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Cauchy sequences and a Meir-Keeler type fixed point theorem in partial metric spaces.

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Abstract: In this paper we prove some new conditions for Cauchy sequences by using the diameter of orbit in partial metric spaces. A fixed point theorem for Meir-Keeler type contractions in this space is established.

Keywords: Partial metric space; Cauchy sequences; fixed point theorem; Meir-Keeler type contraction.

Academic Discipline and Sub-Disciplines

Mathematics, Functional Analysis.

SUBJECT CLASSIFICATION

Functional Analysis

1. Introduction.

The notion of a partial metric space was introduced by G.S. Metthews [10,11] in 1992. The partial metric space is a generalization of the usual metric spaces in which the distance of a point from itself may not be zero. Recently, many authors have been focused on the partial metric spaces and its topological properties. [1, 12, 13].They show that partial metric spaces have many applications both in mathematics and computer science [8, 13]. The concept of Cauchy sequences is very important in functional analysis and especially in fixed point theory.

In [4] we obtained some conditions for equivalent Cauchy sequences and 0-equivalent 0-Cauchy sequences in partial metric spaces.

The Banach contraction principle [14] is the most celebrated fixed point theorem. It is very useful, simple, and classical tool in nonlinear analysis. This principle has many generalizations. For example, in 1969 [2] Meir and Keeler proved a fixed point theorem for the mappings satisfying a (ε-δ) contractive condition. Some generalizations of Meir-Keeler fixed point theorem (see 9, 5, 6) established a class of the contractions called the Mier-Keeler type contraction.

In this paper we will show some conditions about Cauchy sequences in partial metric spaces establish a fixed point theorem for a Meir- Keeler type contraction in these spaces.

2. Preliminaries.

For convenience we start with the following definitions, lemmas, and theorems.

Definition 1. [10] A function $p: X \times X \to R^+$ is a partial metric on X if, for all $x, y, z \in X$, the following conditions hold:

p_1) $x = y$ if and only if $p(x,x) = p(x,y) = p(y,y)$,

p_2) $p(x,x) \leq p(x,y)$

p_3) $p(x,y) = p(y,x)$,

p_4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$

In this case, the pair (X, p) is called a partial metric space.

It is clear that if $p(x,y) = 0$ then from (p_1) and (p_2), $x = y$. But, if $x = y$, $p(x,y)$ may not be 0. As an example of partial metric space we have, (R^+, p) where $p(x,y) = \max\{x,y\}$.

Each partial metric p on X generates a T_0-topology on X, which has as base the family of open p-balls $\{B_p(x,\varepsilon): x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < \varepsilon + p(x,x)\}$ for all $x \in X$ and $\varepsilon > 0$

Definition 2. [10,11] A sequence $\{x_n\}$ in a partial metric space (X, p) is said to be:

(i) p -convergent to a point $x \in X$ if $\lim_{n\to\infty} p(x, x_n) = p(x, x)$;

(ii) p -Cauchy sequence if $\lim_{n,m\to\infty} p(x_m, x_n)$ exists and is finite.

Notice that the limit of sequence in partial metric space is not necessary unique.

Proposition 3. [11] Every partial metric p defines a metric d_p, where

$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ for all $x, y \in X$.

The metric d_p is called the metric associated with partial metric p .

Lemma 1. [10,11]

(1) A sequence $\{x_n\}$ is a p -Cauchy sequence in a partial metric space (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p).

(2) (X, p) is complete if and only if the metric space (X, d_p) is complete.

Lemma 2. **[7]** Let (X, p) be a partial metric space and let (x_n) and (y_n) be sequences in X such that $x_n \to x$ and $y_n \to y$ with respect to d_p. Then $\lim_{n\to\infty} p(x_n, y_n) = p(x, y)$

Definition 4. The sequences (x_n) and (y_n) in a metric space (X, d) are called equivalent if $\lim_{n\to\infty} d(x_n, y_n) = 0$.

Definition 5. The sequences (x_n) and (y_n) in a partial metric space (X, p) are called equivalent if $\lim_{n\to\infty} p(x_n, y_n)$ exists and is finite.

Definition 6. The sequences (x_n) and (y_n) in a partial metric space (X, p) are called equivalent Cauchy if they are Cauchy and equivalent in (X, p).

Definition 7. Let (X, p) be a partial metric space. A sequence $\{x_n\}$ in X is called 0-Cauchy if $\lim_{n,m\to\infty} p(x_m, x_n) = 0$

Definition 8. The sequences (x_n) and (y_n) in a partial metric space (X, p) are called 0-equivalent if $\lim_{n\to\infty} p(x_n, y_n) = 0$.

Definition 9. The sequences (x_n) and (y_n) in a partial metric space (X, p) are called 0-equivalent 0-Cauchy if they are 0-Cauchy and 0-equivalent in (X, p).

Definition 10. Let (X, p) be a partial metric space

i) A subset A in X is called bounded if there exists a real number M>0 such that $p(x, y) \le M$ for all $x, y \in A$;

ii) If A is bounded set of X, then the diameter of A is denoted by δ(A) and is defined by

$\delta(A) = \sup\{p(x, y); x, y \in A\}$

Theorem 2.2.[4] If the sequences (x_n) and (y_n) are equivalent Cauchy in (X, d_p), then they are equivalent Cauchy in partial metric space (X, p).

The example 3 in [4] shows that the converse of the theorem 2.2 is not true.
Also, in [4] we proved some new conditions for equivalent Cauchy sequences in partial metric spaces as follows:

Theorem 2.1. [4] Let (X, p) be a partial metric space and (x_n), (y_n) two sequences in it. If the sequences (x_n), (y_n) satisfy one of the following conditions, then the sequences (x_n), (y_n) are equivalent Cauchy in (X, p).

(1) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall\varepsilon > 0, \exists r \in N, \exists\delta \in (0,+\infty), \exists\varepsilon_0 \in (0,\varepsilon)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow p(x_{i+r}, y_{j+r}) \le \varepsilon_0$ whenever $i, j \in N$

(2) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall\varepsilon > 0, \exists r \in N, \exists\delta \in (0,+\infty)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow \delta_{i+r, j+r} < \varepsilon$, whenever $i, j \in N$

(3) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall n \in N, \exists\alpha_n \in (0,+\infty), \exists r \in N$, such that $\delta_{ij} < \alpha_n \Rightarrow \delta_{i+r, j+r} < \frac{1}{n}$ whenever $i, j \in N$

(4) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall\varepsilon > 0, \exists r \in N, \exists\delta \in (0,+\infty), \exists\varepsilon_0 \in (0,\varepsilon)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow \delta_{i+r, j+r} \le \varepsilon_0$ whenever $i, j \in N$

These conditions in theorem 2.1 are necessary and sufficient for 0-equivalent 0-Cauchy sequences in partial metric spaces as the following theorem shows.

Definition 8. **[7]** Let (X, d) be a partial metric space and T a self-mapping of X .

1. T is called orbitally continuous if

$\lim_{i,j\to\infty} p(T^{n_i}x, T^{n_j}x) = \lim_{i,j\to\infty} p(T^{n_i}x, z) = p(z,z)$ implies $\lim_{i,j\to\infty} p(TT^{n_i}x, TT^{n_j}x) = \lim_{i,j\to\infty} p(TT^{n_i}x, Tz) = p(Tz, Tz)$

for each $x \in X$.

Equivalently, T is orbitally continuous provided that if $T^{n_i}x \to z$ in (X, d_p), then $T^{n_i+1}x \to Tz$ in (X, d_p) for each $x \in X$.

Theorem 2.2. [2] (*Fixed point theorem of Meir-Keler*) Let (X, d) be a metric space and let T be a mapping from X into itself satisfying the following condition:

$$\forall\varepsilon > 0,\ \exists\delta(\varepsilon) > 0 \text{ such that } \varepsilon \le d(x,y) < \varepsilon + \delta(\varepsilon) \Rightarrow d(Tx, Ty) < \varepsilon$$

Then T has a unique fixed point $z \in X$. Moreover, for all $x \in X$, the sequence $\{T^n x\}$ converges to *z.*

3. MAIN RESULTS.

Let (x_n) be a sequence in partial metric space (X, p) .

Define $\delta_{ij}((x_n)) = \sup\{p(x_m, x_k) : m \ge i, k \ge j\}\ \forall (i, j) \in N^2$. (4)

Preposition 3.1. Let (X, p) be a partial metric space and (x_n) a sequence in it. If one $\delta_{i_0 j_0}(\{x_n\})$ is finite than all $\delta_{ij}(\{x_n\})$ are finite.

Proof. Denote $A = \max\{p(x_m, x_{i_0}), 1 \le m \le i_0\}$ and $B = \max\{p(x_k, x_{j_0}) 1 \le k \le j_0\}$

The proof is similar with the proof of preposition 5 in [4] replacing y_k with x_k.

Corollary 3.2. Let (X, p) be a partial metric space and (x_n) a sequence in it. The sequences (x_n) is bounded if and only if $\delta_{11}(\{x_n\})$ is finite.

The proof is similar with the proof of Corollary 6 in [4] replacing y_k with x_k.

Theorem 3.3. Let (X, p) be a partial metric space and (x_n) a sequence in it. If the sequences (x_n) satisfies one of the following conditions, then the sequence (x_n) is Cauchy in (X, p).

(1) The sequences (x_n) is bounded in (X,p) and

$\forall\varepsilon>0,\exists r\in N,\exists\delta\in(0,+\infty),\exists\varepsilon_0\in(0,\varepsilon)$ such that $\delta_{ij}(\{x_n\})\le\varepsilon+\delta\Rightarrow p(x_{i+r},x_{j+r})\le\varepsilon_0$ whenever $i,j\in N$

(2) The sequences (x_n) is bounded in (X,p) and

$\forall\varepsilon>0,\exists r\in N,\exists\delta\in(0,+\infty)$ such that $\delta_{ij}(\{x_n\})\le\varepsilon+\delta\Rightarrow\delta_{i+r,j+r}(\{x_n\})<\varepsilon$, whenever $i,j\in N$

(3) The sequences (x_n) is bounded in (X,p) and

$\forall n\in N,\exists\alpha_n\in(0,+\infty),\exists r\in N$, such that $\delta_{ij}(\{x_n\})<\alpha_n\Rightarrow\delta_{i+r,j+r}<\frac{1}{n}$ whenever $i,j\in N$

(4) The sequences (x_n) is bounded in (X,p) and

$\forall\varepsilon>0,\exists r\in N,\exists\delta\in(0,+\infty),\exists\varepsilon_0\in(0,\varepsilon)$ such that $\delta_{ij}(\{x_n\})\le\varepsilon+\delta\Rightarrow\delta_{i+r,j+r}(\{x_n\})<\varepsilon_0$ whenever $i,j\in N$

Proof.

Let (x_n) be a sequence in (X,p) satisfying (1). Define

$\alpha_n=\delta_{n,n}=\sup\{p(x_i,x_j),i\ge n,j\ge n\}$

The sequences (α_n) is decreasing and positive. Hence it converges and $\lim_{n\to\infty}a_n=\inf\{\alpha_n:n\in N\}=a\ge0$

Suppose that $a>0$. From the condition (1) for $\varepsilon=a>0$ there are $r\in N$, $\varepsilon_0\in(0,\varepsilon)$ and $\delta>0$

such that $\delta_{ij}(\{x_n\})\le\varepsilon+\delta\Rightarrow p(x_{i+r},x_{j+r})\le\varepsilon_0$ whenever $i,j\in N$ (5)

For this $\delta>0$ exists $p\in N$ such that for $n\ge p\Rightarrow\alpha_n<a+\delta=\varepsilon+\delta$

For $i\ge p,j\ge p$ we have $\delta_{ij}(\{x_n\})\le\alpha_p=\delta_{p,p}<\varepsilon+\delta$ .By (5) we have $p(x_{i+r},x_{j+r})\le\varepsilon_0$.

But it is obvious that $i+r=k\ge p+r, j+r=l\ge p+r$, so $p(x_k,x_l)\le\varepsilon_0<\varepsilon=a$, which is a contradiction. Hence we have $\lim_{n\to\infty}a_n=\inf\{\alpha_n:n\in N\}=0$. But $p(x_i,x_j)\le a_{\min\{i,j\}}$

and whereas $\lim_{n\to\infty}a_n=0$ we have $\lim_{i,j\to\infty}p(x_i,x_j)=0$. So the sequence (x_n) is Cauchy.

Furthermore, since $p(x_n,x_n)\le\alpha_n$ and $\lim_{n\to\infty}a_n=0$ hold, then $\lim_{n\to\infty}p(x_n,x_n)=0$.

(2) Let (x_n) be a sequence in (X,p) satisfying (2).

As in theorem 7 in [4], we first shall prove that (2) ⇒ (3) and if (x_n) is satisfying (3) in the same way as in (1) above, we can prove that the sequence (x_n) is Cauchy in (X,p)..

(4). Let (x_n) be a sequence in (X,p) satisfying (4).

It is clear that (4)⇒(2) and by (2) immediately follows that the sequences (x_n) is Cauchy in (X,p).

Remark 3.4. The converse of the theorem 3.3 is not true. For this we can see the following example.

Example 3.5. Let X=R^+ and define a mapping $p:RxR\to R^+$ by $p(x,y)=\max\{x,y\}$ as a partial metric.

The sequence $(x_n)=(\frac{1}{2}-\frac{1}{n})$ is Cauchy in (X,p). But, $\delta_{ij}=\frac{1}{2}$ for $i,j\in N$ and for $\varepsilon=\frac{1}{2}$, for any $\delta>0$ and $r>0$, though $\delta_{ij}=\frac{1}{2}<\varepsilon+\delta$ we have $\delta_{i+r,j+r}=\frac{1}{2}\ge\varepsilon$.

So, the sequence (y_n)= $(\frac{1}{2}-\frac{1}{n})$ do not satisfy the condition (2).

In the same way we can show that this sequence do not satisfy and the conditions (1), (3) and (4).

But if (x_n) is 0-Cauchy sequence then the converse of the theorem 3.3 is true and we can prove the following theorem.

Theorem 3.6. Let (X,p) be a partial metric space and (x_n) a sequence in it. The sequence (x_n) is 0-Cauchy sequence in (X,p) if and only if it satisfies one of the following conditions,

(1) The sequences (x_n) is bounded in (X,p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0,+\infty), \exists \varepsilon_0 \in (0,\varepsilon)$ such that $\delta_{ij}(\{x_n\}) \le \varepsilon + \delta \Rightarrow p(x_{i+r}, x_{j+r}) \le \varepsilon_0$ whenever $i, j \in N$

(2) The sequences (x_n) is bounded in (X,p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0,+\infty)$ such that $\delta_{ij}(\{x_n\}) \le \varepsilon + \delta \Rightarrow \delta_{i+r,j+r}(\{x_n\}) < \varepsilon$, whenever $i, j \in N$

(3) The sequences (x_n) is bounded in (X,p) and

$\forall n \in N, \exists \alpha_n \in (0,+\infty), \exists r \in N,$ such that $\delta_{ij}(\{x_n\}) < \alpha_n \Rightarrow \delta_{i+r,j+r} < \frac{1}{n}$ whenever $i, j \in N$

(4) The sequences (x_n) is bounded in (X,p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0,+\infty), \exists \varepsilon_0 \in (0,\varepsilon)$ such that $\delta_{ij}(\{x_n\}) \le \varepsilon + \delta \Rightarrow \delta_{i+r,j+r}(\{x_n\}) < \varepsilon_0$ whenever $i, j \in N$

Proof.

By the proof of the theorem 3.3 if the sequence (x_n) satisfies one of the conditions (1)-(4) it is Cauchy sequence and $\lim_{i,j\to\infty} p(x_i,x_j) = \lim_{i\to\infty} p(x_i,x_i) = \lim_{i\to\infty} p(x_j,x_j) = 0$. So the sequence (x_n) is 0-Cauchy sequence in (X,p).

Conversely, if (x_n) is a 0-Cauchy sequence in (X,p), then it is a Cauchy sequence with respect to d_p. So, by Definition 1 and 7, we have

$$\lim_{i,j\to\infty} d_p(x_i,x_j) = \lim_{i\to\infty}\left[2p(x_i,x_j) - p(x_i,x_i) - p(x_j,x_j)\right] = 0$$

Therefore, (x_n) is Cauchy in metric space (X,d_p) and as shown in [3] the conditions (1), (2), and (4) are equivalent to being of sequence (x_n) Cauchy sequence in metric space.

So, now we can prove that if sequence (x_n) is 0-Cauchy in (X,p), then it satisfies the condition (3).

By the definition 1 and 7, we have $\lim_{i,j\to\infty} p(x_i,x_j) = \lim_{i\to\infty} p(x_i,x_i) = \lim_{i\to\infty} p(x_j,x_j) = 0$.

Then, for $n \in N$ there is $P \in N$ such that for $i > P, j > P$ we have $p(x_i,x_j) < \frac{1}{n}$ and so $\delta_{PP} < \frac{1}{n}$.

Hence, for $\alpha_n > \frac{1}{n}$, $r = P$ we have $\delta_{ij} < \alpha_n \Rightarrow \delta_{i+r,j+r} < \delta_{PP} < \frac{1}{n}$ whenever $i, j \in N$. So (3) hold.

Let (X,p) be a partial metric space and T a self-mapping define on X. For each $x \in X$, we define the orbit of T by

$O(x) = \{x, Tx, T^2x, T^3x,,, T^nx,,,\}$ and $\delta_{ij} = \sup\left\{p(T^mx, T^ky) : m \ge i, k \ge j\right\}$ $\forall (i,j) \in N^2$.

Theorem 3.6. Let (X,p) be a complete partial metric space and T a self-mapping orbitally continuous define on X. If T satisfies one of the following condition, than T has a unique fixed point $z \in X$ Moreover, $\lim_{n\to\infty} T^nx = z$ for any $x \in X$.

(1) For all $x, y \in X$, the sequences (T^ix) and (T^jy) are bounded in (X,p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0,+\infty), \exists \varepsilon_0 \in (0,\varepsilon)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow p(T^{i+r}x, T^{j+r}y) \le \varepsilon_0$ whenever $i, j \in N$

(2) For all $x, y \in X$, the sequences $(T^i x)$ and $(T^j y)$ are bounded in (X, p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0,+\infty)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow \delta_{i+r, j+r} < \varepsilon$, whenever $i, j \in N$

(3) For all $x, y \in X$, the sequences $(T^i x)$ and $(T^j y)$ are bounded in (X, p) and

$\forall n \in N, \exists \alpha_n \in (0,+\infty), \exists r \in N,$ such that $\delta_{ij} < \alpha_n \Rightarrow \delta_{i+r, j+r} < \frac{1}{n}$ whenever $i, j \in N$

(4) For all $x, y \in X$, the sequences $(T^i x)$ and $(T^j y)$ are bounded in (X, p) and $\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0,+\infty), \exists \varepsilon_0 \in (0,\varepsilon)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow \delta_{i+r, j+r} \le \varepsilon_0$ whenever $i, j \in N$

Prof. Let $x \in X$. We define the iterative sequence $\{x_n\}$ as follows $x_{n+1} = Tx_n$, for $n \in N$.

If there exists $n_o \in N$ such that $x_{n_0} = x_{n_0+1}$ than x_{n_0} is a fixed point of T. Assume then that $x_n \ne x_{n+1}$ for each $n \in N$.

We first shall prove that if T satisfy one of the conditions (1)- (4) the sequence $\{x_n\}$ is a Chauchy sequence.

(1) Suppose T satisfies the condition (1). Substituting $x = x_n$ and $y = x_{n+1}$ in (1) we obtain:

the sequence $\{x_n\}$ is bounded in (X, p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0,+\infty), \exists \varepsilon_0 \in (0,\varepsilon)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow p(T^{i+r}x, T^{j+r}y) \le \varepsilon_0$ whenever $i, j \in N$

but $\delta_{ij} = \sup\{p(T^m x, T^k y) : m \ge i, k \ge j\} = \sup\{p(T^m x_n, T^k x_{n+1}) : m \ge i, k \ge j\} = \sup\{p(T^{m+n}x, T^{k+n+1}x) : m \ge i, k \ge j\}$

$= \sup\{p(x_{m+n}, x_{k+1+n}) : m \ge i, k \ge j\} = \delta_{i+n, j+n+1}(\{x_n\})$

and $p(T^{i+r}x_n, T^{j+r}x_{n+1}) = p(x_{n+i+r}, x_{n+1+j+r})$.

So the sequence $\{x_n\}$ satisfies condition (1) in theorem 3, so it is a Cauchy sequence in (X, p).

(2) Suppose T satisfies the condition (2). We first shall prove that (2) ⇒(3).

For $n \in N$, take $\varepsilon = \frac{1}{n}$ and by (2) we have that exists $r \in N,\ \delta > 0$ and $\alpha_n = \frac{1}{n} + \delta$ such that $\delta_{ij} \le \varepsilon + \delta = \alpha_n \Rightarrow \delta_{ij} < \varepsilon = \frac{1}{n}$ for $i, j \in N$.

(3) Now, suppose T satisfies (3). In the same way as (1), substituting $x = x_n$ and $y = x_{n+1}$ in (3) we obtain:

the sequence $\{x_n\}$ is bounded in (X, p) and

$\forall n \in N, \exists \alpha_n \in (0,+\infty), \exists r \in N,$ such that $\delta_{ij} \le \alpha_n \Rightarrow \delta_{i+r, j+r} \le \frac{1}{n}$ whenever $i, j \in N$

but $\delta_{ij} = \sup\{p(T^m x, T^k y) : m \ge i, k \ge j\} = \sup\{p(T^m x_n, T^k x_{n+1}) : m \ge i, k \ge j\} = \sup\{p(T^{m+n}x, T^{k+n+1}x) : m \ge i, k \ge j\}$

$= \sup\{p(x_{m+n}, x_{k+1+n}) : m \ge i, k \ge j\} = \delta_{i+n, j+n+1}(\{x_n\})$

and $\delta_{i+r, j+r} = \sup\{p(T^m x, T^k y) : m \ge i+r, k \ge j+r\} = \delta_{i+n,+rj+n+1+r}(\{x_n\})$

So the sequence $\{x_n\}$ satisfies condition (3) in theorem 3, so it is a Cauchy sequence in (X, p).

(4) It is clear that (4)⇒(2) and if T satisfies (4) than by (2) the sequence $\{x_n\}$ is a Cauchy sequence in (X, p).

Now, since $\{x_n\}$ is a Cauchy sequence in (X, p), by Lemma 1, it is a Cauchy sequence in the metric space (X, d_p). Since (X, p) is complete, by Lemma 2, it is complete with respect to metric d_p , so there is $z \in X$ such that $x_n \to z$

with respect to d_p . By the orbital continuity of T, we deduce that $x_n \to Tz$ with respect to metric d_p . Hence $z = Tz$ and z is a fixed point of T.

Let $y \in X$, where $y \neq x$. The iterative sequence $\{y_n\}$, where $y_{n+1} = Ty_n$, for $n \in N$ is a Cauchy sequence in (X, p) and $y_n \to z_1$

The sequences $\{x_n\}$ and $\{y_n\}$ satisfy conditions (1)-(4) in theorem 2.1, so they are equivalent Cauchy sequences in (X, p) and as shown in the proof of the theorem 2.1. in [4] we have

$$\lim_{n\to\infty} p(x_n, x_n) = \lim_{n\to\infty} p(x_n, y_n) = \lim_{n\to\infty} p(y_n, y_n) .$$

Also, whereas the sequences $\{x_n\}$ and $\{y_n\}$ converge to z and z_1 respectively with respect to d_p, by Lemma 2, we have $p(z, z_1) = \lim_{n\to\infty} p(x_n, y_n) = 0$ and consequently $z = z_1$, which concludes the proof.

REFERENCE

[1] Altun.I, Sola. F, Simsek. H, Generalized contractions on partial metric spaces. Topology Appl. 157(18), 2010, 2778-2785.

[2] A. Meir and E. Keeler, A theoremon contraction mappings, J.Math.Anal.Appl. 28(1969), 326-329.1,3

[3] Bushati. S,. Disa pohime për vargjet Koshi. Bul.Shk. Univ. Shkodrës, Nr. 53(4), 2001, 5-7.

[4] Duraj. S, Hoxha.E, The equivalent Cauchy sequences in partial metric spaces, Journal of Advances in Mathematics (JAM), vol 12, nr.4, 2016, 6148-6155.

[5] J. Jachymski, Equivalent conditions and the Meir-Keeler type theorems, J.Math.Anal.Appl. 194, (1995), 293-303.1

[6] J. Matkowski, Fixed point theorems for contractive mappings in metric spaces, Casopis Pro Pestovani Matematiky. 105 94), (1980), 341-344.1

[7] Karapinar.E, Romaguera. S, Nonunique fixed point theorems in partial metric spaces. Published by Fac. of Scie. and Math., Univ. of Nis, Serbia, 2013, 1305-1314.

[8] Kopperman. R.,Matthews. S.G, and Pajoohesh .H.: What do partial metrics represent?, Spatial representation: discrete vs. continuous computational models, Dagstuhl Seminar Proceedings, No. 04351, Internationales Begegnungs- und Forschungszentrumf ¨ ur Informatik (IBFI), Schloss Dagstuhl, Germany, (2005).

[9] Lj. B. Ciric, A new fixed point theorem for contractivemappings, Publ. Inst. Math. (Beograd) 30 (44), (1981), 25-27.1

[10] Matthews. S. G., Partial metric topology, Research Report 212, Dept. of Computer Science, University ofWarwick, (1992).

[11] Matthews. S. G., Partial metric topology, Proc. 8th Summer Conference on General Topology and Applications, Ann. New YorkAcad. Sci. 728 (1994). 183–197.

[12] Oltra.S. and Valero. O., Banach's fixed point theorem for partial metric spaces, Rend. Ist. Mat. Univ. Trieste 36 (2004), 17–26.

[13] Romaguera. S. and Valero.O, A quantitative computational model for complete partial metric spaces via formal balls, Math.Struct. Comp. Sci. 19 (2009), 541-563.

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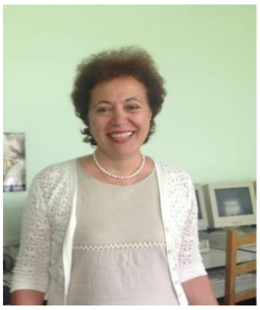

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