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Ideals and some applications of simply open sets

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ABSTRACT

Recently there has been some interest in the notion of a locally closed subset of a topological space. In this paper, we introduce a useful characterizations of simply open sets in terms of the ideal of nowhere dense set. Also, we study a new notion of functions in topological spaces known as dual simply-continuous functions and some of their fundamental properties are investigated. Finally, a new type of simply open sets is introduced.

Keywords and phrases: Ideal; simply open sets; simply continuous; strongly simply continuous and dual simply continuous functions.

1 Introduction

According to Biswas [4] and Neubrunnovaá[23], a subset B of a space (X,τ) is called simply open if it is the union of an open set and a nowhere dense set. In 1969 Biswas [4]introduced the concept of simply continuity and introduced some of its properties. Also, Ewert and Neubrunnova' used simply open set in [13] and [23] to define the concept of simply continuity, i.e. a function $f: X \rightarrow Y$ is simply continuous if the inverse image with respect to f of any open set in Y is simply open in X. Also, Dontchev and Ganster [11] used simply open sets to define the concept of strongly simply continuity, i.e., a function $f: X \rightarrow Y$ is strongly simply continuous if the inverse with respect to f of any semi-open set in Y is simply open in X. This enabled them to produce a decomposition of continuity for functions between arbitrary topological spaces. Let (X,τ) be a topological space. For a subset B of X, the closure and the interior of B with respect to (X,τ) will be denoted by $Cl(B)$ and $Int(B)$, respectively. This paper provides a useful characterizations of simply open sets in terms of the ideal of nowhere dense set. Also, we introduce and study a new notion of functions in topological spaces known as dual simply-continuous functions and investigate some of their fundamental properties.

2 preliminaries

Definition 2.1 A subset A of a topological space (X,τ) is called:

1. $semi$-open [18] if $A \subseteq Cl(Int(A))$,
2. $semi$-closed [9] if $X \setminus A$ is $semi$-open,or equivalently, if $Int(Cl(A)) \subseteq A$.
3. an α-set or α-open [24] if $A \subseteq Int(Cl(Int(A)))$,
4. α-closed [24] if $X \setminus A$ is α-open, or equivalently, if $Cl(Int(Cl(A))) \subseteq A$,
5. preopen [21] if $A \subseteq Int(Cl(A))$,
6. nowhere dense if $Int(Cl(A)) = \emptyset$,
7. regular open [26] if $A = Cl(Int(A))$.

The collection of $semi$-open sets, $semi$-closed sets and α-sets in (X,τ) will be denoted by $SO(X,\tau)$, $SC(X,\tau)$ and τ^{α}, respectively. Njåstad [24] has shown that τ^{α} is a topology on X with the following properties: $\tau \subseteq \tau^{\alpha}$, $(\tau^{\alpha})^{\alpha} = \tau^{\alpha}$ and $A \in \tau^{\alpha}$ if and only if $A = U \setminus N$ where $U \in \tau$ and N is nowhere dense in (X,τ). Hence $\tau = \tau^{\alpha}$ if and only if every nowhere dense set in (X,τ) is closed. Clearly every $\alpha-$set is $semi$-open and every nowhere dense set in (X,τ) is $semi$-closed. Andrijevi´c [2] has observed that $SO(X,\tau^{\alpha}) = SO(X,\tau)$ and that $N \subseteq X$ is nowhere dense in (X,τ^{α}) if and only if N is nowhere dense in (X,τ).

Definition 2.2 A subset A of a topological space (X,τ) is called:

1. δ-set [8] if $Int(Cl(A)) \subseteq Cl(Int(A))$,
2. $semi$-locally closed [28] if A is the intersection of a $semi$-open set and a $semi$-closed set,
3. NDB-set [10] if the boundary of A is nowhere dense,
4. sg-closed [3] if the $semi$-closure of A is included in every $semi$-open superset of A,
5. locally closed [6] if $A = G \cap F$ where G is open and F is closed, or, equivalently, if $A = G \cap Cl(A)$ for some open set U.

We will denote the collections of all locally closed sets and $semi$-locally closed sets of (X,τ) by $LC(X,\tau)$ and $SLC(X,\tau)$, respectively. Note that Stone [27] has used the term FG for a locally closed subset. A dense subset of (X,τ) is locally closed if and only if it is open.

Definition 2.3 Recall that a function $f:(X,\tau) \to (Y,\sigma)$ is called:

1. irresolute [9] if $f^{-1}(V)$ is semi-open in (X,τ) for every semi-open set V of (Y,σ);
2. semi-continuous [18] if $f^{-1}(V)$ is semi-open in (X,τ) for every open set V of (Y,σ);
3. strongly semi-continuous [1], if $f^{-1}(V)$ is open in (X,τ) for every semi-open set V of (Y,σ);
4. simply continuous [4, 13, 23], if $f^{-1}(V)$ is simply-open in (X,τ) for every open set V of (Y,σ).
5. strongly simply-continuous [11], if for every semi-open set V of Y, $f^{-1}(V)$ is simply-open in X;
6. pre $sg-$continous [25] if $f^{-1}(V)$ is $sg-$closed in (X,τ) for every semi-closed set V of (V,σ).

3 Simply-open sets

Definition 3.1 [4, 23]. A subset B of a topological space (X,τ) is called simply-open if $B = G \cup N$, where G is an open set and N is nowhere dense in (X,τ).

By [4], the union and the intersection of two simply open sets is a simply open sets, the complement of a simply open set is a simply open set.

The following proposition is a slight enlargement of Theorem 2.2 from [15].

Proposition 3.1 For a subset $B \subseteq (X,\tau)$ the following conditions are equivalent:

1. B is simply-open.
2. $Fr(B)$ (where $Fr(B) = Cl(B) \setminus Int(B)$) is nowhere dense in X
3. there exist two subsets G and H of X where G is open and H is nowhere dense in X, such that $G \cup H \subseteq B \subseteq Cl(G \cup H)$
4. B is semi-locally closed.
5. B is a δ-set.
6. B is an NDB-set.
7. $B \in LC(X,\tau^{\alpha})$.

Proof. $(1) \Leftrightarrow (2)$: (see [[4],Remark 1])

$(1) \Leftrightarrow (3)$: $(see[r4, Definition 1])$

The implications $(1) \Leftrightarrow (4) \Leftrightarrow (5)$ is given in [15].

$(5) \Leftrightarrow (6)$: Follows from the identity: $Int(Fr(B)) = Int(Cl(B)) \cap Int(Cl(X \setminus B))$

$= Int(Cl(B)) \cap (X \setminus Cl(Int(B)))$

$= Int(Cl(B)) \setminus Cl(Int(B))$.

Remark 3.1 One can deduce that:

open set $\Rightarrow$ semi-open set $\Rightarrow$ simply-open set

Clearly every semi-open and every semi-closed set is simply-open. Conversely, not every simply-open set is semi-open or semi-closed. As shown by the following example.

Example 3.1 Consider the following subset of the real line with the usual topology: $S = (0, \frac{1}{2}) \cup (\frac{1}{2}, 1) \cup \{2\}$. Dontchev and Ganster [11] proved that S is simply-open but neither semi-open nor semi-closed.

Proposition 3.2

1. The family of all simply-open sets in a topological space (X, τ) is an algebra of sets, i.e. it contains the complement of each member as well as the union of each two members.
2. The finite intersection of simply-open sets is also simply-open.

Proposition 3.3 [11] For a subset $B \subseteq (X, \tau)$, the following conditions are equivalent:

1. B is semi-closed.
2. B is sg-closed and simply-open.

Proof. $(1) \Rightarrow (2)$: is clear.

$(2) \Rightarrow (1)$: since B is simply-open, then B can be written as the intersection of a semi-open set S and a semi-closed set F. Since B is sg-closed, we have that $SCl(B)$ is contained in S. Since F is semi-closed, $SCl(B)$ is contained in F. Therefore, $SCl(B) = B$, that is B is semi-closed.

Proposition 3.4 For a topological space (X, τ) the following conditions are equivalent:

1. Every simply-open set is semi-closed,
2. Every open set is regular open,
3. X is locally indiscrete (i.e. every open set is closed),
4. Every simply-open set is α-closed.

Proof. $(1) \Rightarrow (2)$: is in Proposition 2.6 [11].

$(2) \Rightarrow (3)$: is in Theorem 3.3 from [16].

$(3) \Rightarrow (4)$: Let $B \in SMO(X)$, i.e. let $B = G \cup N$, where G is open and N is nowhere dense. By (3), G is closed and hence α-closed. Since N is also α-closed and since the α-open sets form a topology in X, then B is α-closed as well.

$(4) \Rightarrow (1)$: is obvious.

In a topological space (X, τ), a subset B is a V_s- set [7] of (X, τ) if $B = B^{V_s}$, where $B^{V_s} = \cup\{F : F \subseteq B, F^c \in SO(X, \tau)\}$. A topological space (X, τ) is called a semi-R_0- space [19] if every semi-open set contains the semi-closure of each of its singletons.

Theorem 3.1 For a topological space (X, τ) the following conditions are equivalent:

1. Every simply-open subspace is a $V_s -$set,
2. (X,τ) is a semi-$R_0 -$space,
3. Every open subspace is a $V_s -$set.

Proof. From Remark 2.2, $(1)\Rightarrow(2)$ and $(2)\Rightarrow(3)$ are obvious.

$(3)\Rightarrow(1)$ Let $B\subseteq SMO(X,\tau)$, then $B=U\cup N$, where $U\in\tau$ and N is nowhere dense. By (3), U is a $V_s -$set. Since every nowhere dense set is semi-closed, then by Proposition 3.5. [7] B is a $V_s -$set.

Let (X,τ) be a topological space and let us denote by I_n the ideal of nowhere dense subsets of (X,τ). On page 69 in [17] Kuratowski defined a subset $A\subseteq X$ to be open mod I_n if there exists an open set G such that $A\setminus G\in I_n$ and $G\setminus A\in I_n$.

Proposition 3.5 (see page 69 in [17])Let I_n denote the ideal of nowhere dense sets in a space (X,τ). Then

1. open sets are open mod I_n;
2. closed sets are open mod I_n;
3. If A,B are open mod I_n, then $A\cap B, A\cup B$ and $X\setminus A$ are open mod I_n;
4. $A\subseteq X$ is open mod I_n if and only if $A=G\cup N$ where G is open and N is nowhere dense in (X,τ) if and only if A is simply open.

Theorem 3.2 Let A be a subset of a space (X,τ) and let I_n denote the ideal of nowhere dense subsets of (X,τ). Then the following are equivalent:

1. $A\in LC(X,\tau^{\alpha})$;
2. $A\in SLC(X,\tau)$;
3. A is a $\delta -$ set;
4. $A\in SMO(X,\tau)$;
5. A is open mod I_n.

Proof. $(1)\Rightarrow(2)$: Follows from the observation that every $\alpha -$set is semi-open.

$(2)\Rightarrow(3)$: Let $A\in SLC(X,\tau)$, i.e. $A=G\cap F$ where $G\in SO(X,\tau)$ and $F\in SC(X,\tau)$, i.e. $G\subseteq Cl(Int(G))$ and $Int(Cl(F))\subseteq F$. Since $Int(Cl(A))\subseteq Int(Cl(F))\subseteq F$, we have $Int(Cl(A))\subseteq In(F)$. Since $A\subseteq G\subseteq Cl(Int(G))$ we have $Int(Cl(A))\subseteq Cl(Int(G))$. Consequently, $Int(Cl(A))\subseteq Cl(Int(G))\cap Int(F)\subseteq Cl(Int(G)\cap Int(F))=Cl(Int(A))$. Hence A is a $\delta -$ set.

$(3)\Rightarrow(4)$: Assume that $Int(Cl(A))\subseteq Cl(Int(A))$ and let $U=Int(A)$ and $N=A\setminus Int(A)$. We will show that N is nowhere dense. Clearly $Int(Cl(N))\subseteq Int(Cl(A))$, and since $N\cap Int(A)=\varnothing$, we have $Int(Cl(N))\cap Cl(Int(A))=\varnothing$. So $Int(Cl(N))=\varnothing$, i.e. N is nowhere dense.

$(4)\Rightarrow(5)$: See Proposition 2.1 [15].

$(5)\Rightarrow(1)$: Let A be open mod I_n. By Proposition 2.1 [15], $X\setminus A$ is open mod I_n, so $X\setminus A=U\cup N$ where U is open and N is nowhere dense in (X,τ). Hence

$A = (X \setminus A) \cap (X \setminus U) \in LC(X, \tau^{\alpha})$ since $X \setminus N \in \tau^{\alpha}$ and $X \setminus U$ is closed in (X, τ) and consequently closed in (X, τ^{α}).

4 On simply continuous and dual simply continuous functions

Definition 4.1 A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called dual simply-continuous if for every simply open set V of Y, $f^{-1}(V)$ is open in X.

Proposition 4.1 For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following conditions are equivalent:

1. f is simply-continuous ;
2. For every closed set V of Y, $f^{-1}(V)$ is simply-open in X.

Proposition 4.2 For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following conditions are equivalent:

1. f is strongly simply-continuous ;
2. For every semi-closed set V of Y, $f^{-1}(V)$ is simply-open in X.

In 1991, Foran and Liebnitz [14] defined a topological space (X, τ) to be strongly irresolvable if no non-empty open set is resolvable or equivalently if every subset of X is simply-open. In 1969, El'kin [12] defined a topological space (X, τ) to be globally disconnected if every set which can be placed between an open set and its closure is open, i.e. if every semi-open set is open. A semi-door space [29] is a topological space in which every set is either semi-open or semi-closed. Note that a semi-door space is always strongly irresolvable.The relationships between simply-continuous, dual simply-continuous, strongly simply-continuous and other corresponding types of functions are shown in the following **diagram 1:**

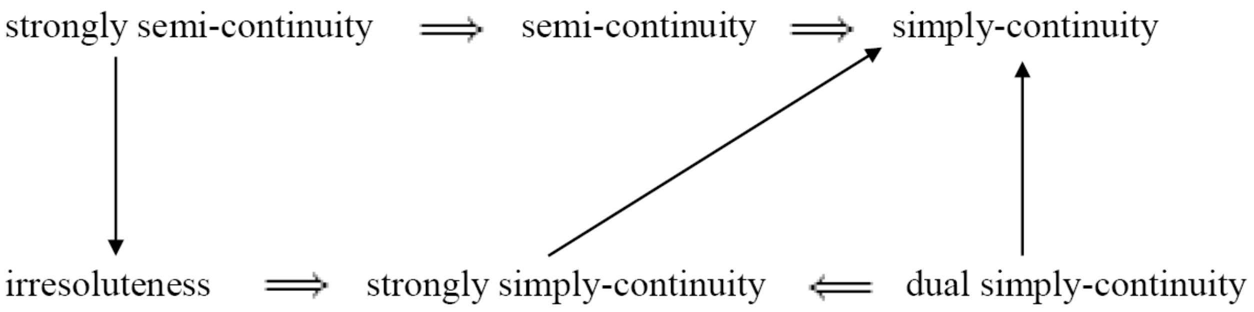
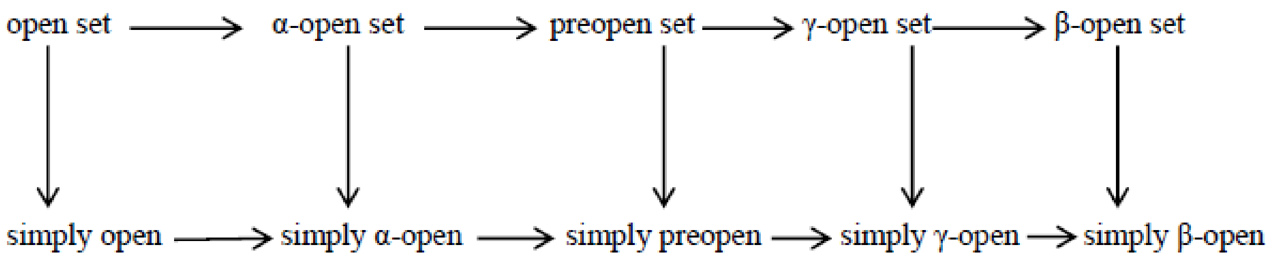


Figure 1:



However, the converses are not true in general as shown by the following examples:

Example 4.1 We will consider example 3.4 from [10]. Let $X = \{a, b, c\}$, $\tau = \{\varnothing, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\varnothing, \{a\}, X\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Clearly, f is simply-continuous but not strongly simply-continuous. Set $V = \{a, b\}$. Note that V is semi-open in σ but V is not simply-open in τ.

Example 4.2 Let $X = \{a, b, c\}$, $\tau = \{\varnothing, \{a\}, \{b\}, \{a, b\}, X\}$ and $f : (X, \tau) \rightarrow (X, \tau)$ be a function defined as follows: $f(a) = f(b) = a$ and $f(c) = c$. As pointed out in [25], f is not pre $sg-$continuous, thus f is not irresolute. But it is easily checked that f is strongly simply-continuous.

Proposition 4.3

1. If (X,τ) is locally indiscrete, then a function $f:(X,\tau)\to(Y,\sigma)$ is irresolute if and only if f is simply-continuous.

2. If (Y,σ) is globally disconnected, then a function $f:(X,\tau)\to(Y,\sigma)$ is strongly simply-continuous if and only if f is simply-continuous.

3. If (X,τ) is strongly irresolvable or, in particular a semi-door space, then every function $f:(X,\tau)\to(Y,\sigma)$ is strongly simply-continuous.

Example 4.3 Let $X=\{a,b,c,d\}$, $\tau=\{\varnothing,\{d\},\{a,d\},X\}$ and $\sigma=\{\varnothing,\{a,d\},\{b,c\},X\}$. Let $f:(X,\tau)\to(X,\sigma)$ defined by: $f(a)=a, f(b)=d, f(c)=b, f(d)=c$. Clearly f is simply continuous but not semi-continuous.

From the above proposition, we have the following decomposition of irresoluteness.

Theorem 4.1 For a function $f:(X,\tau)\to(Y,\sigma)$, the following conditions are equivalent:

1. f is irresolute,
2. f is strongly simply-continuous and pre sg -continuous.

Lemma 4.1 For a topological space (X,τ), we have: a function $f:(X,\tau)\to(Y,\sigma)$ is $\alpha-$continuous if and only if it is both precontinuous and $D(\alpha,p)-$continuous.

Definition 4.2 A function $f:(X,\tau)\to(Y,\sigma)$ is called $\alpha-$continuous [22](resp. precontinuous [21]), if $f^{-1}(V)$ is $\alpha-$set (resp. preopen) for each $V\in\sigma$.

Theorem 4.2 For a function $f:(X,\tau)\to(Y,\sigma)$, the following conditions are equivalent:

1. f is $\alpha-$continuous,
2. f is simply-continuous and precontinuous.

Proof. Evidently, by Lemma 3.1, it is sufficient to prove that every simply-open set belongs to $D(\alpha,p)$. At first we shall show that $B\in D(\alpha,p)$ if and only if $X\setminus B\in D(\alpha,p)$: If $B\in D(\alpha,p)$, then $B\cap Int(Cl(B))=B\cap Int(Cl(Int(B)))$. Thus we obtain $Cl(Int(B))=Cl(Int(Cl(Int(B))))=Cl(Cl(B)\cap Int(Cl(Int(B))))=Cl(B\cap Int(Cl(Int(B))))=Cl(B\cap Int(Cl(B)))$; consequently $Int(Cl(B))=Int(Cl(Int(Cl(B))))=Int(Cl(Int(B)))$. Now let us observe that $Int(Cl(Int(B)))=X\setminus Cl(Int(Cl(X\setminus B)))$ and $Int(Cl(B))=X\setminus Cl(Int(X\setminus B))$. This implies $Cl(Int(Cl(X\setminus B)))=Cl(Int(X\setminus B))$ and consequently $Int(Cl(X\setminus B))=Int(Cl(Int(Cl(X\setminus B))))=Int(Cl(Int(X\setminus B)))$. So $(X\setminus B)\cap Int(Cl(X\setminus B))=(X\setminus B)\cap Int(Cl(Int(X\setminus B)))$, which means $X\setminus B\in D(\alpha,p)$.

Secondly, we observe that every open set belongs to $D(\alpha,p)$ and every nowhere dense set belongs to $D(\alpha,p)$. Therefore, by the above fact, every closed set belongs to $D(\alpha,p)$ and every set of the form $X\setminus N$, where N is nowhere dense, also belongs to $D(\alpha,p)$. Then every simply-open set $U\cup N$ is of the form $X\setminus(X\setminus G)\cap(X\setminus N)$, where $(X\setminus G)\cap(X\setminus N)$ belongs to $D(\alpha,p)$ by Lemma 3.1, thus the set $G\cup N$ belongs to $D(\alpha,p)$.

Definition 4.3 A function $f:(X,\tau)\to(Y,\sigma)$ is called quasi continuous at a point $x\in X$ (see [20]) if fr each neighborhood U of x an each neighborhood open set $G\subseteq U$ such that $f(G)\subseteq V$.

Remark 4.1 It is easy to see that every quasi continuous function is simply continuous.

Definition 4.4 A function $f:(X,\tau)\to(Y,\sigma)$ is called almost quasi continuous at a point $x\in X$ (see [5]) if for each neighborhood V of $f(x)$ and each neighborhood U of x ,the set $f^{-1}(V)\cap U$ is nowhere dense.

Theorem 4.3 A function $f:(X,\tau)\to(Y,\sigma)$ is quasi continuous iff it is almost quasi continuous and simply continuous.

Proof. Follows directly according to Lemma 7 and Theorem 4 of [5].

Theorem 4.4 Let $f:(X,\tau)\to(Y,\sigma)$, $g:(Y,\sigma)\to(Z,\theta)$ be two functions and $g\circ f:(X,\tau)\to(Z,\theta)$ be the composition of f and g . Then the following properties hold: 1. $g\circ f$ is continuous if f is dual simply-continuous and g is simply-continuous,

2. $g\circ f$ is dual simply-continuous if f is continuous and g is dual simply-continuous,

3. $g\circ f$ is strongly semi-continuous if f is dual simply-continuous and g is strongly simply-continuous,

4. $g\circ f$ is strongly semi-continuous if f is dual simply-continuous and g is irresolute,

5. $g\circ f$ is simply-continuous if f is simply-continuous and g is continuous,

6. $g\circ f$ is strongly semi-continuous if f is strongly simply-continuous and g is irresolute,

7. $g\circ f$ is simply-continuous if f is strongly simply-continuous and g is semi-continuous.

5 New types of simply open sets

Definition 5.1 A subset B of a topological space (X,τ) is called simply α -open (resp. simply preopen, simply γ -open, simply β -open) if $B=G\cup N$, where G is an α -open (resp. preopen, γ -open, β -open) set and N is nowhere dense in (X,τ).

Remark 5.1 From the above definition and Definition 2.1, we have the following implications:

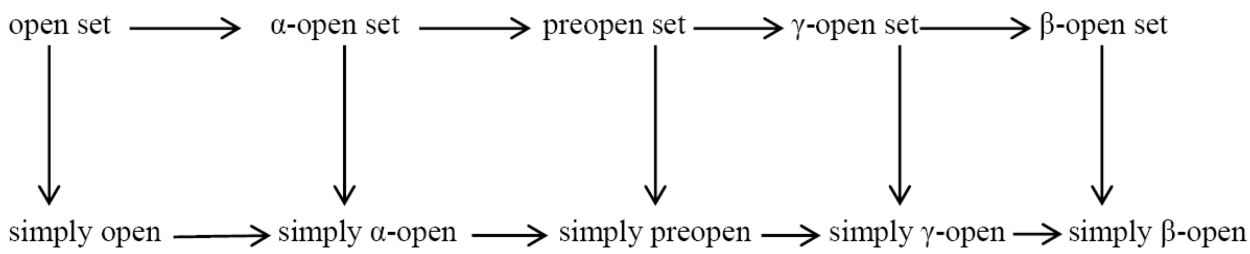
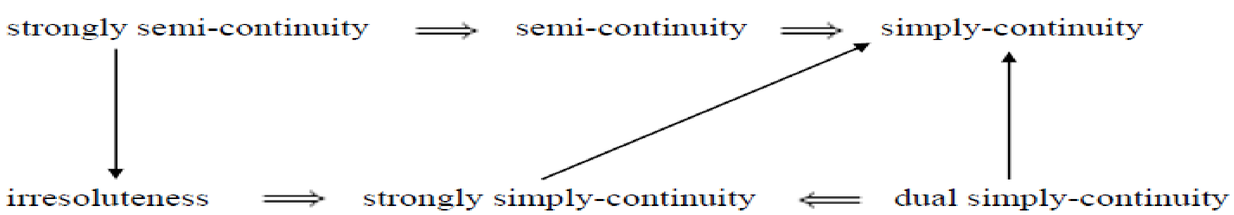


Figure 2:



In the remark above, the relationships can not be reversible as the following examples show.

Example 5.1 Let $X=\{a,b,c,d,e\}$ with a topology τ .

(a) If $\tau=\{\varnothing,\{d\},\{a,b\},\{a,b,d\},\{c,d,e\},X\}$, then

1. $\{c\}$ is simply open but not open,

2. $\{e\}$ is simply α -open but not α -open,

3. $\{a,c\}$ is simply preopen but not preopen,
4. $\{a\}$ is simply preopen but not simply α -open.

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