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Robust H_∞ control for a class of switched nonlinear systems

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Abstract

This article is concerned with the robust H_∞ control problem of a class of switched nonlinear systems with norm-bounded time-varying uncertainties. The system considered in this class is composed of two parts: a uncertain linear switched part and a nonlinear part, which is also switched systems. Under the circumstances, that the H_∞ control problem of all subsystems are not all solvable, the switched feedback control law and the switching law are designed using the average dwell-time method. The corresponding closed-loop switched system is exponentially stable and achieves a weighted L_2-gain.

Keywords: Switched nonlinear cascade systems; H_∞ control; Piecewise Lyapunov function; Average dwell-time method

INTRODUCTION

Switched systems is a special class of hybrid dynamical systems that is composed of a family of continuous or discrete time subsystems and a rule orchestrating the switching between the subsystems. In recent years, there has been increasing interest in the stability analysis and design methodology of switched systems due to their significance both in theory and applications. Such control systems appear in the modeling of chemical processes, transportation systems, computer controlled systems and power systems, etc.

This motivated a large and growing body of research work on a diverse array of issues, including the modeling, optimization, stability analysis, and control, among which the stability issues have been a major focus in studying switched systems (e.g.,[1-15]and the references therein). Among the stability properties, the uniform asymptotic stability is a desirable property which can be guaranteed by a common Lyapunov function[1-5]. But a common Lyapunov function may not exist or is too difficult to find. In this case, the multiple Lyapunov function method [6-8], the single Lyapunov function method[9, 10], and the average dwell-time method[11-13] are developed to study asymptotic stability problem of switched systems under a certain switching law. All these methods and several other methods such as programming method, convex combination method and so on are summarized in the books[14,15].

Due to the uncertainties and nonlinearity are two common phenomenons in practice, the H_∞ control problem for uncertain nonlinear systems is obviously more important and challenging. At present, the research works analyzing the H_∞ control problem are mainly about switched linear systems[16,18]. [16] investigated the disturbance attenuation properties for a class of switched linear systems by using the average dwell-time method incorporated with a piecewise Lyapunov function, and a weighted L_2-gain property is achieved. The stability and L_2-gain analysis for switched linear delay systems was studied in [17]. [18] addressed the L_2-gain analysis for switched systems via multiple Lyapunov functions method. In these papers mentioned above, the switched system studied has no uncertainties and no control input and all the subsystems are stabilisable.

In this paper, the H_∞ control problem for a class of cascade nonlinear switched systems is discussed by the average dwell-time approach incorporated with a piecewise Lyapunov function. The switched system under consideration is composed of a nonlinear part and a uncertain linear part. The piecewise Lyapunov function, the switched feedback controller and the switching law are constructed based on the characteristic of the switched nonlinear cascade system, under which the closed-loop nonlinear switched system is exponentially stable when the disturbance equals to zero, with an disturbance attenuation level γ. Our result is distinct from the existing results, as we don't require the H_∞ control problem of each subsystem is solvable.

The rest of this paper is organised as follows: Section 2 gives the description of the switched system we studied, the preparative knowledge. Section 3 presents the main result. Some conclusions end the paper.

Notation: Given a real matrix M, M^T denotes the transpose of M. $I_{k\times k}$ is the $k\times k$ identity matrix. $L_2[0,\infty)$ denotes the space of square integrable functions on $[0,\infty)$. $\lambda_{max}(P)$ and $\lambda_{min}(P)$ denote the maximum and minimum eigenvalues of P. $\|\cdot\|$ denotes the Euclidean norm. R^n denotes the n-dimensional real Euclidean space. $R^{m\times n}$ is the set of all real $m\times n$ matrix.

Problem statement and preliminaries

2.1. System description

In this paper, we consider the uncertain switched nonlinear system described by

$$\begin{cases} \dot{z} &= f_\sigma(z,\xi), \\ \dot{\xi} &= A_\sigma\xi + \Delta A_\sigma\xi + B_\sigma u_\sigma + \Delta B_\sigma u_\sigma + C_\sigma w, \\ y &= D_\sigma \xi, \end{cases} \tag{1}$$

where $z\in R^{n-d}$, $\xi\in R^d$ are the states, $u_\sigma\in R^m$ is the control input, $w\in R^q$ is the external disturbance input and $w\in L_2[0,\infty)$, $y\in R^p$ is the controlled output. $\sigma(t):[0,\infty]\to I_N=\{1,\ldots,N\}$ is the switching signal, which is a piecewise constant function of time and will be determined later. And $\sigma(t)=i$ means that the ith switched subsystem is activated. A_i, B_i, C_i and $D_i(i\in I_N)$ are constant matrices of appropriate dimensions that describe the nominal systems. $f_i(z,\xi)$ are smooth vector fields, and we have $f_i(0,0)=0$. ΔA_i and ΔB_i are uncertain time-varying matrices denoting the uncertainties in the system matrices and having the following form

$$[\Delta A_i, \Delta B_i] = E_i\Gamma[F_{1i}, F_{2i}], \qquad i\in I_N. \tag{2}$$

where $E_i\in R^{d\times l}$, $F_{1i}\in R^{k\times d}$, and $F_{2i}\in R^{k\times m}$ are given constant matrices which characterize the structure of uncertainty, and F_{2i} is of full column rank. Γ is the norm-bounded time-varying uncertainty, i.e.

$$\Gamma=\Gamma(t)\in\left\{\Gamma(t):\Gamma(t)^T\Gamma(t)=I_{k\times k},\Gamma(t)\in R^{l\times k}, the elements of\ \Gamma(t)\ are Lebesgue measurable\right\}.$$

There are several reasons for assuming that the system uncertainties have the structures given in (2), which can been found in [19].

The following lemma is given on the Input-to-state stability of switched nonlinear systems.

Consider the nonlinear switched systems described by equations of the form

$$\dot{x} = f_\sigma(x,v). \tag{3}$$

where $\sigma(t):[0,\infty]\to I_N=\{1,\ldots,N\}$ is the switching signal, which is a piecewise constant function of time. $f_i(x,v)$ are smooth vector fields, the set of measurable function $v:[0,\infty)\to R^l$ is the input.

Let $I_N=I_s\cup I_u$ such that $I_s\cap I_u=\varnothing$. Where not all subsystems of the system (3) are ISS, but only for a subset I_s of I_N. Denote by $T^u(\tau,t)$ the total activation time of the systems in I_u and by $T^s(\tau,t)$ the total activation time of the systems in I_s during the time interval $[\tau,t)$, where $0\le\tau\le t$. Clearly, $T^s(\tau,t)=t-\tau-T^u(\tau,t)$. Then, we choose a scalar $\lambda^*\in(0,\lambda_s)$. Motivated by the idea in [23], we propose the switching law satisfying the following condition:

$$\inf_{t>\tau}\frac{T^s(\tau,t)}{T^u(\tau,t)}\ge\frac{\lambda_u+\lambda^*}{\lambda_s-\lambda^*} \tag{4}$$

Lemma 1: Consider the switched system (3). Suppose exist functions $\alpha_1,\alpha_2,\phi_1\in K_\infty$, continuously differentiable functions $V_p:R^n\to R$ and constants $\lambda_s,\lambda_u>0,\mu\ge1$ such that

$$\alpha_1(|x|)\le V_p(x)\le\alpha_2(|x|),$$

$$\frac{\partial V_p}{\partial x} f_p(x,u) \le -\lambda_s V_p(x) + \phi_1(|u|), \qquad \forall p \in I_s,$$

$$\frac{\partial V_p}{\partial x} f_p(x,u) \le \lambda_u V_p(x) + \phi_1(|u|), \qquad \forall p \in I_u,$$

$$V_p(x) \le \mu V_q(x), \qquad x \in R^n$$

If there exist constants $\tau_0, \rho \ge 0$ such that

$$\rho < \frac{\lambda_s}{\lambda_s + \lambda_u}$$

$$\forall t \ge \tau \ge 0: \quad T^u(t,\tau) \le \tau_0 + \rho(t-\tau)$$

and if σ is a switched signal with average dwell-time

$$\tau_a > \frac{\ln \mu}{\lambda_s(1-\rho) - \lambda_u \rho} \tag{5}$$

then the switched system (3) is ISS.

Definition 1: System (1) is said to be globally exponentially stabilizable via switching if there exist a switching signal $\sigma(t)$ and an associate switched state feedback $u_\sigma = K_\sigma \xi$ such that the corresponding closed-loop system (1) with $w(t) \equiv 0$ is globally exponentially stable for all admissible uncertainties.

Consider the switched system

$$\begin{cases} \dot{x} &= A_\sigma x + B_\sigma w, \\ y &= C_\sigma x, \end{cases} \tag{6}$$

where $x \in R^n$, w, y, σ are the same as stated in (1), A_i, B_i, C_i $(1 \le i \le N)$ are known constant matrices.

Definition 2: System (6) is said to have a $e^{-\lambda t} - weighted$ L_2-gain over $\sigma(t)$, from the disturbance input $w(t)$ to the controlled output $y(t)$, if the following inequality holds for each $\sigma(t)$ and some real-valued function $\beta(t)$ with $\beta(0) = 0$

$$\int_0^\infty e^{-\lambda t} y^T(t) y(t) dt \le \gamma^2 \int_0^\infty w^T(t) w(t) dt + \beta(x(0)), \qquad w(t) \in L_2[0, +\infty), \tag{7}$$

along the solutions to (6). Where $x(0) \ne 0$ is the initial state.

The aim of the paper is to find a switched state feedback controller and a class of average dwell-time based switching laws, such that the corresponding closed-loop system (3) is globally exponentially stable with $w(t) = 0$ and has a $e^{-\lambda t}$-weighted L_2-gain under the designed switching law.

The following lemma will be used in the development of the main results.

Lemma 2:[20] Given any constant $\lambda > 0$ and any matrices M, Γ, N of compatible dimensions, then

$$2x^T M \Gamma N x \le \frac{1}{\lambda} x^T M M^T x + \lambda x^T N^T N x.$$

for all $x \in R^n$, where Γ is an uncertain matrix satisfying $\Gamma^T \Gamma \le I$.

Main results

This section presents the sufficient condition for the stabilization and $e^{-\lambda t}$-weighted L_2-gain of switched system (1). The switching law satisfying one average dwell time and the switched state feedback controller are also designed.

Theorem 1: Given any constant $\gamma > 0$, suppose that the switched system(1) satisfies the following conditions

(i) if there exist constants $\varepsilon_i > 0$, $\lambda_s > 0$, $\lambda_u > 0$, such that the following inequalities

$$\begin{aligned} &A_i^T P_i + P_i A_i + \varepsilon_i^{-2} P_i E_i E_i^T P_i + \gamma^{-2} P_i C_i C_i^T P_i + \varepsilon_i^2 F_{1i}^T F_{1i} + D_i^T D_i + \lambda_s P_i \\ &+ I - (\varepsilon_i^{-1} P_i B_i + \varepsilon_i F_{1i}^T F_{2i})(F_{2i}^T F_{2i})^{-1} (\varepsilon_i^{-1} P_i B_i + \varepsilon_i F_{1i}^T F_{2i})^T < 0, \qquad i \in I_s \end{aligned} \tag{8}$$

and

$$\begin{aligned} &A_i^T P_i + P_i A_i + \varepsilon_i^{-2} P_i E_i E_i^T P_i + \gamma^{-2} P_i C_i C_i^T P_i + \varepsilon_i^2 F_{1i}^T F_{1i} + D_i^T D_i - \lambda_u P_i \\ &+ I - (\varepsilon_i^{-1} P_i B_i + \varepsilon_i F_{1i}^T F_{2i})(F_{2i}^T F_{2i})^{-1} (\varepsilon_i^{-1} P_i B_i + \varepsilon_i F_{1i}^T F_{2i})^T < 0, \qquad i \in I_u \end{aligned} \tag{9}$$

$$P_i \le \mu P_j, i, j = 1, \ldots, N \tag{10}$$

have positive definite solutions P_i

(ii) there exists smooth positive definite function $W(i), i \in I_N$ and positive numbers $\alpha_1, \alpha_2, \alpha_3, \lambda_s, \lambda_u$, such that for $\forall z \in R^{n-d}$, and $i \in I_N$, we have

$$\alpha_1 \|z\|^2 \le w_i(z) \le \alpha_2 \|z\|^2 \tag{11}$$

$$\begin{cases} \dfrac{\partial w_i(z)}{\partial z} f_i(z,0) \le -\lambda_s w_i(z), & \forall i \in I_s, \\ \dfrac{\partial w_i(z)}{\partial z} f_i(z,0) \le \lambda_u w_i(z), & \forall i \in I_u, \end{cases} \tag{12}$$

$$\left\| \frac{\partial w_i(z)}{\partial z} \right\| \le \alpha_3 \|z\| \tag{13}$$

.

Then, the closed-loop system (1) with $w(t) = 0$ is globally exponentially stable and has a $e^{-\lambda t}$-weighted L_2-gain under arbitrary switching law satisfying the average dwell time

$$\tau_a \ge \tau_a^* = \frac{\ln \mu}{\lambda_s (1-\rho) - \lambda_u \rho}. \tag{14}$$

and the corresponding switched state feedback controller is given by

$$u_i = -(F_{2i}^T F_{2i})^{-1} (\varepsilon_i^{-2} B_i^T P_i + F_{2i}^T F_{1i}) \xi. \tag{15}$$

Proof: For switched system (1), define the following piecewise Lyapunov function candidate

$$V(z,\xi) = K w_\sigma(z) + \xi^T P_\sigma \xi. \tag{16}$$

where $w_i, i = 1...., N$ are the solutions of inequalities (11)-(13), positive scalar K will be defined later.

Then, based on Lemma 1 and Lemma 2, when the ith subsystem is activated, the time derivative of $V(z,\xi)$ along the trajectory of the switched system (1) is

$$\dot{V} = K \frac{\partial w_i(z)}{\partial z} f_i(z,\xi) + [A_i \xi + \Delta A_i \xi + B_i u_i + \Delta B_i u_i + C_i w]^T P_i \xi$$

$$+\xi^T P_i[A_i\xi+\Delta A_i\xi+B_iu_i+\Delta B_iu_i+C_iw]$$

$$=K\frac{\partial w_i}{\partial z}f_i(z,0)+K\frac{\partial w_i}{\partial z}[f_i(z,\xi)-f_i(z,0)]+\xi^T(A_i^TP_i+P_iA_i)\xi+2\xi^TP_i\Delta A_i\xi+2\xi^TP_iB_iu_i$$

$$+2\xi^TP_i\Delta B_iu_i+2\xi^TP_iC_iw$$

I. When $i\in I_s$, we obtain

$$\dot V\le -K\lambda_s w_i(z)+LK\alpha_3\|z\|\|\xi\|+\xi^T(A_i^TP_i+P_iA_i)\xi+2\xi^TP_iE_i\Gamma(F_{1i}\xi+F_{2i}u_i)+2\xi^TP_iB_iu_i$$

$$+2\xi^TP_iC_iw$$

$$\le -K\lambda_s w_i(z)+LK\alpha_3\|z\|\|\xi\|+\xi^T(A_i^TP_i+P_iA_i+\varepsilon_i^{-2}P_iE_iE_i^TP_i)\xi$$

$$+\varepsilon_i^2(F_{1i}\xi+F_{2i}u_i)^T(F_{1i}\xi+F_{2i}u_i)+2\xi^TP_iB_iu_i+2\xi^TP_iC_iw$$

$$\le -K\lambda_s w_i(z)+LK\alpha_3\|z\|\|\xi\|+\xi^T(A_i^TP_i+P_iA_i+\varepsilon_i^{-2}P_iE_iE_i^TP_i+\varepsilon_i^2F_{1i}^TF_{1i})\xi$$

$$+2\varepsilon_i^2\xi^TF_{1i}^TF_{2i}u_i+\varepsilon_i^2u_i^TF_{2i}^TF_{2i}u_i+2\xi^TP_iB_iu_i+2\xi^TP_iC_iw$$

$$\le -K\lambda_s w_i(z)+LK\alpha_3\|z\|\|\xi\|+\xi^T(A_i^TP_i+P_iA_i+\varepsilon_i^{-2}P_iE_iE_i^TP_i+\varepsilon_i^2F_{1i}^TF_{1i})\xi$$

$$+[\varepsilon_iu_i+(F_{2i}^TF_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})^T\xi]^T(F_{2i}^TF_{2i})[\varepsilon_iu_i+(F_{2i}^TF_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})^T\xi]$$

$$-\xi^T[(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})(F_{2i}^TF_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})^T]\xi+2\xi^TP_iC_iw.$$

From (15), we obtain

$$\dot V\le -K\lambda_s w_i(z)+LK\alpha_3\|z\|\|\xi\|+\xi^T\{A_i^TP_i+P_iA_i+\varepsilon_i^{-2}P_iE_iE_i^TP_i+\varepsilon_i^2F_{1i}^TF_{1i}$$

$$-[(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})(F_{2i}^TF_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})^T]\}\xi+2\xi^TP_iC_iw.$$

It is easy to calculate that

$$\dot V+y^Ty-\gamma^2w^Tw$$

$$\le -K\lambda_s w_i(z)+LK\alpha_3\|z\|\|\xi\|+\xi^T\{A_i^TP_i+P_iA_i+\varepsilon_i^{-2}P_iE_iE_i^TP_i+\varepsilon_i^2F_{1i}^TF_{1i}$$

$$-[(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})(F_{2i}^TF_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})^T]\}\xi+2\xi^TP_iC_iw$$

$$+\xi^TD_i^TD_i\xi-\gamma^2w^Tw$$

$$\le -K\lambda_s w_i(z)+LK\alpha_3\|z\|\|\xi\|+\xi^T\{A_i^TP_i+P_iA_i+\varepsilon_i^{-2}P_iE_iE_i^TP_i+\varepsilon_i^2F_{1i}^TF_{1i}$$

$$+D_i^TD_i+\gamma^{-2}P_iC_iC_i^TP_i-[(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})(F_{2i}^TF_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i+\varepsilon_iF_{1i}^TF_{2i})^T]\}\xi$$

$$-(\gamma^{-1}C_i^TP_i\xi_\gamma w)^T(\gamma^{-1}C_i^TP_i\xi_\gamma w)$$

From (8), we know

$$\dot V+y^Ty-\gamma^2w^Tw$$

$$\le -K\lambda_s w_i(z)+LK\alpha_3\|z\|\|\xi\|-\lambda_s\xi^TP_i\xi-\|\xi\|^2$$

$$\le -K\lambda_s w_i(z) - \lambda_s \xi^T P_i \xi + \frac{L^2K^2\alpha_3^2}{4}\|z\|^2 - (\|\xi\| - \frac{LK\alpha_3}{2}\|z\|)^2$$

$$\le -(K\lambda_s - \frac{L^2K^2\alpha_3^2}{4\alpha_1})w_i(z) - \lambda_s \xi^T P_i \xi.$$

Choose $0 < K < \frac{4\lambda_s\alpha_1}{L^2\alpha_3^2}$, we have

$$\dot{V} + y^T y - \gamma^2 w^T w \le -\hat{\lambda}_s V. \tag{17}$$

where $\hat{\lambda}_s = \min\{K\lambda_s - \frac{L^2K^2\alpha_3^2}{4\alpha_1}, \lambda_s\}$.When $w(t) = 0$, from the above inequality, we obtain

$$\dot{V} \le -\hat{\lambda}_s V, \tag{18}$$

II. When $i \in I_u$, from(9) and the above proof of inequality, we obtain

$$\dot{V} + y^T y - \gamma^2 w^T w$$

$$\le K\lambda_u w_i(z) + LK\alpha_3\|z\|\|\xi\| + \lambda_u \xi^T P_i \xi - \|\xi\|^2$$

$$\le K\lambda_s w_i(z) + \lambda_u \xi^T P_i \xi + \frac{L^2K^2\alpha_3^2}{4}\|z\|^2 - (\|\xi\| - \frac{LK\alpha_3}{2}\|z\|)^2$$

$$\le (K\lambda_u + \frac{L^2K^2\alpha_3^2}{4\alpha_1})w_i(z) + \lambda_u \xi^T P_i \xi.$$

For $\forall K > 0$, we have

$$\dot{V} + y^T y - \gamma^2 w^T w \le \hat{\lambda}_u V. \tag{19}$$

Where $\hat{\lambda}_u = \max\{K\lambda_u + \frac{L^2K^2\alpha_3^2}{4\alpha_1}, \lambda_u\}$. When $w(t) = 0$, from the above inequality, we obtain

$$\dot{V} \le \hat{\lambda}_u V, \tag{20}$$

Moreover, from (10) and (16), it is easy to get

$$V_i(t) \le \mu V_j(t), \quad i, j \in I_N. \tag{21}$$

For arbitrary $t > 0$, denote $t_0 \le t_1 \le t_2 \le \ldots \le t_k \ldots \le t_{N_\sigma(0,t)}$ as the switching instants of $\sigma(t)$ over the interval $(0,t)$, then

$$V(t) \le e^{\lambda_u T^u(0,t) - \lambda_s T^s(0,t)} V(t_{N_\sigma(0,t)}) \le \mu e^{\lambda_u T^u(0,t) - \lambda_s T^s(0,t)} V(t^-_{N_\sigma(0,t)})$$

$$\le \ldots \le \mu^{N_\sigma(0,t)} e^{\lambda_u T^u(0,t) - \lambda_s T^s(0,t)} V(0) = e^{N_\sigma(0,t)\ln\mu - \lambda_u T^u(0,t) - \lambda_s T^s(0,t)} V(0).$$

If τ_a satisfies (14), i.e. for arbitrary $N_0 > 0$

$$N_\sigma(0,t) \le N_0 + \frac{t}{\tau_a}, \ \tau_a \ge \tau_a^* = \frac{\ln\mu}{\lambda_s(1-\rho) - \lambda_u\rho}, \tag{22}$$

then, we have

$$N_\sigma(0,t)\ln\mu \le N_0 \ln\mu + t[\lambda_s(1-\rho)-\lambda_u\rho]. \tag{23}$$

Thus

$$V(t) \le e^{N_0\ln\mu - t[\lambda_u\rho-\lambda_s(1-\rho)]+\lambda_u T^u(0,t)-\lambda_s T^S(0,t)}V(0). \tag{24}$$

and by (4), we obtain

$$\begin{aligned} V(t) &\le e^{N_0\ln\mu - t[\lambda_u\rho-\lambda_s(1-\rho)]-\lambda^*(T^S+T^u)}V(0) \\ &\le e^{N_0\ln\mu - t[\lambda^*+\lambda_u\rho-\lambda_s(1-\rho)]}V(0). \end{aligned} \tag{25}$$

Based on (11) and (16), we know that there exist constants $\lambda_1>0$, $\lambda_2>0$ such that

$$\lambda_1\|\xi\|^2 + Ka_1\|z\|^2 \le V(t) \le \lambda_2\|\xi\|^2 + Ka_2\|z\|^2,$$

where $\lambda_1=\min\{\lambda_{\min}(P_i)\,|\,i\in I_N\}$, $\lambda_2=\max\{\lambda_{\max}(P_i)\,|\,i\in I_N\}$.

Let $b_1=\min\{\lambda_1, Ka_1\}$, $b_2=\max\{\lambda_2, Ka_2\}$, we have

$$b_1(\|\xi\|^2+\|z\|^2)\le V(t)\le b_2(\|\xi\|^2+\|z\|^2). \tag{26}$$

Combining (25) and (26) gives

$$\|z(t),\xi(t)\| \le \sqrt{\frac{b_1}{b_2}}\mu^{\frac{N_0}{2}} e^{-\frac{\lambda^*+\lambda_u\rho-\lambda_s(1-\rho)}{2}t}\|z(0),\xi(0)\|. \tag{27}$$

Hence, the globally exponential stability of the closed-loop system (1) with $w(t)=0$ for any $N_0>0$ follows from **Definition 1.**

From (17) and (19) ,we know that the piecewise Lyapunov function candidate (16) satisfies

$$V(t)\le\begin{cases} e^{-\lambda_0(t-t_{N_\sigma(0,t)})}V(t_{N_\sigma(0,t)})-\int_{t_{N_\sigma(0,t)}}^{t} e^{-\lambda_0(t-\tau)}[y^T(\tau)y(\tau)-\gamma^2 w^T w(T)]d\tau & i\in I_s \\ e^{\lambda_0(t-t_{N_\sigma(0,t)})}V(t_{N_\sigma(0,t)})-\int_{t_{N_\sigma(0,t)}}^{t} e^{-\lambda_0(t-\tau)}[y^T(\tau)y(\tau)-\gamma^2 w^T w(T)]d\tau & i\in I_u \end{cases}$$

Since $V(t_i)\le\mu V(t_i^-)$ holds on every switching point t_i according to (21), we obtain by induction that

$$\begin{aligned} V(t) &\le \mu^{N_\sigma(0,t)}e^{\lambda_0 T^u(0,t)-\lambda_0 T^S(0,t)}V(0)-\int_0^t \mu^{N_\sigma(\tau,t)}e^{\lambda_0 T^u(\tau,t)-\lambda_0 T^S(\tau,t)}[y^T(\tau)y(\tau) \\ &-\gamma^2 w^T(\tau)w(\tau)]d\tau \\ &= e^{\lambda_0 T^u(0,t)-\lambda_0 T^S(0,t)+N_\sigma(0,t)\ln\mu}V(0)-\int_0^t e^{\lambda_0 T^u(\tau,t)-\lambda_0 T^S(\tau,t)+N_\sigma(\tau,t)\ln\mu}[y^T(\tau)y(\tau) \\ &-\gamma^2 w^T(\tau)w(\tau)]d\tau \end{aligned}$$

From $T^s(\tau,t)=t-\tau-T^u(\tau,t)$, we obtain

$$V(t)\le e^{c-\lambda_0+N_\sigma(0,t)\ln\mu}V(0)-\int_0^t e^{c-\lambda_0(t-\tau)+N_\sigma(\tau,t)\ln\mu}[y^T(\tau)y(\tau)-\gamma^2 w^T(\tau)w(\tau)]d\tau$$

where $c = 2\lambda_0 T^u$.

Multiplying both sides of the above inequality by $e^{-N_\sigma(0,t)\ln\mu}$, results in

$$e^{-N_\sigma(0,t)\ln\mu}V(t) \le e^{c-\lambda_0 t}V(0) - \int_0^t e^{c-\lambda_0(t-\tau)-N_\sigma(0,\tau)\ln\mu}[y^T(\tau)y(\tau) - \gamma^2 w^T(\tau)w(\tau)]d\tau. \quad (28)$$

Moreover, in view of $V(t) > 0$, the following inequality follows from (23)

$$\int_0^t e^{c-N_0\ln\mu-\lambda_0(t-\tau)-[\lambda_s(1-\rho)-\lambda_u\rho]\tau} y^T(\tau)y(\tau)d\tau \le e^{c-\lambda_0 t}V(0) + \gamma^2\int_0^t e^{c-N_0\ln\mu-\lambda_0(t-\tau)}(\tau)w(\tau)d\tau. \quad (29)$$

Integrating both sides of (29) from $t=0$ to ∞ and rearranging the double-integral area, we obtain

$$\int_0^\infty e^{-[\lambda_s(1-\rho)-\lambda_u\rho]\tau} y^T(\tau)y(\tau)\mu^{-N_0}\left(\int_\tau^\infty e^{c-\lambda_0(t-\tau)}dt\right)d\tau$$

$$\le \int_0^\infty e^{c-\lambda_0 t}V(0)dt + \gamma^2\int_0^\infty w^T(\tau)w(\tau)\mu^{-N_0}\left(\int_\tau^\infty e^{c-\lambda_0(t-\tau)}dt\right)d\tau,$$

i.e.

$$\frac{1}{\lambda_0}\mu^{-N_0}\int_0^\infty e^{-[\lambda_s(1-\rho)-\lambda_u\rho]\tau} y^T(\tau)y(\tau)d\tau \le \frac{1}{\lambda_0}V(0) + \frac{\gamma^2}{\lambda_0}\mu^{-N_0}\int_0^\infty w^T(\tau)w(\tau)d\tau,$$

which is equivalent to

$$\int_0^\infty e^{-[\lambda_s(1-\rho)-\lambda_u\rho]\tau} y^T(\tau)y(\tau)d\tau \le \mu^{N_0}V(0) + \gamma^2\int_0^\infty w^T(\tau)w(\tau)d\tau. \quad (30)$$

From Definition 3 we know that the closed-loop switched system achieves a $e^{-\lambda t}$-weighted L_2-gain.

Remark 1: Applying Shur complement formula, the first matrix inequality of condition (i) can be easily transformed into the LIMs form. The second inequality of condition (i) is trivial, as long as we let $\mu = \sup_{i,j\in I_N}\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}$.

Remark 2: When $\mu \equiv 1$, $\tau_a^* \equiv 0$, and (16) becomes a common Lyapunov function for switched system (1). In this case, the stabilization and robust H_∞ control problem can be solved under arbitrary switching law.

Remark 3: Condition (ii) implies that the second part of the switched system (1) is uniformly exponentially stable. Since the second part of the switched system (1) has a lower dimension, its Lyapunov function is relatively easier to find than that of the whole switched system. A number of methods are available for finding the common Lyapunov function for such switched systems [3, 4].

When the switched system (1) with $I_u = \varnothing$. We have the following Corollary.

Corollary 1: Given any constant $\gamma > 0$, suppose that the switched system (1) satisfies the following conditions

(i) if there exist constants $\varepsilon_i > 0$, $\lambda_s > 0$, such that the following inequalities

$$\begin{aligned}&A_i^T P_i + P_i A_i + \varepsilon_i^{-2}P_i E_i E_i^T P_i + \gamma^{-2}P_i C_i C_i^T P_i + \varepsilon_i^2 F_{1i}^T F_{1i} + D_i^T D_i + \lambda_s P_i \\ &+ I - (\varepsilon_i^{-1}P_i B_i + \varepsilon_i F_{1i}^T F_{2i})(F_{2i}^T F_{2i})^{-1}(\varepsilon_i^{-1}P_i B_i + \varepsilon_i F_{1i}^T F_{2i})^T < 0, \qquad i \in I_s\end{aligned} \quad (31)$$

$$P_i \le \mu P_j, i, j = 1,..., N \tag{32}$$

have positive definite solutions P_i

(ii) there exists smooth positive definite function $W(i), i \in I_N$ and positive numbers $\alpha_1, \alpha_2, \alpha_3, \lambda_s$, such that for $\forall z \in R^{n-d}$, and $i \in I_N$, we have

$$\alpha_1 \|z\|^2 \le w_i(z) \le \alpha_2 \|z\|^2 \tag{33}$$

$$\frac{\partial w_i(z)}{\partial z} f_(i)(z,0) \le -\lambda_s w_i(z), \qquad \forall i \in I_s \tag{34}$$

$$\left\| \frac{\partial w_i(z)}{\partial z} \right\| \le \alpha_3 \|z\| \tag{35}$$

Then, the closed-loop system (1) with $w(t) = 0$ is globally exponentially stable and has a $e^{-\lambda t}$-weighted L_2-gain under arbitrary switching law satisfying the average dwell time

$$\tau_a \ge \tau_a^* = \frac{\ln \mu}{\delta}, \quad \delta \in [0, \lambda_s). \tag{36}$$

and the corresponding switched state feedback controller is given by

$$u_i = -(F_{2i}^T F_{2i})^{-1} (\varepsilon_i^{-2} B_i^T P_i + F_{2i}^T F_{1i}) \xi. \tag{37}$$

Proof: The proof process is similar to that of Theorem 1.

Conclusions

In this paper, the stabilization and robust H_∞ control problem for a class of uncertain switched nonlinear cascade systems with external disturbances input is investigated. The sufficient conditions guaranteeing the existence of the switched state feedback controller are presented, the corresponding average-dwell time based switching law has been simultaneously designed. With the switched state feedback controller the closed-loop switched system is globally exponentially stable and achieves a $e^{-\lambda t}$-weighted L_2 gain under the designed switching law. The stabilization problem and L_2-gain analysis for the same class of switched nonlinear cascade systems when both parts are respectively stabilizable under two different average-dwell time deserves further study.

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