha_3 = 4$). When k = 1, then

Using eq. (15)

$$\mu_{1:3} = B\left(1, \sum_{i=1}^{n} \alpha_i - 1\right) = B\,(1, \alpha_1 + \alpha_2 + \alpha_3 - 1) = 0.1111$$

Using eq. (14)

$$\mu_{2:3} = \sum_{j=2}^{3} (-1)^{j-2} \binom{j-1}{1} I_j(1) = I_2(1) - 2\, I_3(1)$$

$= [B\,(1, \alpha_1 + \alpha_2 - 1) + B\,(1, \alpha_1 + \alpha_3 - 1) + B\,(1, \alpha_2 + \alpha_3 - 1)] - 2\,B\,(1, \alpha_1 + \alpha_2 + \alpha_3 - 1) = 0.3111$

and using eq. (16)

$$\mu_{3:3} = \sum_{j=1}^{3} (-1)^{j-1} I_j(1) = I_1(1) - I_2(1) + I_3(1) = 0.9111$$

Where $I_1(1) = B\,(1, \alpha_1 - 1) + B\,(1, \alpha_2 - 1) + B\,(1, \alpha_3 - 1)$

3.2. Moments of order statistic from INID exponential Lomax random variables

Theorem 3: for $1 \le r \le n$ and k = 1, 2, …,

$$I_j(k) = \sum_{1\le i_1<i_2<\ldots<i_j\le n} \ldots \sum \frac{k\,\beta^k}{\alpha} \sum_{L=0}^{k-1} (-1)^L \binom{k-1}{L} \frac{\Gamma\left(\frac{k-L}{\alpha}\right)}{\left(\sum_{t=1}^{j} \lambda_{i_t}\right)^{\left(\frac{k-L}{\alpha}\right)}} \tag{17}$$

Proof: By using Theorem 1 and eq. (10), then

$$I_j(k) = \sum_{1\le i_1<i_2<\ldots<i_j\le n} \ldots \sum k \int_{-\beta}^{\infty} x^{k-1} \prod_{t=1}^{j} [1 - F_{EL_i}(x)]\, dx$$

$$= \sum_{1\le i_1<i_2<\ldots<i_j\le n} \ldots \sum k \int_{-\beta}^{\infty} x^{k-1} e^{-\left(\frac{\beta}{x+\beta}\right)^{-\alpha} \sum_{t=1}^{j} \lambda_{i_t}} dx$$

Let, $y=\left(\frac{\beta}{x+\beta}\right)^{-\alpha}$, $y(-\beta)=0$ ، $y(\infty)=\infty$, $x=\beta(y^{\frac{1}{\alpha}}-1)$ and $dx=\frac{\beta}{\alpha}y^{\frac{1}{\alpha}-1}dy$, then

$$I_j(k)=\sum_{1\le i_1<i_2<\ldots<i_j\le n}\cdots\sum \frac{k\ \beta^k}{\alpha}\int_0^{\infty} y^{\frac{k-1}{\alpha}}(1-y^{-1/\alpha})^{k-1}\ e^{-y\sum_{t=1}^{j}\lambda_{i_t}}\ y^{\frac{1}{\alpha}-1}dy$$

$$=\sum_{1\le i_1<i_2<\ldots<i_j\le n}\cdots\sum \frac{k\ \beta^k}{\alpha}\sum_{L=0}^{k-1}(-1)^L\binom{k-1}{L}\int_0^{\infty} y^{\left(\frac{k-L}{\alpha}-1\right)}\ e^{-y\sum_{t=1}^{j}\lambda_{i_t}}\ dy$$

$$=\sum_{1\le i_1<i_2<\ldots<i_j\le n}\cdots\sum \frac{k\ \beta^k}{\alpha}\sum_{L=0}^{k-1}(-1)^L\binom{k-1}{L}\frac{\Gamma\left(\frac{k-L}{\alpha}\right)}{\left(\sum_{t=1}^{j}\lambda_{i_t}\right)^{\left(\frac{k-L}{\alpha}\right)}}$$

Result 2: Substituting eq.(17) in eq.(9), the k^th^ moments of the r^th^ order statistics from INID exponential Lomax can be written as:

$$\mu_{r:n}^{(k)}=\sum_{j=n-r+1}^{n}(-1)^{j-(n-r+1)}\binom{j-1}{n-r}\sum_{1\le i_1<i_2<\ldots<i_j\le n}\cdots\sum\frac{k\ \beta^k}{\alpha}\sum_{L=0}^{k-1}(-1)^L \times\frac{\Gamma\left(\frac{k-L}{\alpha}\right)}{\left(\sum_{t=1}^{j}\lambda_{i_t}\right)^{\left(\frac{k-L}{\alpha}\right)}} \qquad (18)$$

Also, the k^{th} moment of the smallest order statistic $x_{1:n}$ from INID exponential Lomax random variables can be written as:

$$\mu_{1:n}^{(k)}=\frac{k\ \beta^k}{\alpha}\sum_{L=0}^{k-1}(-1)^L\binom{k-1}{L}\frac{\Gamma\left(\frac{k-L}{\alpha}\right)}{\left(\sum_{i=1}^{n}\lambda_i\right)^{\left(\frac{k-L}{\alpha}\right)}} \qquad (19)$$

and the k^{th} moment of the largest order statistic $x_{n:n}$ from INID exponential Lomax random variables can be written as:

$$\mu_{n:n}^{(k)}=\sum_{j=1}^{n}(-1)^{j-1}\sum_{1\le i_1<i_2<\ldots<i_j\le n}\cdots\sum\frac{k\ \beta^k}{\alpha}\sum_{L=0}^{k-1}(-1)^L\binom{k-1}{L}\frac{\Gamma\left(\frac{k-L}{\alpha}\right)}{\left(\sum_{t=1}^{j}\lambda_{i_t}\right)^{\left(\frac{k-L}{\alpha}\right)}} \qquad (20)$$

Example (2)

Let n = 3, $X_1 \sim EL\ (\beta = 3, \alpha = 2, \lambda_1 = 1)$, $X_2 \sim EL\ (\beta = 3, \alpha = 2, \lambda_2 = 2)$ and $X_3 \sim EL\ (\beta = 3, \alpha = 2, \lambda_3 = 3)$. when k = 1, then

Using eq. (19)

$$\mu_{1:3}=\frac{2}{3}.\frac{\Gamma(0.5)}{\left(\sum_{i=1}^{3}\lambda_i\right)^{0.5}}=1.0853$$

Using eq. (18), $\mu_{2:3} = I_2(1) - 2\, I_3(1) = 1.8817$

Where

$$I_2(1)=\frac{2}{3}\Gamma(.5)\left[\frac{1}{(\lambda_1+\lambda_2)^{0.5}}+\frac{1}{(\lambda_1+\lambda_3)^{0.5}}+\frac{1}{(\lambda_2+\lambda_3)^{0.5}}\right]=4.0524 \text{ and}$$

$$I_3(1)=\frac{2}{3}.\frac{\Gamma(.5)}{(\lambda_1+\lambda_2+\lambda_3)^{0.5}}=1.0853$$

and using eq. (20), $\mu_{3:3} = I_1(1) - I_2(1) + I_3(1) = 3.1053$
Where

$$I_1(1) = \frac{3}{2}\Gamma(.5)\left[\frac{1}{\lambda_1^{0.5}} + \frac{1}{\lambda_2^{0.5}} + \frac{1}{\lambda_3^{0.5}}\right] = 6.0724$$

3.3. Moments of order statistics from INID exponential Pareto random variables

Theorem 4: for $1 \le r \le n$ and $k = 1, 2, \ldots$,

$$I_j(k) = \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum \frac{k\ p^k}{\theta} \frac{\Gamma\left(\frac{k}{\theta}\right)}{\left(\sum_{t=1}^{j} \lambda_{i_t}\right)^{\frac{k}{\theta}}} \tag{21}$$

Proof: on applying Theorem 1 and using eq.(10), Then

$$I_j(k) = \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum k \int_0^\infty x^{k-1} \prod_{t=1}^{j} [1 - F_{EP_i}(x)]\ dx$$

$$= \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum k \int_0^\infty x^{k-1} e^{-(x/p)^\theta \sum_{t=1}^{j} \lambda_{i_t}} dx$$

Let $y = \left(\frac{x}{p}\right)^\theta$, $y(0) = 0$, $y(\infty) = \infty$, $x = p\ y^{\frac{1}{\alpha}}$ and $dx = \frac{p}{\theta} y^{\frac{1}{\alpha}-1} dy$, then

$$I_j(k) = \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum \frac{k\ p^k}{\alpha} \int_0^\infty y^{\frac{k}{\theta}-1} e^{-y \sum_{t=1}^{j} \lambda_{i_t}} dy$$

$$= \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum \frac{k\ p^k}{\theta} \cdot \frac{\Gamma\left(\frac{k}{\theta}\right)}{\left(\sum_{t=1}^{j} \lambda_{i_t}\right)^{\frac{k}{\theta}}}$$

Result 3: Substituting eq.(21) in eq.(9), the k^{th} moments of the r^{th} order statistic from INID exponential Pareto random variables can be written as:

$$\mu_{r:n}^{(k)} = \sum_{j=n-r+1}^{n} (-1)^{j-(n-r+1)} \binom{j-1}{n-r} \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum \frac{k\ p^k}{\theta} \cdot \frac{\Gamma\left(\frac{k}{\theta}\right)}{\left(\sum_{t=1}^{j} \lambda_{i_t}\right)^{\frac{k}{\theta}}} \tag{22}$$

Also, we can obtain the k^{th} moment of $x_{1:n}$ and $x_{n:n}$ from INID exponential Pareto as follows:

$$\mu_{1:n}^{(k)} = \frac{k\ p^k}{\theta} \cdot \frac{\Gamma\left(\frac{k}{\theta}\right)}{\left(\sum_{i=1}^{n} \lambda_i\right)^{\frac{k}{\theta}}} \tag{23}$$

and

$$\mu_{n:n}^{(k)} = \sum_{j=1}^{n} (-1)^{j-1} \sum_{1 \le i_1 < i_2 < \ldots < i_j \le n} \cdots \sum \frac{k\ p^k}{\theta} \cdot \frac{\Gamma\left(\frac{k}{\theta}\right)}{\left(\sum_{t=1}^{j} \lambda_{i_t}\right)^{\frac{k}{\theta}}} \tag{24}$$

Example (3)

Let n = 5, $X_1 \sim EP$ (P = 2, θ = 1, λ_1 = 1), $X_2 \sim EP$ (P = 2, θ = 1, λ_2 = 2),

$X_3 \sim EP$ (P = 2, θ = 1, λ_3 = 3), $X_4 \sim EP$ (P = 2, θ = 1, λ_4 = 4) and

$X_5 \sim EP$ (P = 2, θ = 1, λ_5 = 5). when k = 1, then

Using eq. (23)

$$\mu_{1:5} = 2.\frac{\Gamma(1)}{\left(\sum_{i=1}^{5} \lambda_i\right)} = 0.1333$$

Using eq. (22)

$$\mu_{2:5} = \sum_{j=4}^{5} (-1)^{j-4} \binom{j-1}{3} I_j(1) = I_4(1) - 4 \; I_5(1) = 0.3119$$

Where

$$I_4(1) = 2\left[\frac{\Gamma(1)}{\lambda_1+\lambda_2+\lambda_3+\lambda_4} + \frac{\Gamma(1)}{\lambda_1+\lambda_2+\lambda_3+\lambda_5} + \frac{\Gamma(1)}{\lambda_1+\lambda_2+\lambda_4+\lambda_5} + \frac{\Gamma(1)}{\lambda_1+\lambda_3+\lambda_4+\lambda_5} + \frac{\Gamma(1)}{\lambda_2+\lambda_3+\lambda_4+\lambda_5}\right]$$

$= 0.8451$ and

$$I_5(1) = 2.\frac{\Gamma(1)}{\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5} = 0.1333$$

and using eq. (24)

$$\mu_{5:5} = \sum_{j=1}^{5} (1)^{j-1} I_j(1) = I_1(1) - I_2(1) + I_3(1) - I_4(1) + I_5(1) = 0.6969$$

Where for example. When n = 5, then

$$I_2(k) = \frac{kp^k}{\theta} \sum_{1 \le i_1 < i_2 \le 5} \cdots \sum \frac{\Gamma(\frac{k}{\theta})}{\left(\sum_{t=1}^{2} \lambda_{i_t}\right)^{(k/\theta)}}$$

$$= \frac{kp^k}{\theta}\Gamma(\frac{k}{\theta})\left[\frac{1}{(\lambda_1+\lambda_2)^{(k/\theta)}} + \frac{1}{(\lambda_1+\lambda_3)^{(k/\theta)}} + \frac{1}{(\lambda_1+\lambda_4)^{(k/\theta)}} + \frac{1}{(\lambda_1+\lambda_5)^{(k/\theta)}} + \frac{1}{(\lambda_2+\lambda_3)^{(k/\theta)}}\right.$$

$$\left. + \frac{1}{(\lambda_2+\lambda_4)^{(k/\theta)}} + \frac{1}{(\lambda_2+\lambda_5)^{(k/\theta)}} + \frac{1}{(\lambda_3+\lambda_4)^{(k/\theta)}} + \frac{1}{(\lambda_3+\lambda_5)^{(k/\theta)}} + \frac{1}{(\lambda_4+\lambda_5)^{(k/\theta)}}\right]$$

4. Conclusion

In this paper, exact moments of the r^{th} order statistic form independent and nonidentically distributed random variables for the Lomax, exponential Lomax and exponential Pareto distribution are derived using the BAT technique. Some numerical examples are presented to illustrate the theorems and results of this study.

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