ISSN 2347-1921
Volume 13 Number 4
Journal of Advance in Mathematics

On the ωb − compactspace

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ABSTRACT:

The main aim of our paper is introduced new conceptofcompactspaceis called ωb-compact Space, for this aim,the conceptof b-compact space and ω-compactspace introduced. and findthat everyis relationshipsamong compact, b-compact, ω-compact spaces. andthe converse is not not true in generals.and wedefine nearlyωb-compact space and we prove some resultsabout subject.

Keywords: b − open, ; ω − open; ωb − open; ωb − compact , nearly ωb-compact

1.Introduction and Preliminaries

The concept of ω-open sets in topological spaces was introduced in1982 byHdeib[1], In1996 Andrjivic [2] gave a new typeof generalized open setin topological spacecalled b-open sets,In 2008,Noiri,Al-Omari and Noorani [3] introduced the conceptof ωb −open and the complement of an ωb − open set is said to be ωb − closed [11] the intersection of all ωb − closed sets of X, containing A is calledthe ωb-closure of A and is denoted by $\overline{A}^{\omega b}$.The union of all ωb − open sets. of X contained in A is calledtheωb-interior of A and is denoted by $A^{\circ\omega b}$, In [14] Burbaki studied

the concept of compact space.In this workwe introduced definition theconcept of ωb-compact

space, ωb-cluster point,and we introduced definitionωb-converge point,ωb-accumulatepoint, and we prove some theoremabout subject .

Definition (1.1): [1]

A subsetA is said to be ω −open set if for each x∈A ,there existsan open setU_x such that $x \in U_x$ and $U_x - A$ is countable,thecomplement of ω-open set is called ω-closed the family of ω-open sets denoted by $\omega O(X)$.

Definition (1.2): [2]

LetX be topological spaceA is called b-open set in X, iff $\overline{\mathring{A}} \cup \overline{A}^{\circ}$ the complementof b-open set is

Called b-closedand it is easy to see that A is b-closedsetiff$\overline{\mathring{A}} \cap \overline{A}^{\circ} \subseteq A$,thefamily of b-opensets

denoted by $BO(X)$.

Definition(1.3): [3]

A subset A of a space Xis said to be ωb-open,if for every $x \in A$,there exists a b-open subset$U_x \subseteq X$containing x,such that$U_x - A$ is countable,thecomplement of an ωb-opensubsetis said

to be ωb-closed,thefamily of ωb-open sets denoted by $\omega bO(X)$.

Definition (1.4): [4]

Let $f: X \to Y$ be a function of a space X into a space Y,thenf is called an open function,iff(A) is an open set inY,for every open.setA in X.

Definition (1.5): [4]

Let $f: X \to Y$be a function ofa spaceX into a space Y,then f is called an closed function,if f(A) is an closed set in Y,for every closed set A in X.

Definition (1.6):[5]

Let X be a topological space and $A \subseteq X$,A is called regular open set in X,if $A = \overline{A}^{\circ}$,the

Complement ofregular open set is called regular closed,and it is easy to see that A is regular closed if $A = \overline{A^{\circ}}$.

Definition (1.7): [4]

Let $f: X \to Y$ be a function ofa space X into a space Y,then f is called a continuousfunction,iff$^{-1}(A)$ is an openset in X, for every openset A in Y.

Definition (1.8): [8]

A space X is called T_2-space (Hausdorff space) if for each $x \neq y$ in X,there existsdisjoint an open sets U and V such that $x \in U, y \in V$.

Definition (1.9): [6]

A function $f:(D,\geq) \longrightarrow X$, from a directset $(D,\geq)$ to anon-empty set Xis called a net on X,and it denoted by $\{X_\alpha\}_{\alpha \in D}$ $\forall \alpha \in D \ \exists X_\alpha \in X \ni f(\alpha) = X_\alpha$

Definition (1.10): [7]

ATopological space X issaid to be compact,if everyopen cover of X,has a finite sub cover.

Theorem (1.11): [7]

1- Every closed subset of a compact space is compact.

2- In any topological space the intersection of a compact subset with closedsubset is compact.

3- Every compact subset of a Hausdorff spaceis closed.

Definition (1.12): [8]

Atopological space X is said to be b-compact,ifevery b-open cover of X,hasafinite sub cover.

Remark (1.13): [8]

It is clear that every b-compact space is compact,however, butthe converse is not true in general as the following example shows.

Example (1.14): [8]

Let B be an infinite set such that $a \notin B, X = B \cup \{a\}$,let $\tau = \{X, \emptyset, \{a\}\}$be atopologyon X,such that $(X,\tau)$is compact space,where it is not ab-compact since$\{\{a,b\}: b \in B\}$ is a b-open cover of X, which has no finite sub cover.

Definition (1.15): [9]

Atopological space X is said to beω-compact,if every ω-open cover of X,has a finite sub cover.

Definition (1.16): [10]

A topological spaceX is called nearly compact if for every regular open cover ofX,hasfinite sub cover .

2 − ωb − compact space

Definition (2.1):

Afunction $f: X \longrightarrow Y$ is said to be ωb-open,for every open subsetAof X,if $f(A)$ is an ωb-open set inY.

Definition (2.2):

A function $f: X \longrightarrow Y$ is said to beωb-closed,for every closedsubset A of X, if f(A) is an ωb − closed set inY,

Definition (2.3):

Let Xbe a space and $A \subseteq X$,theintersection of all ωb-closed sets of X containingAis called ωb-closure of Adefinedby $\overline{A}^{\omega b} = \cap \{B: B \ \omega b\text{– closed in} X \text{ and } A \subseteq B\}$

Definition (2.4):

Let X be a space and $A \subseteq X$,the union of all ωb-open sets of X containing A is called ωb-Interior of A denoted by $A^{\circ\omega b}$ or $\omega b - In(A) A^{\circ\omega b} = \cup \{B: B \ \omega b - \text{open in} X \text{and } B \subseteq A\}$.

Definition (2.5):

Let Xbe topological space and $A \subseteq X$,A is called regular-ωb-open set in Xif $A = \overline{A}^{\omega b \, \circ\omega b}$ the complement of regular-ωb-open setis called regular-ωb-closed and it is easy to see that A is

regular-ωb-closed set if $A = \overline{A^{\circ\omega b}}^{\omega b}$.

Definition (2.6):

Let$f$: $X \rightarrow Y$ be a function of a space X into a space Y then f is called an ωb-continuous function if $f^{-1}(A)$ is an ωb-open set in X, for every open set$A$ in Y.

Definition (2.7):

Let $f: X \rightarrow Y$ be a function of a topological space(X, τ) into a topological space (Y, τ'),then f iscalled an ωb-irresolutefunction if $f^{-1}(A)$ is an ωb-open set in X,for every ωb-open set$A$ in Y.

Definition (2.8):

A space X is called $\omega b T_2$-space (ωb-Hausdorff space) if for each$x \neq y$in X,there exists disjoint an ωb-open sets U, V such that $x \in U, y \in V$

Definition (2.9):

Atopological space X is said to be ωb-compact,if every ωb-open cover ofX,hasafinite subcover

Remark (2.10):

1- It is clear that every ωb-compact space is compact.

2- It is clear that every ω-compact space is compact. but the converse is not true in general as following example shows:

Example (2.11):

Let X= R withthe topology, $\tau = \{X, \emptyset, Q, Q^c\}$ then $(X, \tau)$is compact space,but it is not aωb-compact,since the family $\{Q \cup X - x \notin Q$ is ωb-open cover of $\mathcal{R}$,thus X= $\cup Q \cup X$,but it is has no finite sub cover.

Definition (2.12):

A topological space$X$ is said to be nearlyωb-compact if every ωb-regular open cover of X,hasfinite sub cover .

Remark (2.13):

1- Every b-compact is not true in general ω-compact

2- Every b-compact is not true in general ωb-compact as the following

Example (2.14):

Let $X = Z$,be the integer number with topological ,$\tau = \{X, \emptyset, Z^+, Z^-\}$,then $BO(X)$=$\{A \subseteq X: 0 \notin A\} \cup \{X\}$,thus X is b-compact,since $\omega o(X) = \omega BO(X) =$

$\{A: A \subseteq X\}$,therefore X is notω-compact and ωb-compact

Remark (2.15):

1- Every ω-compact is not true in general b-compact.

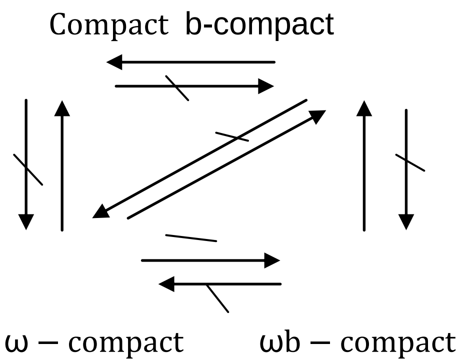
2- Every ω-compact is not true in general ωb-compact. as the following

Example (2.16)

Let B is an un countable, X = B∪{a}, $a \notin B$ and,$\tau = \{\emptyset, X, \{a\}\}$, then $\omega o(X) = \{\emptyset, X, \{a\}\} \cup \{G \subseteq X : G^c \text{ is finite }\}$, thus X is $\omega-$compact, since

$BO(X) = \omega BO(X) = \{\{a, b\}: b \in B\}$, then X is not b-compact and ωb-compact

The following diagram shows the relations among the difference types of compact space.



Theorem (2.17):

Let $f: X \to Y$be an onto,ωb-continuous function,ifX is ωb-compact then Y is compact.

Proof

Let $\{G_\lambda: \lambda \in I\}$be an open cover of Y,then $\{f^{-1}(G_\lambda): \lambda \in I\}$ is an ωb-open cover of X,since X is ωb-compact,thusX has finite sub cover say$\{f^{-1}(G_{\lambda i}): i = 1,2,..,n\}$, and $G_{\lambda i} \in \{G_\lambda: \lambda \in I\}$

,hence $\{G_{\lambda i}: i = 1,2,\dots,n\}$is afinite sub cover of Y, therefore Y is compact.

Proposition (2.18):

For any topological spaceX,the following statement are equivalent:

1-X is ωb-compact.

2- Every family of ωb-closed sets $\{V_\alpha: \alpha \in \Lambda\}$ of X,such that $\bigcap_{\alpha\in\Lambda} V_\alpha = \emptyset$,then there exists

a finite subset$\Lambda_0 \subseteq \Lambda$ such that $\bigcap_{\alpha\in\Lambda_0} V_\alpha = \emptyset$.

Proof

(1)→(2)

Assume that X is ωb-compact,let$\{V_\alpha: \alpha \in \Lambda\}$be a family of ωb-closed subset of X, such that $\bigcap_{\alpha\in\Lambda} V_\alpha = \emptyset$, then thefamily $\{X - V_\alpha: \alpha \in \Lambda\}$is ωb-open coverof theωb-compact(X, τ)there

exists a finite subsetΛ_0 of Λ,thus $X = \bigcup\{X - V_\alpha: \alpha \in \Lambda_0\}$ therefore $\emptyset = X - \bigcup\{X - V_\alpha: \alpha \in \Lambda_0\}$

$= \bigcap\{X - (X - V_\alpha): \alpha \in \Lambda_0\} = \bigcap\{V_\alpha: \alpha \in \Lambda_0\}$

(2)→(1)

Let $U = \{U_\alpha: \alpha \in \Lambda\}$ be an ωb-open coverof the space (X, τ),then $X - \{U_\alpha: \alpha \in \Lambda\}$ is a family of ωb-closed subset of(X, τ) with $\bigcap \{X - U_\alpha: \alpha \in \Lambda\} = \emptyset$by assumpmption,there exists a

a finite subset $\Lambda_0$of Λ,hence $\bigcap\{X - U_\alpha: \alpha \in \Lambda_0\} = \emptyset$,so $X = X - \bigcap\{X - U_\alpha: \alpha \in \Lambda_0\} = \bigcup\{U_\alpha: \alpha \in \Lambda_0\}$,therefore X is ωb-compact.

Proposition (2.19):

If $f: X \to Y$ is ωb-irresolute function, and X is ωb-compact space,then $f(X)$ is $\omega b-$compact.

Proof

Let $\{B_\lambda: \lambda \in I\}$ be an ωb-open cover of $f(X)$,then $f(X) \subseteq \bigcup_{\lambda\in I} B_\lambda$such that $f^{-1}(f(X)) \subseteq$

$f^{-1}(\bigcup_{\lambda\in I} B_\lambda) = \bigcup_{\lambda\in I} f^{-1}(B_\lambda) \subseteq X$,thus $X = \bigcup_{\lambda\in I} f^{-1}(B_\lambda)$since $B_\lambda$is ωb-open set inY,∀

$\lambda \in I$ and, since $f$is ωb-irresolute hence $f^{-1}(B_\lambda)$ is ωb-openset in X,$\forall \lambda \in I$, $\{f^{-1}(B_\lambda): \lambda \in I\}$

is ωb − open cover of X, sinceX is ωb − compact space $\exists \lambda_1 \lambda_2, \dots \lambda_n \in I$ such that

$X = \bigcup_{i=1}^{n} f^{-1}(B_{\lambda i}), f(X) = \bigcup_{i=1}^{n} f(f^{-1}(B_{\lambda i})) \subseteq \bigcup_{i=1}^{n} B_{\lambda i}$,therefore $f(X)$ is ωb-compact .

Definition (2.20):

A subset Bof a topological spaceX,is said to be ωb-compact relative to X,if every cover of B by ωb − open sets of X,has finite sub cover of B,the subset B is ωb-compact,if it is ωb-compact as a subspace.

Theorem (2.21):

The following statements are equivalent,for any topological space

1- X isωb-compact .

2- Every anyfamily Fofωb-open sets, ifnofinite subfamilyofF coversX,thenF doesnot cover X .

3- Every anyfamily F ofωb-closed sets,if F satisfiesthefinite intersectioncon-dition then$\bigcap\{A: A \in F\} \neq \emptyset$

4- Every anyfamilyF of subsetsofX,if F satisfies the finite intersectioncon-dition then$\bigcap\{\overline{A}^{\omega b}: A \in F\} \neq \emptyset$

Proof

(1)⇔(2) and(2)⇔(3) are obvious (3) ⇒(4) if F⊂P(X) satisfies the finiteintersection condition,then$\bigcap\{\overline{A}^{\omega b}: A \in F\}$isa family ofωb-closedsets which

,obviously satisfies the finite intersectioncondition.(4)⇒(3)

Follows fromthefact that$A=\overline{A}^{\omega b}$ for every ωb-closed set A .

Proposition (2.22):

Let Ybe $\omega b-\text{open subspace of a space}$ X,and $B\subseteq Y$,then B is ωb-compact set inY,iff B is ωb-compact in X.

Proof

Let B an ωb-compact set inY,and let $\{V_\alpha:\alpha\in\Lambda\}$ be $\omega b-\text{open cover of}B$ in X,then$B\subseteq\cup_{\alpha\in\Lambda}V_\alpha$,since $B\subseteq Y, B\subseteq\cup\{Y\cap V_\alpha:\alpha\in\Lambda\}$since $Y\cap V_\alpha$ is $\omega b-\text{open}$ relative to Y,thus

$\{Y\cap V_\alpha:\alpha\in\Lambda\}$ is $\omega b-\text{opencover of B relative to Y, we have}$ B$\subseteq(Y\cap V_\alpha)\cup\ldots\cup(Y\cap V_{\alpha_n})$,therefore B is ωb-compact in X .

Conversely:

Let B be ωb-compact set in X,and let$\{U_\alpha:\alpha\in\Lambda\}$ be an $\omega b-\text{open cover of B}$ in Y,then $B\subseteq\cup_{\alpha\in\Lambda}U_\alpha$,thus there exists $V_\alpha$is$\omega b-\text{open}$ relative toX,such that $U_\alpha=Y\cap V_\alpha,\forall\alpha\in\Lambda$, hence

$B\subseteq\cup_{\alpha\in\Lambda}V_\alpha$where$\{V_\alpha:\alpha\in\Lambda\}\omega b-\text{open cover of B relative to X}$,since B is $\omega b-\text{compact}$

set inX ,$\exists\alpha_1,\alpha_2,\ldots\alpha_n\in\Lambda$ such that $\exists B\subseteq\cup_{i=1}^{n}V_{\alpha_i}$ since $B\subseteq Y, B\subseteq Y\cap\{V_{\alpha_1}\cup V_{\alpha_2},..\cup V_{\alpha_n}\}=(Y\cap V_{\alpha_1})\cup\ldots\cup(Y\cap V_{\alpha_n})$since $Y\cap V_{\alpha_i}=U_i$,therefore Bis ωb-compact inY.

Theorem(2.23):

For any topological spaceX, the following statement are equivalent :

1-X is nearly ωb-compact.

2- Every ωb-open cover $\mu=\{V_\alpha:\alpha\in\Lambda\}$ of X,there exists a finite subset$\Lambda_0\subseteq\Lambda$ suchthat . $X=\overline{\cup_{\alpha\in\Lambda o}}^{\omega b\circ\omega b}$.

Proof

(1)→(2)

Let $\mu=\{V_\alpha:\alpha\in\Lambda\}$ be ωb-open cover of X, then $\left\{\overline{V_\alpha}^{\omega b\circ\omega b}:\alpha\in\Lambda\right\}$ is ωb-regularopen cover of the nearlyωb-compact space X, thus there exists a finite subset$\Lambda_0\subseteq\Lambda$ such that $X=\cup_{\alpha\in\Lambda_0}\overline{V_{\alpha\in\Lambda o}}^{\omega b\circ\omega b}$.

(2)→(1)

It is clear since ωb-regular open set is ωb-open.

Definition (2.24):

A point x∈ X is said to be ωb-cluster point of a net $\{X_\alpha\}_{\alpha\in\Delta}$ if $\{X_\alpha\}_{\alpha\in\Delta}$ isFrequently in every ωb-open set containing x .we denote by ωb-cp$\{X_\alpha\}_{\alpha\in\Delta}$ the set of all ωb-cluster pointsof a net $\{X_\alpha\}_{\alpha\in\Delta}$.

Theorem (2.25):

Atopological spaceX is ωb-compact,iffeach net$\{X_\alpha\}_{\alpha\in\Delta}$in X, hasat least one ωb-cluster point.

Proof

Let X be a ωb-compactspace,assume that thereexists somenet$\{x_\alpha\}_{\alpha\in\Delta}$ in X ,such that

ωb-cp$\{x_\alpha\}_{\alpha\in\Delta}$is empty,letx∈X,then there exist G(x)∈ωBO(X,x)is not frequently,thus

there exist $\alpha(x)\in\Delta$,such that$x_\lambda\notin G(x)$,whenever,$\lambda\geq\alpha(x),\lambda\in\Delta$,thefamily$\{G(x):x\in X\}$is acover ofX byωb-open sets and has afinite subcoversay,$\{G_k:k=1,2,\ldots n\}$where, $G_k=G(x_k)$for $k=1,2,\ldots n$,$\{x_k:k=1,2,\ldots n$,let us take □∈Δ,henceα≥αxk,for every k∈*1,2,…*□;foreveryλ∈Δsuch that λ≥□ we have ,xλ∉Gx,k= 1,2,….n, hence$x_\lambda\notin X$ which isa contradiction.

Conversely:

IfX is not ωb-compactthen there exists$\{G_i:i\in I\}$acover of X , by ωb-open sets

whichhas no finite subcover;let$P(I)$be the family of every finite subsetsof I

clear$(P(I),\subseteq)$ is adirected set, foreach $J\in\mathcal{J}$wemay choose $x_j\in X-\cup\in\{G_i:i\in J\}$

let usconsider thenet$\{x_j\}_{j\in p(I)}$ byhypothesisthesetωb-cp$\{x_j\}_{j\in p(I)}$ isnonempty,let $x\in\omega b-cp\{x_j\}_{j\in p(I)}$andlet$i_0\in$ I,hence $x\in G_{i0}$,bythe definition ofωb-clusterpoint

ISSN 2347-1921
Volume 13 Number 4
Journal of Advance in Mathematics

,for each $J \in P(I)$ thus there exists $J^* \in P(I)$ such that $J \subset J^*$ and $x_{j^*} \in G_{i0}$ for $J = \{i_0\}$,

There exists $J^* \in P(I)$ such that $i_0 \in J^*$ and $x_{j^*} \in G_{i0}$ but $x_{j^*} \in X - \cup \{G_i : i \in j^*\}$

$\subset X - G_{i0}$ is contradiction, therefore X is ωb − compact .

In the following we will give a characterization of ωb −compact, by means of filterbases, let us we recall that a nonempty family $\mathcal{F}$ a of subsets of X, is said to be a filterbase on X, if $\emptyset \notin \mathcal{F}$ and each intersection of two members of $\mathcal{F}$ contains third member of $\mathcal{F}$, notice that each chain in the family of every filterbase on X. has an upper bound, the union of every members of the chain then by Zorn's lemma, the family of every filterbases on X, has at least one maximal element Similarly, the family of every filterbases on X, containing a given filterbase $\mathcal{F}$ has at least one maximal element

Definition (2.26):

A filterbase $\mathcal{F}$ on a topological space X, is said to be:

1- ωb-converge to a point $x \in X$, if for each ωb-open set U containing x, there exists $B \in \mathcal{F}$ such that $B \subset U$.

2- ωb-accumulate at $x \in X$, if $U \cap B \neq \emptyset$ for every ωb-open set U containing x and every $B \in \mathcal{F}$

Lemma (2.27):

If a maximal filterbase $\mathcal{F}$ ωb-accumulate at, $x \in X$, then $\mathcal{F}$ ωb-converge to x .

Proof

Let $\mathcal{F}$ be a maximal filterbase with ωb-accumulate at $x \in X$, if $\mathcal{F}$ is not ωb-converge to x, then there exists a ωb-open set U_0 containing x, such that $U_0 \cap B \neq \emptyset$ nd $(X\text{-}U_0) \cap B \neq \emptyset$ for every $B \in \mathcal{F}$, thus $\mathcal{F} \cup \{U_0 \cap B : B \in \mathcal{F}\}$ is a filterbase which contains $\mathcal{F}$, which is contradiction

Theorem (2.28):

Let X be topological space, then following statements are equivalent:

1- X is ωb-compact.

2- Every maximal filterbase ωb-converges to some points of X .

3- Every filter base ωb-accumulates at some points of X .

Proof

(1) ⇒(2)

Let $\mathcal{F}_0$ be a maximal filterbase on X, suppose that $\mathcal{F}_0$ is not ωb-converges to any point of X, then by lemma (2.27), $\mathcal{F}_0$ is not ωb-accumulates at any point of X, for each $x \in X$, then there exists a ωb-open set U_x containing x and $B_x \in \mathcal{F}_0$ hence $U_x \cap B_x = \emptyset$ the family

$\{U_x : x \in X\}$ is a cover of X, by ωb-open sets , by (1) thus there exists a finite subset $\{x_1, x_2, \ldots, x_n\}$ of X, hence $X = \cup \{U_{xk} : k\, 1,2,..\}$, since $\mathcal{F}_0$ is a filterbase , there exists $B_0 \in \mathcal{F}_0$

such that $B_0 \subset \cap \{B_{x_k} : k = 1,2,\ldots n\} = X - \cup \{U_{x_k} : k = 1,2,..\}$, hence $B_0 = \emptyset$

which is contradiction.

(2) ⇒ (3)

Let $\mathcal{F}$ be a filterbase on X, then there exists a maximal filterbase $\mathcal{F}_0$, hence $\mathcal{F} \subset \mathcal{F}_0$ by (2), $\mathcal{F}_0$ is ωb-converges to some point $x_0 \in X$, let $B \in \mathcal{F}$ for every $U \in BO(X, x_0)$ thus there exists $B_U \in \mathcal{F}_0$ such that $B_u \subset U$, hence $U \cap B \neq \emptyset$, Since it is contains the member $B_U \cap B$ of $\mathcal{F}_0$, this that $\mathcal{F}$ ωb-accumulates at x_0.

(3) ⇒ (1)

Let $\{V_i : i \in I\} = \emptyset$ be any family of ωb-closed sets such that $\cap \{V_i : i \in I\} = \emptyset$ we prove that there exists a finite subset I_0 of I, hence $\cap \{V_i : i \in I\}$ by theorem (2.21)(1), let P(I) be the family of finite subsets of I , assume that $\cap \{V_i : i \in J\} = \emptyset$ for every $J \in P(I)$....** thus the family $\mathcal{F} = \{\cap \{V_i : i \in J\} : J \in P(I)\}$ is a filter base on X by (3), $\mathcal{F}$ is ωb-accumulates to some point $x_0 \in X$, Since $\{X - V_i : I \in I\}$ is a cover of X, there exists $i_0 \in I$ hence $x_0 \in X - V_{i0}$, $X - V_{i0}$ is ωb −

open set contains x_0, $V_{i0} \in \mathcal{F}$ and $(X - V_{i0}) \cap V_{i0} = \emptyset$ which is contradiction withthe fact that $\mathcal{F}$ωb-accumulatesat x_0 shows that(∗∗)is false.

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