



On time fractional Cahn-Allen equation

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ABSTRACT

In [1], Ozkan Güner et al. obtained some exact solutions of the time fractional Cahn-Allen equation.

By using the method proposed in [10], we have tested these solutions and have found that they are not the solutions of this equation.

Keywords: time fractional Cahn-Allen equation; exact solution, Exp-function method; First integral method

SUBJECT CLASSIFICATION 35J05, 35J10, 33E12, 33R11, 35A22

INTRODUCTION

In [1], Ozkan Güner et al. studied the following time fractional Cahn-Allen equation:

$$D_t^\alpha u - u_{xx} - u + u^3 = 0, \tag{1}$$

where $0 < \alpha \leq 1$, and

$$D_t^\alpha u(x, t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\tau)^{-\alpha} [u(x, \tau) - u(x, 0)] d\tau. \tag{2}$$

Where D_t^α denotes Jumarie's modified Riemann–Liouville fractional derivative [2]. Eq. (1) arises in many scientific applications such as quantum mechanics and plasma physics [3-8]. They obtained some analytical exact solutions by using Exp-function method, the (G'/G) -expansion method and First integral method[3]. However, we have observed that these solutions are not true. Here we list two exact solution obtained in [1] as follows:

$$u(x, t) = \frac{1}{2} + \frac{1}{2} \tanh\left(\frac{\sqrt{2}}{2} x + \frac{3}{2\Gamma(1+\alpha)} t^\alpha\right), \tag{3}$$

$$u(x, t) = \frac{1}{2} - \frac{1}{2} \tanh\left(\frac{\sqrt{2}}{2} x - \frac{3}{2\Gamma(1+\alpha)} t^\alpha\right). \tag{4}$$

In section 2, by the method proposed in [10], we will prove that the functions (3) and (4) are not the solutions of the Eq. (1).

ANALYSIS AND RESULTS

By Eq. (2), we can rewrite the Eq. (1) as:



$$\frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\tau)^{-\alpha} (u(x,\tau) - u(x,0)) d\tau = u_{xx} + u - u^3. \quad (5)$$

For simplicity, we choose $\alpha = 0.5$ in Eq.(1) for checking the obtained solutions (3) and (4).

If the function (4) is a solution of the fractional differential Eq. (1), then the function

$$u(x,t) = \frac{1}{2} - \frac{1}{2} \tanh\left(\frac{\sqrt{2}}{2}x - \frac{3}{2\Gamma(1.5)}t^{0.5}\right) \quad (6)$$

satisfies the following equation:

$$\frac{1}{\Gamma(0.5)} \frac{d}{dt} \int_0^t (t-\tau)^{-0.5} (u(x,\tau) - u(x,0)) d\tau = u_{xx} + u - u^3. \quad (7)$$

Integrating both sides of the eq. (7) with respect to t from 0 to 1, we have

$$\int_0^1 (1-\tau)^{-0.5} (u(x,\tau) - u(x,0)) d\tau = \Gamma(0.5) \int_0^1 (u_{xx} + u - u^3) dt. \quad (8)$$

Take $x = 0$ in Eq.(8), we have:

$$\begin{aligned} & \int_0^1 \frac{1}{2} (1-\tau)^{-0.5} \tanh\left(\frac{3\tau^{0.5}}{2\Gamma(1.5)}\right) d\tau \\ &= \Gamma(0.5) \int_0^1 (u_{xx} + u - u^3) \Big|_{x=0} dt. \end{aligned} \quad (9)$$

By Maple software, we obtain that left side of Eq.(9) approximately equals 0.832725 and right approximately equals 0.087684.

Thus, the function (4) is not a solution of the Eq.(1). Similarly we can prove that the function (3) does not satisfy Eq. (1).

DISCUSSION AND CONCLUSIONS

Different from integer-order differential equation, for a given fractional differential equation, it is very difficulty to test whether or not a function satisfies it. In this paper, by using the method proposed in [11], we have tested the functions (3) and (4) and have found that they are not the solutions of the Eq. (1).

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