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Intuitionistic Fuzzy Dot BCK/BCI – Algebras

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ABSTRACT

In this paper we apply the concept of intuitionistic fuzzy set to dot BCK-sub algebra. The notion of an intuitionistic fuzzy dot BCK-sub algebra is introduced and some interesting properties are investigated. Then we study the homomorphism between intuitionistic fuzzy dot BCK- subalgebras.

Keywords

Fuzzy sets; intuitionistic fuzzy sets; BCK-algebra; intuitionistic fuzzy dot BCK-sub algebra; homomorphism.

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INTRODUCTION

The study of BCK-algebras was initiated by Imai and Iseki in 1966 as a generalization of set-theoretic difference and proportional calculus. In the same year Iseki introduced BCI – algebras as a super class of the class of BCK-algebras. In particular BCK/BCI – algebras are non-classical logic algebras and they are algebraic formulations of BCK-system. The concept of intuitionistic fuzzy set was introduced by K.T.Atanassov[1], as a generalization of the notion of fuzzy set. In this paper, we introduced the concept of intuitionistic fuzzy dot BCK-sub algebras and study this structure. We state and prove some theorem in intuitionistic fuzzy dot BCK-sub algebras. Also we introduce the concept of homomorphism in intuitionistic fuzzy dot BCK-sub algebras and established some results.

Preliminaries: In this section, we first review some definitions and properties which will be used in the sequel.

1.1 **Definition [1]:** Let X be a non-empty set. A fuzzy subset A of X is a function

$$A: X \to [0,1].$$

1.2 **Definition [1]:** An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form

$$A = \{< x, \mu_A(x), \lambda_A(x) > | x \in X\}$$

where $\mu_A: X \to [0,1]$ and $\lambda_A: X \to [0,1]$ defined the degree of membership and the degree of non-membership of the element $x \in X$ respectively, and

$$0 \le \mu_A(x) + \lambda_A(x) \le 1 \text{ for all } x \in X.$$

1.3 Definition [1]: For every two intuitionistic fuzzy sets $A =< x, \lambda_A, \lambda_A > and$ $B =< x, \mu_B, \lambda_B >$ in X, define the following operations.

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\lambda_A(x) \ge \lambda_B(x)$ $forall$ $x \in X.$

(ii) A=B if and only if $A \subseteq B$ and $B \subseteq A,$

(iii) $A^c = \{< x, \lambda_A(x), \mu_A(x) > | x \in X\},$

(iv) $A \cap B = \{< x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\} > | x \in X\},$

(v) $A \cup B = \{< x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\} > | x \in X\},$

(vi) $\blacksquare A = \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X\},$

(vii) $\Delta A = \{< x, 1 - \lambda_A(x), \lambda_A(x) > | x \in X\},$

1.4 Definition [3]: A non-empty set X with a constant 0 and a binary operation $*$ is called a BCC algebra if it satisfies the following conditions.

(i) $((x * y) * (z * y)) * (x * z) = 0$

(ii) $x * x = 0$

(iii) $0 * x = 0$

(iv) $x * 0 = x$

(v) $x * y = 0$ and $y * x = 0$ $then$ $x = y$ for all $x, y, z \in X.$

1.5 Definition [3]: A BCC algebra X is said to be BCK algebra if

$$(x * y) * z = (x * z) * y \text{ for all } x, y, z \in X.$$

1.6 Definition: A nonempty subset S of a BCK algebra X is called a sub algebra of X if it is closed under the BCK operation.

1.7 Definition [2]: A mapping $f: X \to Y$ of BCK –algebra is called a homomorphism if

$$f(x * y) = f(x) * f(y) \text{ for all } x, y \in X.$$

1.8 Definition [4]: Let X be a BCK-algebra. An intuitionistic fuzzy subset A of X is said to be an intuitionistic fuzzy BCK/BCI-sub algebra if

(i) $\mu_A(x*y) \geq \min\{\mu_A(x), \mu_A(y)\}$

(ii) $\lambda_A(x*y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$ $for\ all\ x, y \in X.$

For the sake of simplicity, we just write $A = <\mu_A, \lambda_A>$ instead of

$$A = \{<x, \mu_A(x), \lambda_A(x)> | x \in X\}.$$

2. Results on intuitionistic fuzzy dot BCK/BCI-sub algebra

2.1 Definition: Let X be a BCK-algebra. An intuitionistic fuzzy subset A of X is said to be an intuitionistic fuzzy dot(IFD)**[5]** BCK/BCI-sub algebra if

(i) $\mu_A(x*y) \geq \mu_A(x).\mu_A(y)$

(ii) $\lambda_A(x*y) \leq \lambda_A(x) + \lambda_A(y) \leq 1$ $for\ all\ x, y \in X.$

2.2 Example: Let X = {0, a, b, c } be a set with the following Cayley table

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then $(X, *, 0)$ is a BCK – algebra.

Define an IFS $A = <\mu_A, \lambda_A>$ in X by

$\mu_A(0) = 0.8, \mu_A(a) = 0.5, \mu_A(b) = \mu_A(c) = 0.3,$

$\lambda_A(0) = 0.1, (a) = 0.3\ \ and\ \ \lambda_A(b) = \lambda_A(c) = 0.4.$

Then $A = <\mu_A, \lambda_A>$ is intuitionistic fuzzy dot BCK-subalgebra of X.

2.3 Theorem: If A = $<\mu_A, \lambda_A>$ is a IFD BCK-subalgebra of X, then for all $x \in X$

$\mu_A(0) \geq (\mu_A(x))^2$ and

$\lambda_A(0) \leq 2\lambda_A(x)$

Proof: For all $x \in X$, we have $x * x = 0$

Now, $\mu_A(0) = \mu_A(x*x)$

$\geq \mu_A(x).\mu_A(x)$

$= (\mu_A(x))^2$

Hence $\mu_A(0) \geq (\mu_A(x))^2$

Also, $\lambda_A(0) = \lambda_A(x*x)$

$$\leq \lambda_A(x) + \lambda_A(x)$$
$$= 2\lambda_A(x)$$

Hence $\lambda_A(0) \leq 2\lambda_A(x)$

2.4 Theorem: Let A be a IFD BCK – subalgebra of X. If there exists a sequence $\{x_n\}$ in X, such that $\lim_{n\to\infty}(\mu_A(x_n))^2 = 1$ and $\lim_{n\to\infty}(2\lambda_A(x_n)) = 0$ Then $\mu_A(0) = 1$ and $\lambda_A(0) = 0$.

Proof: By theorem 2.3, we have

$$\mu_A(0) \geq (\mu_A(x))^2 \quad \text{for all } x \in X.$$
$$\mu_A(0) \geq (\mu_A(x_n))^2 \quad \text{for every positive integer n}$$

Therefore, $1 \geq \mu_A(0) \geq \lim_{n\to\infty}\mu_A(x_n)^2 = 1$

Which implies that $\mu_A(0) = 1$

Also, $\lambda_A(0) \leq 2\lambda_A(x)$ for all $x \in X$.

$$\lambda_A(0) \leq 2\lambda_A(x_n) \quad \text{for every positive integer n}$$

Therefore, $0 \leq \lambda_A(0) \leq \lim_{n\to\infty}(2\lambda_A(x_n)) = 0$

Which implies that $\lambda_A(0) = 0$.

2.5 Theorem: Let $A =< \mu_A, \lambda_A >$ and $B =< \mu_B, \lambda_B >$ are intuitionistic fuzzy dot BCK – subalgebras of X. Then $A \cap B$ is a intuitionistic fuzzy dot BCK–subalgebra of X.

Proof: Let $x, y \in A \cap B$

Then $x, y \in A \text{ and } B$

Since A and B are intuitionistic fuzzy dot BCK – subalgebras of X, we have

$$(\mu_A \cap \mu_B)(x * y) = \min\{\mu_A(x * y), \mu_B(x * y)\}$$
$$\geq \min\{\mu_A(x).\mu_A(y), \mu_B(x).\mu_B(y)\}$$
$$\geq (\min\{\mu_A(x).\mu_B(x)\}).(\min\{\mu_A(y).\mu_B(y)\})$$
$$= (\mu_A \cap \mu_B)(x).(\mu_A \cap \mu_B)(y)$$

Therefore, $(\mu_A \cap \mu_B)(x * y) \geq (\mu_A \cap \mu_B)(x).(\mu_A \cap \mu_B)(y)$

Also, $(\lambda_A \cap \lambda_B)(x * y) = \max\{\lambda_A(x * y), \lambda_B(x * y)\}$

$$\leq \max\{\lambda_A(x) + \lambda_A(y), \lambda_B(x) + \lambda_B(y)\}$$
$$\leq (\max\{\lambda_A(x), \lambda_B(x)\}) + (\max\{\lambda_A(y), \lambda_B(y)\})$$
$$= (\lambda_A \cap \lambda_B)(x) + (\lambda_A \cap \lambda_B)(y)$$

Therefore, $(\lambda_A \cap \lambda_B)(x * y) \leq (\lambda_A \cap \lambda_B)(x) + (\lambda_A \cap \lambda_B)(y)$

Hence $A \cap B$ is an intuitionistic fuzzy dot BCK-subalgebra of X.

2.6 Corollary: If $\{A_i | i \in N\}$ be a family of intuitionistic fuzzy dot BCK-subalgebra of X then $\bigcap_{i\in N} A_i$ is also an intuitionistic fuzzy dot BCK-subalgebra of X.

2.7 Theorem: If $A =< \mu_A, \lambda_A >$ is IFD BCK –subalgebra of X then $\blacksquare A$ is also IFD BCK- subalgebra of X.

Proof: It is sufficient to show that $1 - \mu_A(x)$ satisfies condition (ii) in definition 2.1.

For $x, y \in X$,

$$(1-\mu_A)(x*y) = 1-\mu_A(x*y)$$
$$\leq 1-(\mu_A(x).\mu_A(y))$$
$$\leq (1-\mu_A(x)) + (1-\mu_A(y))$$
$$= (1-\mu_A)(x) + (1-\mu_A)(y)$$

Therefore, $(1-\mu_A)(x*y) \leq (1-\mu_A)(x) + (1-\mu_A)(y)$.

Hence ■$A = < \mu_A, 1-\mu_A >$ is IFD BCK-subalgebra of X.

2.8 Theorem: Let $A = < \mu_A, \lambda_A >$ be an IFD BCK-subalgebra of X. Then $U(\mu_A;1) = \{x \in X | \mu_A(x) = 1\}$ and $L(\lambda_A;0) = \{x \in X | \lambda_A(x) = 0\}$ are either empty or subalgebras of X.

Proof: Let $x, y \in U(\mu_A;1)$

Then $\mu_A(x*y) \geq \mu_A(x).\mu_A(y)$

$\mu_A(x*y) \geq 1$

Which implies that $\mu_A(x*y) = 1$.

Therefore, $x*y \in U(\mu_A;1)$

Hence $U(\mu_A;1)$ is a subalgebra of X.

Also, let $x, y \in L(\lambda_A;0)$

Then $\lambda_A(x*y) \leq \lambda_A(x) + \lambda_A(y)$

$\lambda_A(x*y) \leq 0$

Which implies that $\lambda_A(x*y) = 0$.

Therefore, $x*y \in L(\lambda_A;0)$

Hence $L(\lambda_A;0)$ is a subalgebra of X.

3. IFD BCK-subalgebra of X under homomorphism:

3.1 Theorem: Let $f: X \to Y$ be a one to one function. Then the homomorphic image of an intuitionistic fuzzy dot BCK-subalgebra of X is an intuitionistic fuzzy dot BCK-subalgebra.

Proof: Let $f: X \to Y$ be a BCK homomorphism from X into Y and B = f(A), where A is intuitionistic fuzzy dot BCK-subalgebra of X.

We have to prove that B is IFD BCK-subalgebra of Y.

Now, for f(x),f(y) in Y,

$$\mu_B(f(x)*f(y)) = \mu_B(f(x*y))$$
$$= \mu_A(x*y)$$
$$\geq \mu_A(x).\mu_A(y)$$
$$= \mu_B(f(x)).\mu_B(f(y))$$

Which implies that $\mu_B(f(x)*f(y)) \geq \mu_B(f(x)).\mu_B(f(y))$

Also, $\lambda_B(f(x)*f(y)) = \lambda_B(f(x*y))$

$$= \lambda_A(x * y)$$
$$\leq \lambda_A(x) + \lambda_A(y)$$
$$= \lambda_B(f(x)) + \lambda_B(f(y))$$

Which implies that $\lambda_B(f(x) * f(y)) \leq \lambda_B(f(x)) + \lambda_B(f(y))$

Hence B is an IFD BCK-subalgebra of Y.

3.2 Theorem: (The homomorphic preimage of an IFD BCK – subalgebra is an IFD BCK-subalgebra.

Proof: Let $f: X \rightarrow Y$ be a BCK homomorphism from X into Y and let B = f(A) where B is an IFD BCK-subalgebra of X. We have to prove that A is IFD BCK-subalgebra ofX.

Nov for $x, y \in X$,

$$\mu_A(x * y) = \mu_B(f(x * y))$$
$$= \mu_B(f(x) * f(y))$$
$$\geq \mu_B(f(x)).\mu_B(f(y))$$
$$= \mu_A(x).\mu_A(y)$$

Which implies that $\mu_A(x * y) \geq \mu_A(x).\mu_A(y)$

Also, $\lambda_A(x * y) = \lambda_B(f(x * y))$
$$= \lambda_B(f(x) * f(y))$$
$$\leq \lambda_B(f(x)) + \lambda_B(f(y))$$
$$= \lambda_A(x)+\lambda_A(y)$$

Which implies that $\lambda_A(x * y) \leq \lambda_A(x)+\lambda_A(y)$

Hence, A is IFD BCK-subalgebra of X.

Conclusion:

In the present paper, we have introduced the concept of intuitionistic fuzzy dot sub algebras of BCK/BCI-algebras and investigated some of their useful properties. In our opinion, these definitions and important results can be extended to some other fuzzy algebraic systems.

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