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Transient behavior of $M^{[X]}/G/1$ Queueing Model With State Dependent Arrival, Balking and Bernoulli Vacation

Dr.G. Ayyappan
Professor
Pondicherry Engineering College,Pondicherry
ayyappanpec@hotmail.com
S.Shyamala
Assistant Professor
Arunai Engineering College,Thiruvannamalai
subramaniyanshyamala@gmail.com

Abstract

This paper analyzes single server queueing system wherein the arrival units are in batches following compound Poisson process with varying arrival rates for different states. The arrival rates differ for different states like idle state, busy state and vacation period. The server provides service one by one and after completion of a service, the server either takes a vacation of random length with probability p $(0 \leq p \leq 1)$ or continue to serve the next customer with probability 1-p, if any. Both service time and vacation time follow general distribution. In addition to this one of the customer impatience, balking is also discussed. We have obtained time dependent solution followed by the steady state analysis with some performance measures of our model.

Key Words

Batch arrival; Balking; state dependent; Bernoulli schedule; Probability generating function; Transient state; Steady state.

AMS Subject Classification : 60K25,60K30

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1.Introduction

The research on queueing theory has been tremendously increased due to the vast applications of queueing theory in the decision making processes in various fields like manufacturing, bio-sciences, population studies, health sectors, production systems, computer networks, communication systems, banking sectors etc. Most recently, research studies on queue with server vacations have been increased as an important area of queueing theory and have been studied extensively and successfully because of their various applications. When the system is empty, the server becomes idle and this idle time may be utilized by the server for being encaged for other purposes. During this time, the server is not available and such time is known as vacation. During the last three or four decades, queueing theorists are interested in studying queueing models with vacations immensely, because of their applicability and theoretical structures in real life situations such as manufacturing and production systems, computer and communication systems, service and distribution systems, etc. Levy(1975)studied about the utilization of idle of M/G/1 model. Later, the same author(1976) discussed Markovian multi server with server vacations.Doshi(1986) has given an extensive survey of vacation queues with various vacation policies. Keilson et al.(1987) studied non-Markovian queue with vacations. Choudhury(2002) analyzed batch arrival single server with vacation under single vacation policy.

The study of queueing models with customer impatience has also plays a significant role in queueing theory. Haight (1957) is the first who studied about the concept of customer behavior called balking, which deals the reluctance of a customer to join a queue upon arrival, since then a remarkable attention has been given on many queueing models with customer impatience. Madan (2012) analyzed the steady state batch arrival queueing system with balking and re service in a Vacation queue, having two types of heterogeneous services. Kumar (2012) have studied Markovian queueing model with balking and reneging. The study of queueing models with state dependent arrival rates with modified Bernoulli vacations is contributed by Madhu Jain et al. (2010). Singh et al.(2012) analyzed M/G/1 queueing model with state dependent arrival and deterministic vacation.

Transient state measures are very important to track down the functioning of the system at any instant of time. Takagi (1990) studied the time-dependent analysis of an M/G/1 vacation models with exhaustive service. Madan (1992) has obtained time dependent solution of M/G/1 model with compulsory vacation. Krishna kumar et al. (2002) have obtained time dependent solution of many of queueing models.

In this paper we present an analysis of the transient state behavior of a queueing system with different arrival rates for different states. After completion of each service, the server may go on vacation. The vacations follow a Bernoulli distribution, that is, after a service completion, the server may go for a vacation with probability p $(0 \le p \le 1)$ or may continue to serve the next customer,if any, with probability 1 - p. Also balking is included by assuming that the batch arrival units may decide not to join the system (balks) by estimating the duration of waiting time for a service to get completed or by witnessing the long length of the queue. The customers are served one by one on a first come - first served basis.

The rest of the paper has been organized as follows: in section 2, the mathematical description of our model has been found, in section 3,the transient solution of the system has been derived, in section 4, the steady state analysis has been discussed and in section 5 some concluding remarks has been given.

2. Mathematical Description of the Queueing Model

To describe the required queueing model, we assume the following .

- Let $\lambda c_i dt; i = 1, 2, 3...$ be the first order probability of arrival of 'i' customers in batches in the system during a short period of time(t,t+dt) where $0 \le c_i \le 1, \sum_{i=1}^{\infty} c_i = 1, \lambda > 0$ is the mean arrival rate of batches.

- There is a single server which provides service following a general(arbitrary) distribution with distribution function B(v) and density function b(v). Let $\mu(x)dx$ be the conditional probability density function of service completion during the interval (x,x+dx] given that the elapsed service time is x, so that

$$\mu(x) = \frac{b(x)}{1 - B(x)} \tag{1}$$

and therefore

$$b(v) = \mu(v) e^{-\int_0^v \mu(x)dx} \tag{2}$$

- We assume that $(1 - a_1); (0 \le a_1 \le 1)$ is the probability that an arriving customer balks during the period when the server is busy and $(1 - a_2); (0 \le a_2 \le 1)$ is the probability that an arriving customer balks during the period when the server is on vacation.

- As soon as a service is completed, the server may take a vacation of random length with probability p (or) he may stay in the system providing service with probability 1-p, where $0 \le p \le 1$.

- The vacation time of the server follows a general(arbitrary)with distribution function V(s)and the density function v(s). Let $\nu(x)$ dx be the conditional probability of a completion of a vacation during the interval (x,x+dx] given that the elapsed vacation time is x so that

$$\nu(x) = \frac{\upsilon(x)}{1 - V(x)} \tag{3}$$

and therefore

$$\upsilon(s) = \nu(s) e^{-\int_0^s \nu(x)dx} \tag{4}$$

- Various stochastic processes involved in the queueing system are assumed to be independent of each other.

3. Definitions and Equations governing the System

We let,

(i) $P_n(x,t)$ = Probability that at time 't' the server is active providing service and there are 'n' $(n \ge 0)$ customers in the queue excluding the one being served and the elapsed service time for this customer is x. Consequently $p_n(t)$ denotes the probability that at time 't' there are 'n' customers in the queue excluding the one customer in service irrespective of the value of x.

(ii) $V_n(x,t)$ = probability that at time 't', the server is on vacation with elapsed vacation time x, and there are 'n' $(n \ge 0)$ customers waiting in the queue for service. Consequently $V_n(x,t)$ denotes the probability that at time 't' there are 'n' customers in the queue and the server is on vacation irrespective of the value of x.

(iv) Q(t)= probability that at time 't' there are no customers in the system and the server is idle but available in the system.

(v) The customers are served according to the first come -first served queue discipline.

3.Transient Solution of the Model

The model is then, governed by the following set of differential-difference equations.

$$\frac{\partial}{\partial t} P_n(x,t) + \frac{\partial}{\partial x} P_n(x,t) + (\lambda_2 + \mu(x)) P_n(x,t) = \lambda_2 (1 - a_1) P_n(x,t) + a_1 \lambda_2 \sum_{i=1}^{n-1} c_i P_{n-i}(x,t); n \ge 1 \tag{5}$$

$$\frac{\partial}{\partial t} V_n(x,t) + \frac{\partial}{\partial x} V_n(x,t) + (\lambda_3 + \nu(x)) V_n(x,t) = \lambda_3 (1 - a_2) V_n(x,t) + a_2 \lambda_3 \sum_{i=1}^{n-1} c_i V_{n-i}(x,t); n \ge 1 \tag{6}$$

$$\frac{\partial}{\partial t} V_0(x,t) + \frac{\partial}{\partial x} V_0(x,t) + (\lambda_3 + \nu(x)) V_0(x,t) = \lambda_3 (1 - a_2) V_0(x,t) \tag{7}$$

$$\frac{d}{dt} Q(t) = -\lambda_1 Q(t) + \lambda_1 (1 - a_1) Q(t) + \int_0^\infty V_0(x,t) \nu(x) dx + (1 - p) \int_0^\infty P_0(x,t) \mu(x) dx \tag{8}$$

The above equations are to be solved subject to the following boundary conditions

$$P_n(0,t) = (1 - p) \int_0^\infty P_{n+1}(x,t) \mu(x) dx + \int_0^\infty V_{n+1}(x,t) \nu(x) dx + \lambda_1 a_1 c_{n+1} Q(t); n \ge 1 \tag{9}$$

$$V_n(0,t) = p \int_0^\infty P_n(x,t) \mu(x) dx; n \ge 0 \tag{10}$$

Assuming there are no customers in the system initially so that the server is idle.

$$V_0(0) = 0; V_n(0) = 0; Q(0) = 1; P_n(0) = 0, n = 0,1,2,... \tag{11}$$

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Now we define the probability generating functions as follows

$$P_q(x,z,t)=\sum_{n=0}^{\infty} z^n P_n(x,t) \tag{12}$$

$$P_q(z,t)=\sum_{n=0}^{\infty} z^n P_n(t) \tag{13}$$

$$V_q(x,z,t)=\sum_{n=0}^{\infty} z^n V_n(x,t) \tag{14}$$

$$V_q(z,t)=\sum_{n=0}^{\infty} z^n V_n(t) \tag{15}$$

$$C(z)=\sum_{n=1}^{\infty} c_n z^n \tag{16}$$

which are convergent inside the circle given by $|z|\le 1$ and define the Laplace transform of a function f(t) as

$$\overline{f}(s)=\int_0^\infty f(t)e^{-st}dt \tag{17}$$

Taking Laplace transforms of equations (5) to (8) and using the probability generating function defined above.

$$\frac{\partial}{\partial x}\overline{P}_n(x,s)+(s+\lambda_2 a_1+\mu(x))\overline{P}_n(x,s)=\lambda_2 a_1\sum_{i=1}^{n-1}\overline{P}_{n-i}(x,s); n\ge 1 \tag{18}$$

$$\frac{\partial}{\partial x}\overline{V}_n(x,s)+(s+\lambda_3 a_2+\nu(x))\overline{V}_n(x,s)=\lambda_3 a_2\sum_{i=1}^{n-1}\overline{V}_{n-i}(x,s); n\ge 0 \tag{19}$$

$$s\overline{Q}(s)=1-\lambda_1 a_1\overline{Q}(s)+\int_0^\infty \overline{V}_0(x,s)\nu(x)dx+(1-p)\int_0^\infty \overline{P}_0(x,s)\mu(x)dx \tag{20}$$

for the boundary conditions

$$\overline{P}_n(0,s)=(1-p)\int_0^\infty \overline{P}_{n+1}(x,s)\mu(x)dx+\int_0^\infty \overline{V}_{n+1}(x,s)\nu(x)dx+\lambda_1 a_1 c_{n+1}\overline{Q}(s); n\ge 1 \tag{21}$$

$$\overline{V}_n(0,s)=p\int_0^\infty \overline{P}_n(x,s)\mu(x)dx; n\ge 0 \tag{22}$$

multiply equation (18) by z^n implies

$$\frac{\partial}{\partial x}\overline{P}_q(x,z,s)+(s+\lambda_2 a_1(1-C(z))+\mu(x))\overline{P}_q(x,z,s)=0 \tag{23}$$

performing similar operations to equation (19)

$$\frac{\partial}{\partial x}\overline{V}_q(x,z,s)(s+\lambda_3 a_2(1-C(z))+\nu(x))\overline{V}_q(x,z,s)=0 \tag{24}$$

For the boundary conditions, we multiply equation (21) by z^{n+1}, sum over n from 1 to ∞ and use generating function defined above,we get

$$z\overline{P}_q(0,z,s)=(1-p)\int_0^\infty \overline{P}_q(x,z,s)\mu(x)dx+\int_0^\infty \overline{V}_q(x,z,s)\nu(x)dx$$

$$+\lambda_1 a_1 (C(z)-1)\bar{Q}(s)+(1-s\bar{Q}(s)) \tag{25}$$

Similarly multiply equation (22) by z^n and sum over n from 0 to ∞ and use generating function defined above

$$\bar{V}_q(0,z,s) = p\int_0^\infty \bar{P}_q(x,z,s)\mu(x)dx \tag{26}$$

Integrating equation(23) from 0 to x yields

$$\bar{P}_q(x,z,s) = \bar{P}_q(0,z,s)e^{-(s+\lambda_2 a_1(1-C(z)))x-\int_0^x \mu(t)dt} \tag{27}$$

where $\bar{P}_q(0,z,s)$ is given by equation(25)

Again integrating equation (27) by parts with respect to x yields

$$\bar{P}_q(z,s) = \bar{P}_q(0,z,s)\left[\frac{1-\bar{B}(s+\lambda_2 a_1(1-C(z)))}{s+\lambda_2 a_1(1-C(z))}\right] \tag{28}$$

where

$$\bar{B}(s+\lambda_2 a_1(1-C(z))) = \int_0^\infty e^{-(s+\lambda_2 a_1(1-C(z)))x} dB(x) \tag{29}$$

is Laplace - Stieltjes transform of the service time B(x).

Now multiplying both sides of equation (27) by $\mu(x)$ and integrating over x, we get

$$\int_0^\infty \bar{P}_q(x,z,s)\mu(x)dx = \bar{P}_q(0,z,s)\bar{B}(s+\lambda_2 a_1(1-C(z))) \tag{30}$$

Similarly solving equation (24)

$$\bar{V}_q(x,z,s) = \bar{V}_q(0,z,s)e^{-(s+\lambda_3 a_2(1-C(z)))x-\int_0^x \mu(t)dt} \tag{31}$$

Again integrating equation (31) by parts with respect to x yields

$$\bar{V}_q(z,s) = \bar{V}_q(0,z,s)\left[\frac{1-\bar{V}(s+\lambda_3 a_2(1-C(z)))}{s+\lambda_3 a_2(1-C(z))}\right] \tag{32}$$

Where

$$\bar{V}(s+\lambda_3 a_2(1-C(z))) = \int_0^\infty e^{-(s+\lambda_3 a_2(1-C(z)))x} dV(x) \tag{33}$$

is Laplace - Stieltjes transform of the vacation time V(x).

Now multiplying both sides of equation (31) by $\nu(x)$ and integrating over x, we get

$$\int_0^\infty \bar{V}_q(x,z,s)\nu(x)dx = \bar{V}_q(0,z,s)\bar{V}(s+\lambda_3 a_2(1-C(z))) \tag{34}$$

substituting by the value of $\bar{V}_q(0,z,s)$ from (26) in equation (34), we get

$$\int_0^\infty \bar{V}_q(x,z,s)\nu(x)dx = p\bar{P}_q(0,z,s)\bar{B}(s+\lambda_2 a_1(1-C(z)))\bar{V}(s+\lambda_3 a_2(1-C(z))) \tag{35}$$

Also equation (32) becomes

$$\bar{V}_q(z,s) = p\bar{P}_q(0,z,s)\bar{B}(s+\lambda_2 a_1(1-C(z)))\left[\frac{1-\bar{V}(s+\lambda_3 a_2(1-C(z)))}{s+\lambda_3 a_2(1-C(z))}\right] \quad (36)$$

Now using (30) and (35) in equation (25) and solving for $\bar{P}_q(0,z,s)$ we get

$$\bar{P}_q(0,z,s) = \frac{[(1-s\bar{Q}(s))+\lambda_1 a_1(C(z)-1)\bar{Q}(s)]}{Dr} \quad (37)$$

where

Dr = $z-(1-p)\bar{B}(s+\lambda_2 a_1(1-C(z)))-p\bar{B}(s+\lambda_2 a_1(1-C(z)))\bar{V}(s+\lambda_3 a_2(1-C(z)))$

substituting the value of $\bar{P}_q(0,z,s)$ from equation (37)in to equations (28)and (36)

$$\bar{P}_q(z,s) = \frac{[(1-s\bar{Q}(s))+\lambda_1 a_1(C(z)-1)\bar{Q}(s)]\left[\frac{1-\bar{B}(s+\lambda_2 a_1(1-C(z)))}{(s+\lambda_2 a_1(1-C(z)))}\right]}{Dr} \quad (38)$$

$$\bar{V}_q(z,s) = \frac{p[(1-s\bar{Q}(s))+\lambda_1 a_1(C(z)-1)\bar{Q}(s)]\left[\frac{1-\bar{V}((s+\lambda_3 a_2(1-C(z))))}{s+\lambda_3 a_2(1-C(z))}\right]}{Dr} \quad (39)$$

where Dr is given above

4.Steady State Solution

In this section we shall derive the steady state probability distribution for our queueing model. To define the steady state probabilities, suppress the argument't' where ever it appears in the time dependent analysis by using well known Tauberian property

$$Lt_{s\to 0} s\bar{f}(s) = Lt_{t\to\infty} f(t) \quad (40)$$

multiplying both sides of equation (38),(39) and applying equation(40) and simplifying, we get

$$P_q(z) = \frac{\left(\frac{\lambda_1}{\lambda_2}\right)Q[\bar{B}(\lambda_2 a_1(1-C(z)))-1]}{z-(1-p)\bar{B}(\lambda_2 a_1(1-C(z)))-p\bar{B}(\lambda_2 a_1(1-C(z)))\bar{V}((\lambda_3 a_2(1-C(z))))} \quad (41)$$

$$V_q(z) = \frac{p\left(\frac{\lambda_1 a_1}{\lambda_3 a_2}\right)Q\bar{B}(\lambda_2 a_1(1-C(z)))[\bar{V}((\lambda_3 a_2(1-C(z))))-1]}{z-(1-p)\bar{B}(\lambda_2 a_1(1-C(z)))-p\bar{B}(\lambda_2 a_1(1-C(z)))\bar{V}((\lambda_3 a_2(1-C(z))))} \quad (42)$$

Let $W_q(z)$ denotes the probability generating function of queue size irrespective of the state of the system. Then adding(41)and(42) we get

$$W_q(z) = P_q(z)+V_q(z) \quad (43)$$

$$W_q(z) = \frac{\frac{\lambda_1}{\lambda_2}Q[\bar{B}(\lambda_2 a_1(1-C(z)))-1]+p\frac{\lambda_1 a_1}{\lambda_3 a_2}Q\bar{B}(\lambda_2 a_1(1-C(z)))[\bar{V}((\lambda_3 a_2(1-C(z))))-1]}{z-(1-p)\bar{B}(\lambda_2 a_1(1-C(z)))-p\bar{B}(\lambda_2 a_1(1-C(z)))\bar{V}((\lambda_3 a_2(1-C(z))))} \quad (44)$$

In order to obtain Q, we use the normalization condition

$$W_q(1)+Q=1 \quad (45)$$

we see that z=1, $W_q(z)$ is indeterminate of the form 0/0. We apply L'Hospitals rule in equation (44)

$$W_q(1)=\frac{\lambda_1 a_1 QE(I)[E(S)+pE(V)]}{1-E(I)[\lambda_2 a_1 E(S)+p\lambda_3 a_2 E(V)]} \quad (46)$$

where $\bar{B}(0)=1, \bar{V}(0)=1, -B'(0)=E[B]$ the mean service time and $-V'(0)=E[V]$ the mean vacation time. Using equation (46) in equation (45)

$$Q=\frac{1-E(I)[\lambda_2 a_1 E(S)+P\lambda_3 a_2 E(V)]}{1+E(I)[(\lambda_1-\lambda_2)a_1 E(S)+p(\lambda_1 a_1-\lambda_3 a_2)E(V)]} \quad (47)$$

and the the utilization factor ρ of the system is given by

$$\rho=1-Q \quad (48)$$

where $\rho<1$ is the stability condition under which the steady state exists, equation (47) gives the probability that the server is idle. Substitute Q from equation(47) in equation (44) $W_q(z)$ have been completely and explicitly determined which is the the probability generating function of the queue size.

The average queue size.

Let L_q denote the mean number of customers in the queue under the steady state, then

$$L_q=\frac{d}{dz}W_q(z)|_{z=1}$$

since this formula gives 0/0 form, then we write $W_q(z)=\frac{N(z)}{D(z)}$ where N(z) and D(z) are the numerator and denominator of the right hand side of equation (49) respectively , then we use

$$L_q=\frac{D'(1)N''(1)-N'(1)D''(1)}{2[D'(1)]^2} \quad (49)$$

where primes and double primes in equation (49) denote first and second derivation at z=1 respectively. Carrying out the derivatives at z=1 , we have

$$N'(1)=-\lambda_1 a_1 QE(I)[E(S)+pE(V)] \quad (50)$$

$$N''(1)=-Q\lambda_1[E(I)]^2\left\{\lambda_2 a_1^2 E(S^2)+2pa_1 a_2\lambda_2 E(S)E(V)+p\lambda_3 a_2^2 E(V^2)\right\}$$
$$+Q\lambda_1 E(I(I-1))a_1[E(S)+pE(V)] \quad (51)$$

$$D'(1)=1-E(I)[\lambda_2 a_1 E(S)+p\lambda_3 a_2 E(V)] \quad (52)$$

$$D''(1)=-[E(I)]^2\left\{\lambda_2^2 a_1^2 E(S^2)+P\lambda_3^2 a_2^2 E(V^2)+2\lambda_2\lambda_3 a_1 a_2 pE(S)E(V)\right\}$$
$$-E(I(I-1))[\lambda_2 a_1 E(S)+p\lambda_3 a_2 E(V)] \quad (53)$$

where $E(S^2)$ and $E(V^2)$ is the second moment of the vacation time and Q has been found in equation (47). Then if we substitute the values of $N'(1), N''(1), D'(1)$ and $D''(1)$ from equations(50), (51), (52) and (53) into (49) equation we obtain L_q in a closed form.

Mean waiting time of a customer could be found by using Little formula.

$$W_q = \frac{L_q}{\lambda_{eff}};\lambda_{eff} = \lambda_1 Q + \lambda_2 P_q(1) + \lambda_3 V_q(1) \qquad (54)$$

Conclusion

We have analyzed batch arrival with different arrival rates for different states like idle state, busy state and vacation period which reflects many real life situations and it is useful for design and development stages in communication, computer networks and manufacturing field. Also, we have considered one of the customer impatience, balking which can also be seen in most of real life problems. We have obtained the transient analysis followed by the steady state behavior for getting the stability condition for the prescribed queueing model. the performance measures like average number of customer, average waiting time of a customer have been obtained.

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