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COINCIDENCE AND COMMON FIXED POINT THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACES

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Abstract: In this paper, as an application of occasionally weakly compatible mappings, we prove common fixed point theorems under contractive conditions that extend the scope of the study of common fixed point theorems from the class of weakly compatible mappings to a wider class of mappings. Our results generalize the results of Pant *et al.* [9] for four self maps in intuitionistic fuzzy metric space.

Key words: Occasionally Weakly Compatible Maps, Weakly Compatible Maps, Intuitionistic fuzzy metric space.

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1. Introduction:

Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [13]. In 2004, Park [10] introduced and discussed a notion of intuitionistic fuzzy metric spaces, which is based both on the idea of intuitionistic fuzzy sets and the concept of a fuzzy metric space given by George *et al.* [4]. Using the idea of intuitionistic fuzzy sets, Alaca et al. [1] defined the notion of IFM-space as Park [10] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil et al. [7]. In 2006, Turkoglu et al. [12] studied the notion of compatible mappings in intuitionistic fuzzy metric space.

In 1986, Jungck [5] introduced the notion of compatible maps for a pair of self mappings. However, the study of common fixed points of non-compatible maps is also very interesting and this condition has further been weakened by introducing the notion of weakly compatible mappings by Jungck et al. [6]. The concept of weakly compatible mappings is most general as each pair of compatible mappings is weakly compatible but the reverse is not true.

Al-Thagafi et al. [2] introduced the notion of occasionally weakly compatible mappings which is more general than the concept of weakly compatible maps.

In this paper, as an application of occasionally weakly compatible mappings, we prove common fixed point theorems under contractive conditions that extend the scope of the study of common fixed point theorems from the class of weakly compatible mappings to a wider class of mappings. Our results generalize the results of Pant et al. [9] for four self maps in intuitionistic fuzzy metric space.

2. Preliminaries.

The concepts of triangular norms (*t*-norms) and triangular conorms (*t*-conorms) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [8] in study of statistical metric spaces.

Definition 1. [11] A binary operation $*$: $[0,1]\times[0,1] \longrightarrow [0,1]$ is continuous *t*-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0,1]$;

(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d \in [0,1]$.

Definition 2. [11] A binary operation $\Diamond$: $[0,1]\times[0,1] \longrightarrow [0,1]$ is continuous *t*-conorm if $\Diamond$ satisfies the following conditions:

(i) $\Diamond$ is commutative and associative;

(ii) $\Diamond$ is continuous;

(iii) $a \Diamond 0 = a$ for all $a \in [0,1]$;

(iv) $a \Diamond b \le c \Diamond d$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d \in [0,1]$.

Alaca et al. [1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous *t*-norm and continuous *t*-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [7] as :

Definition 3. [1] A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous *t*-norm, $\Diamond$ is a continuous *t*-conorm and M, N are fuzzy sets on $X^2\times[0, \infty)$ satisfying the following conditions:

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(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;

(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;

(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(vi) for all $x, y \in X$, $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous;

(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;

(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;

(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(xi) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(xii) for all $x, y \in X$, $N(x, y, .) : [0, \infty)\to[0, 1]$ is right continuous;

(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 4.[1] Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1\text{-}M, *, \Diamond)$ *such* that t-norm $*$ and t-conorm $\Diamond$ are associated as

$x \Diamond y = 1-((1- x) * (1- y))$ for all $x, y \in X$.

Remark 5.[1] In intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$, $M(x, y, *)$ is non-decreasing and $N(x, y, \Diamond)$ is non-increasing for all $x, y \in X$.

Alaca, Turkoglu and Yildiz [1] introduced the following notions:

Definition 6.[1] Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0.$

Definition 7. [1] An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in *X* is convergent.

Example 8.[1] Let *X* = $X = \left\{\frac{1}{n} : n \in N\right\} \cup \{0\}$ and let * be the continuous *t*-norm and ◊ be the continuous *t*-conorm defined by *a* * *b* = *ab* and *a* ◊ *b* = min{1, *a*+*b*} respectively, for all $a, b \in [0,1]$. For each $t \in (0, \infty)$ and $x, y \in X$, define (*M, N*) by

$$M(x, y, t) = \begin{cases} \frac{t}{t + |x - y|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{and} \quad N(x, y, t) = \begin{cases} \frac{|x - y|}{t + |x - y|}, & t > 0 \\ 1, & t = 0. \end{cases}$$

Clearly, $(X, M, N, *, \Diamond)$ is complete intuitionistic fuzzy metric space.

Definition 9.[12] **A** pair of self mappings (*f, g*) of a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be compatible if

$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$ whenever $\{x_n\}$ is a sequence in *X* such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some *z* ∈ *X*.

Thus the mappings *f* and *g* will be non-compatible if there exists at least one sequence $\{x_n\}$ such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some *z* ∈ *X* but either

$\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1, \lim_{n\to\infty} N(fgx_n, gfx_n, t) \neq 0$ or the limit does not exist.

Definition 10.[5] Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. *f* and *g* be self maps on *X*. A point *x* ∈ *X* is called a coincidence point of f and g iff *fx* = *gx*. In this case, *w* = *fx* = *gx* is called a point of coincidence of *f* and *g*.

In 1996, Jungck [5] introduced the notion of weakly compatible maps as follows:

Definition 11.[5] **A** pair of self mappings (*f, g*) of a metric space is said to be weakly compatible if they commute at the coincidence points i.e. *fu* = *gu* for some *u* ∈ *X* , then *fgu* = *gfu.*

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 12.[2] Two self mappings *f* and *g* of intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$

are said to be occasionally weakly compatible (owc) iff there is a point *x* ∈ *X* which is coincidence point of *f* and *g* at which *f* and *g* commute.

Lemma 13.[9] Let *f* and *g* be self maps on an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ and let *f* and *g* have a unique point of coincidence, *w* = *fx* = *gx*, then *w* is the unique common fixed point of *f* and *g*.

Proof: Since *f* and *g* are owc, there exists a point $x \in X$ such that *fx* = *gx* = *w* and *fgx* = *gfx*. Thus, *ffx* = *fgx* = *gfx*, which says that *ffx* is also a point of coincidence of *f* and *g*. Since the point of coincidence *w* = *fx* is unique by hypothesis, *gfx* = *ffx* = *fx*, and *w* = *fx* is a common fixed point of *f* and *g*.

Moreover, if *z* is any common fixed point of *f* and *g*, then *z* = *fz* =*gz* = *w* by the uniqueness of the point of coincidence.

Lemma 14.[12]. Let $(X, M, N, *, \Diamond)$ be intuitionistic fuzzy metric space and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0,1)$,

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t).$$

Then *x* = *y*.

Remark 15.[1] In intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$, $M(x, y, t)$ is non-decreasing and $N(x, y, t)$ is non-increasing for all $x, y \in X$.

Lemma 16.[12] Let $(X, M, N, *, \Diamond)$ be intuitionistic fuzzy metric space and $\{x_n\}$ be a sequence in *X*. If there exists a number $k \in (0,1)$ such that:

$$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t) \text{ and } N(y_n, y_{n+1}, kt) \leq N(y_{n-1}, y_n, t)$$

for all $t > 0$ and *n* = 1, 2,3, ... then $\{x_n\}$ is a Cauchy sequence in *X*.

In our results, $(X, M, N, *, \Diamond)$ will denote an intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\Diamond$ defined by $t * t \geq t$ and $(1-t)\Diamond(1-t) \leq (1-t)$ for all $t \in [0,1]$.

3. Main Result:

Theorem 17. Let the pairs (*A, S*) and (*B, T*) are occasionally weakly compatible self mappings on an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ satisfying:

(3.1) for any $x, y \in X$, $t > 0$ such that:

$$\begin{pmatrix} [1 + aM(Sx, Ty, kt)] \\ * M(Ax, By, kt) \end{pmatrix} \geq a \min \begin{Bmatrix} M(Ax, Sx, kt) * \\ M(By, Ty, kt), \\ M(Ax, Ty, 2kt) * \\ M(By, Sx, 2kt) \end{Bmatrix} + \begin{Bmatrix} M(Sx, Ty, t) \\ * M(Ax, Sx, t) \\ * M(By, Ty, t) \\ * M(Ax, Ty, 2t) \\ * M(By, Sx, 2t) \end{Bmatrix}$$

and

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$$\begin{pmatrix}[1+aN(Sx,Ty,kt)] \\ \Diamond N(Ax,By,kt)\end{pmatrix} \le a\max\begin{Bmatrix}N(Ax,Sx,kt)\Diamond \\ N(By,Ty,kt), \\ N(Ax,Ty,2kt)\Diamond \\ N(By,Sx,2kt)\end{Bmatrix} + \begin{Bmatrix}N(Sx,Ty,t) \\ \Diamond N(Ax,Sx,t) \\ \Diamond N(By,Ty,t) \\ \Diamond N(Ax,Ty,2t) \\ \Diamond N(By,Sx,2t)\end{Bmatrix}$$

where $0<k<1$ and with fixed constant $a\in(-1,0]$.

Then *A, S, B* and *T* have a unique common fixed point in *X*.

Proof: As the pairs (*A, S*) and (*B, T*) are occasionally weakly compatible, there exist points *u*, *v* ∈ *X* such that *Au = Su, ASu = SAu* and *Bv = Tv, BTv = TBv.*

First, we show that *Au = Bv.*

For this, take *x = u* and *y = v* in (3.1),

$$\begin{pmatrix}[1+aM(Su,Tv,kt)] \\ *M(Au,Bv,kt)\end{pmatrix} \ge a\min\begin{Bmatrix}M(Au,Su,kt)* \\ M(Bv,Tv,kt), \\ M(Au,Tv,2kt)* \\ M(Bv,Su,2kt)\end{Bmatrix} + \begin{Bmatrix}M(Su,Tv,t) \\ *M(Au,Su,t) \\ *M(Bv,Tv,t) \\ *M(Au,Tv,2t) \\ *M(Bv,Su,2t)\end{Bmatrix}$$

$$\begin{pmatrix}[1+aM(Au,Bv,kt)] \\ *M(Au,Bv,kt)\end{pmatrix} \ge a\min\begin{Bmatrix}1*1, \\ M(Au,Bv,2kt)* \\ M(Bv,Au,2kt)\end{Bmatrix} + \begin{Bmatrix}M(Au,Bv,t)*1 \\ *1*M(Au,Bv,2t) \\ *M(Bv,Au,2t)\end{Bmatrix}$$

$$\begin{pmatrix}[1+aM(Au,Bv,kt)] \\ *M(Au,Bv,kt)\end{pmatrix} \ge a\begin{bmatrix}M(Au,Bv,kt) \\ *M(Bv,Bv,kt)\end{bmatrix} + \begin{Bmatrix}M(Au,Bv,t)*1 \\ *1*M(Au,Bv,t)*M(Bv,Bv,t) \\ *M(Bv,Au,t)*M(Au,Au,t)\end{Bmatrix}$$

$$\begin{pmatrix}[1+aM(Au,Bv,kt)] \\ *M(Au,Bv,kt)\end{pmatrix} \ge aM(Au,Bv,kt)+M(Au,Bv,t)$$

$$M(Au,Bv,kt)+aM(Au,Bv,kt) \ge aM(Au,Bv,kt)+M(Au,Bv,t)$$

$$M(Au,Bv,kt) \ge M(Au,Bv,t)$$

and

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$$\begin{pmatrix}[1+aN(Su,Tv,kt)] \\ \Diamond N(Au,Bv,kt)\end{pmatrix} \le a\max\begin{Bmatrix} N(Au,Su,kt)\Diamond \\ N(Bv,Tv,kt), \\ N(Au,Tv,2kt)\Diamond \\ N(Bv,Su,2kt)\end{Bmatrix} + \begin{Bmatrix} N(Su,Tv,t) \\ \Diamond N(Au,Su,t) \\ \Diamond N(Bv,Tv,t) \\ \Diamond N(Au,Tv,2t) \\ \Diamond N(Bv,Su,2t)\end{Bmatrix}$$

$$\begin{pmatrix}[1+aN(Au,Bv,kt)] \\ \Diamond N(Au,Bv,kt)\end{pmatrix} \le a\max\begin{Bmatrix} 1\Diamond 1, \\ N(Au,Bv,2kt)\Diamond \\ N(Bv,Au,2kt)\end{Bmatrix} + \begin{Bmatrix} N(Au,Bv,t)\Diamond 1 \\ \Diamond 1\Diamond N(Au,Bv,2t) \\ \Diamond N(Bv,Au,2t)\end{Bmatrix}$$

$$\begin{pmatrix}[1+aN(Au,Bv,kt)] \\ \Diamond N(Au,Bv,kt)\end{pmatrix} \le a\begin{bmatrix} N(Au,Bv,kt) \\ \Diamond N(Bv,Bv,kt)\end{bmatrix} + \begin{Bmatrix} N(Au,Bv,t)\Diamond 0 \\ \Diamond 0\Diamond N(Au,Bv,t)\Diamond N(Bv,Bv,t) \\ \Diamond N(Bv,Au,t)\Diamond N(Au,Au,t)\end{Bmatrix}$$

$$\begin{pmatrix}[1+aN(Au,Bv,kt)] \\ \Diamond N(Au,Bv,kt)\end{pmatrix} \le aN(Au,Bv,kt) + N(Au,Bv,t)$$

$$N(Au,Bv,kt) + aN(Au,Bv,kt) \le aN(Au,Bv,kt) + N(Au,Bv,t)$$

$$N(Au,Bv,kt) \le N(Au,Bv,t)$$

By Lemma 14, this gives, $Au = Bv$. Therefore, $Au = Bv = Su = Tv$. If there is another point z such that $Az = Sz$, then again by using inequality (3.1), it follows that $Az = Sz = Bv = Tv$ that is $Az = Au$. Hence $w = Au = Su$ is unique point of coincidence of A and S. By Lemma 13, w is the unique common fixed point of A and S *i.e.* $Aw = Sw = w$. Similarly, there is unique point $z \in X$ such that $z = Bz = Tz$. Now, we claim that $w = z$. For this, put $x = w$ *and* $y = z$ in (3.1), we have

$$\begin{pmatrix}[1+aM(Sw,Tz,kt)] \\ *M(Aw,Bz,kt)\end{pmatrix} \ge a\min\begin{Bmatrix} M(Aw,Sw,kt)* \\ M(Bz,Tz,kt), \\ M(Aw,Tz,2kt)* \\ M(Bz,Sw,2kt)\end{Bmatrix} + \begin{Bmatrix} M(Sw,Tz,t) \\ *M(Aw,Sw,t) \\ *M(Bz,Tz,t) \\ *M(Aw,Tz,2t) \\ *M(Bz,Sw,2t)\end{Bmatrix}$$

$$\begin{pmatrix}[1+aM(w,z,kt)] \\ *M(w,z,kt)\end{pmatrix} \ge a\min\begin{Bmatrix} M(w,w,kt)* \\ M(z,z,kt), \\ M(w,z,2kt)* \\ M(z,w,2kt)\end{Bmatrix} + \begin{Bmatrix} M(w,z,t) \\ *M(w,w,t) \\ *M(z,z,t) \\ *M(w,z,2t) \\ *M(z,w,2t)\end{Bmatrix}$$

$$\begin{pmatrix}M(w,z,kt) \\ +aM(w,z,kt)\end{pmatrix} \ge a\min\begin{Bmatrix} 1*1, \\ M(w,z,kt) \\ *M(z,z,kt)\end{Bmatrix} + \begin{Bmatrix} M(w,z,t) \\ *1*1 \\ *M(w,z,t) \\ *M(z,z,t)\end{Bmatrix}$$

$$M(w,z,kt) + aM(w,z,kt) \ge aM(w,z,kt) + M(w,z,t)$$

$$M(w,z,kt) \ge M(w,z,t)$$

and

$$\begin{pmatrix} [1+aN(Sw,Tz,kt)] \\ \Diamond N(Aw,Bz,kt) \end{pmatrix} \leq a \max \begin{Bmatrix} N(Aw,Sw,kt) \\ \Diamond N(Bz,Tz,kt), \\ N(Aw,Tz,2kt) \\ \Diamond N(Bz,Sw,2kt) \end{Bmatrix} + \begin{Bmatrix} M(Sw,Tz,t) \\ \Diamond N(Aw,Sw,t) \\ \Diamond N(Bz,Tz,t) \\ \Diamond N(Aw,Tz,2t) \\ \Diamond N(Bz,Sw,2t) \end{Bmatrix}$$

$$\begin{pmatrix} [1+aN(w,z,kt)] \\ \Diamond N(w,z,kt) \end{pmatrix} \leq a \max \begin{Bmatrix} N(w,w,kt) \\ \Diamond N(z,z,kt), \\ N(w,z,2kt) \\ \Diamond N(z,w,2kt) \end{Bmatrix} + \begin{Bmatrix} N(w,z,t) \\ \Diamond N(w,w,t) \\ \Diamond N(z,z,t) \\ \Diamond N(w,z,2t) \\ \Diamond N(z,w,2t) \end{Bmatrix}$$

$$\begin{pmatrix} N(w,z,kt) \\ +aN(w,z,kt) \end{pmatrix} \leq a \max \begin{Bmatrix} 0\Diamond 0, \\ N(w,z,kt) \\ \Diamond N(z,z,kt) \end{Bmatrix} + \begin{Bmatrix} N(w,z,t) \\ \Diamond 0\Diamond 0 \\ \Diamond N(w,z,t) \\ \Diamond N(z,z,t) \end{Bmatrix}$$

$$N(w,z,kt) + aN(w,z,kt) \leq aN(w,z,kt) + N(w,z,t)$$

$$N(w,z,kt) \leq N(w,z,t)$$

Thus, by Lemma 14, we have *w* = *z*. Hence, *w* is unique common fixed point of *A, S, B* and *T* in *X*.

On taking *a* = 0, we have the following result:

Corollary 18. Let the pairs (*A, S*) and (*B, T*) are occasionally weakly compatible self mappings on an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ satisfying:

(3.2) for any $x, y \in X$, $t > 0$ such that:

$$M(Ax,By,kt) \geq \begin{Bmatrix} M(Sx,Ty,t) * M(Ax,Sx,t) * M(By,Ty,t) * \\ M(Ax,Ty,2t) * M(By,Sx,2t) \end{Bmatrix}$$

and

$$N(Ax,By,kt) \leq \begin{Bmatrix} N(Sx,Ty,t) \Diamond N(Ax,Sx,t) \Diamond N(By,Ty,t) \\ \Diamond N(Ax,Ty,2t) \Diamond N(By,Sx,2t) \end{Bmatrix}$$

where $0 < k < 1$. Then, *A, S, B* and *T* have a unique common fixed point in *X*.

On taking *A* = *B* and *S* = *T* in Theorem 17, we get the following result:

Corollary 19. Let (*A, S*) be occasionally weakly compatible self mappings on an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ satisfying:

(3.3) for any $x, y \in X$, $t > 0$ such that:

$$\begin{pmatrix}[1+aM(Sx,Sy,kt)]\\ *M(Ax,Ay,kt)\end{pmatrix}\geq a\min\begin{Bmatrix}M(Ax,Sx,kt)*\\ M(Ay,Sy,kt),\\ M(Ax,Sy,2kt)*\\ M(Ay,Sx,2kt)\end{Bmatrix}+\begin{Bmatrix}M(Sx,Sy,t)\\ *M(Ax,Sx,t)\\ *M(Ay,Sy,t)\\ *M(Ax,Sy,2t)\\ *M(Ay,Sx,2t)\end{Bmatrix}$$

and

$$\begin{pmatrix}[1+aN(Sx,Sy,kt)]\\ \Diamond N(Ax,Ay,kt)\end{pmatrix}\leq a\max\begin{Bmatrix}N(Ax,Sx,kt)\\ \Diamond N(Ay,Sy,kt),\\ N(Ax,Sy,2kt)\\ \Diamond N(Ay,Sx,2kt)\end{Bmatrix}+\begin{Bmatrix}N(Sx,Sy,t)\\ \Diamond N(Ax,Sx,t)\\ \Diamond N(Ay,Sy,t)\\ \Diamond N(Ax,Sy,2t)\\ \Diamond N(Ay,Sx,2t)\end{Bmatrix}$$

where $0<k<1$ and with fixed constant $a\in(-1,0]$.

Then, *A* and *S* have a unique common fixed point in *X*.

On taking *a* = 0 in the above result, we have the following result:

Corollary 20. Let (*A, S*) be occasionally weakly compatible self mappings on an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ satisfying:

(3.4) for any $x,y\in X$, $t>0$ and $k\in(0,1)$ such that:

$$M(Ax,Ay,kt)\geq\begin{Bmatrix}M(Sx,Sy,t)*M(Ax,Sx,t)*M(Ay,Sy,t)\\ *M(Ax,Sy,2t)*M(Ay,Sx,2t)\end{Bmatrix}$$

and

$$N(Ax,Ay,kt)\leq\begin{Bmatrix}N(Sx,Sy,t)\Diamond N(Ax,Sx,t)\Diamond N(Ay,Sy,t)\\ \Diamond N(Ax,Sy,2t)\Diamond N(Ay,Sx,2t)\end{Bmatrix}$$

Then, *A* and *S* have a unique common fixed point in *X*.

4. References

[1] C. Alaca, D. Turkoglu, and C. Yildiz, Fixed points in intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals,* **29**(2006), 1073-1078.

[2] M.A. Al-Thagafi and N. Shahzad, Generalized I-nonexpansive selfmaps and invariant approximations, *Acta Math. Sinica,* **24**(5)(2008), 867-876.

[3] K. Atanassov, Intuitionistic Fuzzy sets, *Fuzzy sets and system*, **20**(1986) 87-96.

[4] A. George and P. Veeramani, On some results in fuzzy metric spaces, *Fuzzy Sets and Systems* **64** (1994), 395–399.

[5] G. Jungck, Compatible mappings and common fixed points. *Internat. J. Math. Sci*. **9**(1986), 771–779.

[6] G.Jungck and B.E.Rhoades, Fixed point Theorems for occasionally weakly compatible mappings, *Fixed point theory*, **7**(2006), 286-296.

[7] I. Kramosil and J. Michalek, Fuzzy metric and Statistical metric spaces, *Kybernetica,* **11**, (1975) 326-334.

[8] K. Menger, Statistical metrics, *Proc. Nat. Acad. Sci.* (USA), **28** (1942), 535- 537.

[9] B.D. Pant and S. Chauhan, Fixed point theorem for occasionally weakly compatible mappings in Menger Space, *Mathematiqki Vesnikcs* (Accepted for Publication).

[10] J. H. Park, Intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals,* **22**(2004) 1039-1046.

[11] B. Schweizer and A. Sklar, Probabilistic Metric Spaces, *North Holland Amsterdam,* 1983.

[12] D. Turkoglu, C. Alaca and C. Yildiz, Compatible maps and Compatible maps of type (α) and (β) in intuitionistic fuzzy metric spaces, *Demonstratio Math*. **39** (3) (2006), 671-684.

[13] L.A. Zadeh, Fuzzy sets, *Infor. and Control.,* **8** (1965), 338-353.