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Simply Fuzzy generalized open and closed sets

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Abstract

In this paper, we introduced new concepts, Simply open set, Simply fuzzy open set, Simply fuzzy continuous, Simply fuzzy open function(resp. fuzzy -- closed), simply fuzzy open cover and simply fuzzy compact. We study some of their properties.

Key Words: Simply open set; fuzzy topological space and Near continuous mappings.

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1. Introduction

The concept of fuzzy set and fuzzy set operations were first introduced by L.A. Zadeh in his classical paper [8] in the year 1965. Subsequently several researches have worked on topology using fuzzy set and, subsets naturally plays a very significant role in the study of the topology introduced by C.L. Chang [3]. G. and P. Sundaram [4] introduced and studied generalized closed fuzzy sets in fuzzy topology. K.K.Azad [7] introduced semi closed fuzzy sets in the year 1981. H.Maki, T. Fukutake, M. Kojima and H. Harada [6] introduced semi generalized closed fuzzy sets (briefly fsg closed) in fuzzy topological space in the year 1998.Also in [10], [2] Singal, Rajvanshi and Bin Shahna have introduced the concept of fuzzy $\alpha-$ open sets. Several notions based on fuzzy $\alpha-$ open(closed) sets and fuzzy $\alpha-$ continuous mapping have been studied. Moreover, the study also included the relationship between those concepts and some other weaker forms of fuzzy open sets and fuzzy continuous mappings The aim of this paper is to introduce a new concepts, namely simply open, simply fuzzy open, simply open. Further more we introduce and study a new concepts namely simply fuzzy continuous mappings and also introduced simply fuzzy compactness, simply connectedness.

Definition 1.1 A map $f : X \to Y$ is said to be fuzzy μ^* irresolute map [12] if the inverse image of every μ^* fuzzy open set in Y is fuzzy μ^* open set in $X.$

Definition 1.2 A map $f : X \to Y$ is said to be: Fuzzy continuous map [4] if $f^{-1}(G)$ is fuzzy open (fuzzy closed) set in X for each fuzzy open (fuzzy closed) set G in $Y.$

Definition 1.3 Fuzzy $\alpha-$ continuous map [10] if $f^{-1}(G)$ is fuzzy $\alpha-$ open (fuzzy $\alpha-$ closed) set in X for each fuzzy open (fuzzy closed) set G in Y .

Definition 1.4 A subset A of topological space (X,τ) is called: Simply open [1] if $A = G \cup N$ where G is open set and N is nowhere dense and we denote the class of simply

open sets of X and Y by $\overset{M}{S}O(X)$ and $\overset{M}{S}O(Y).$

Definition 1.5 A subset A of topological space (X,τ) is called: $\alpha-$ open [11] if $A \subseteq \operatorname{int}(cl(\operatorname{int} A).$

Definition 1.6 A fuzzy set A in a fuzzy topological space X is called fuzzy $\alpha-$ open set [5] if $A \le \operatorname{int}(cl(\operatorname{int} A).$

Definition 1.7 A mapping $f : (X,\tau) \to (Y,\sigma)$ is said to be Simply irresolute map [11] (for short, $\overset{M}{S}-$ irresolute) if $f^{-1}(G) \in \overset{M}{S}O(X)$ for every $G \in \overset{M}{S}O(Y).$

Definition 1.8 A mapping $f : (X,\tau) \to (Y,\sigma)$ is said to be simply continuous map [1] $f^{-1}(G) \in \overset{M}{S}O(X)$ for every $G \in \sigma.$

Theorem 1.1 Let $f :(X,\tau) \to (Y,\sigma)$ and $g :(Y,\sigma) \to (Z,\eta)$ be two maps [12]. Then the following are hold:

1) If f is surjective fuzzy μ^* open map (resp. fuzzy μ^* closed) and g is simply fuzzy open map. Then $g \circ f$ is μ^* continuous.
2) If $g \circ f$ is fuzzy μ^* continuous map .Then g is μ^* open.
3) If $g \circ f$ is fuzzy μ^* open map and g is surjective simply fuzzy continuous map. Then f is μ^* open.

Definition 1.9 A subset A of a topological space (X,τ) is said to be simply* compact [9] (briefly $\overset{M^*}{S}$ -compact X if every $\overset{M^*}{S}-$ open cover of has a finite sub cover.

2. Simply fuzzy open set

Definition 2.1 The interior of A as the joining of all the open subset contained in $A,$ denoted by $A^{\circ},$ i.e,

$A^{\circ} = \vee\{B \in \delta \ : \ B \le A\}.$

Definition 2.2 The closure of A as the meeting of all the closed subset contained A, denoted by $\bar{A}$ i.e ,

$\bar{A} = \wedge\{B \in \delta^{\backslash} \ : \ B \ge A\}.$

Definition 2.3 A subset B of a topological space (X, τ) is called simply open set if $\left(\bar{B}\right)^{\circ} \subseteq \left(\overline{\overset{\circ}{B}}\right).$

Definition 2.4 A subset B of a fuzzy topological space (X, τ) is called fuzzy simply open set if $\mu\left(\bar{B}\right)^{\circ} \le \mu\left(\overline{\overset{\circ}{B}}\right).$

$B = \{(a,0.3),(b,0.5),(c,0.9)\}, \tau^{\backslash} = \{1,0,B^{\backslash}\}, B^{\backslash} = (a,0.7),(b,0.5),(c,0.1), \bar{B} = \{(a,0.7),(b,0.5),(c,0.1)\} \therefore \left(\bar{B}\right)^{\circ} = 0$

(1) $, \because B^{\circ} = \{(a,0.3),(b,0.5),(c,0.9)\} \therefore \left(\overline{B^{\circ}}\right) = 1.$ Then we get $\mu\left(\bar{B}\right)^{\circ} \le \mu\left(\overline{\overset{\circ}{B}}\right).$ Therefore B is called simply open.

Example 2.2 Let A be a fuzzy subset of X , $A = \{(a,0.2),(b,0.6),(c,0.3)\}$ and $\tau = \{1,0,\{(a,0.3),(b,0.5),(c,0.9)\}\}$, $\tau^{\backslash} = \{1,0,\{(a,0.7),(b,0.5),(c,0.1)\}\}$, then $\bar{A} = 1 \therefore \left(\bar{A}\right)^{\circ} = 1, \because A^{\circ} = 0 \therefore \overline{A^{\circ}} = 0.$ Then we get $\mu\left(\bar{A}\right)^{\circ} \not\le \mu\left(\overline{\overset{\circ}{A}}\right).$ Therefore A is not simply open

Remark 2.1 Any fuzzy subset of X is not simply fuzzy open set .

Remark 2.2 Any fuzzy open set in fuzzy topological space (X, τ) is simply fuzzy open set .

3. Simply fuzzy continuous mappings

Definition 3.1 A map $f \ : \ X \to Y$ is said to be simply fuzzy continuous map if the inverse image of every fuzzy open set in Y is simply fuzzy open set in X .

Definition 3.2 A map $f \ : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy open (resp. fuzzy -closed) map if the image of fuzzy open set (resp. fuzzy closed set) in X is fuzzy open (resp. fuzzy closed) set in Y .

Theorem 3.1 A map $f \ : \ X \to Y$ is said to be simply fuzzy continuous map if the inverse image of every fuzzy closed set in Y is simply fuzzy closed set in X .

Proof: let G is fuzzy closed set in Y. Then G^{C} is fuzzy open set in Y , since f is simply fuzzy continuous map. Then $f^{-1}(Y - G) = X - f^{-1}(G)$ is simply fuzzy open. Then $f^{-1}(G)$ is simply closed set, therefore a map f is simply fuzzy continuous map.

Theorem 3.2 If $f \ : \ X \to Y$ is a simply fuzzy continuous map and $f \ : \ X \to Y$ is fuzzy continuous map. Then $g \circ f \ : X \to Z$ is simply fuzzy continuous map.

Proof: Let $g \ : \ Y \to Z$ is fuzzy continuous map. Then for any fuzzy open set G in Z , then $f^{-1}(G)$ is fuzzy open set in Y , and since f is simply fuzzy continuous map. Thus $f^{-1}(g^{-1}(G)) = (G \circ f)^{-1}(G)$ is fuzzy open set in X is simply fuzzy open set in X , therefore $g \circ f$ is simply fuzzy continuous map.

Proposition 3.1 If $g \circ f$ is fuzzy open and g simply fuzzy continuous map. Then f is simply fuzzy open map .

Proof: Since $g \circ f$ is fuzzy open map. Then for every fuzzy open set G in X , then $(g \circ f)(G)$ is fuzzy open set in Z . Since g simply fuzzy continuous map, then $g^{-1}(g \circ f)(G) = f(G)$ simply fuzzy hence f is simply fuzzy open map.

Theorem 3.3 Let $f : X \to Y$ be a map from a fuzzy topological space X into a fuzzy topological space Y. Then

1) f is simply fuzzy continuous map .

2) The inverse image of each fuzzy open set in Y is simply fuzzy open set in X.

Proof: Obvious from the definition.

Definition 3.3 A map $f : X \to Y$ is said to be fuzzy simply irresolute map if the inverse image of every simply fuzzy open set in Y is simply fuzzy open set in X.

Theorem 3.4 A map $f : X \to Y$ is said to be fuzzy simply irresolute map if the inverse image of every simply fuzzy closed set in Y is simply fuzzy closed set in X.

Proof: Let G is fuzzy closed set in Y .Then $Y - G$ is simply fuzzy open set in Y since f is simply fuzzy irresolute. Then $f^{-1}(Y - G) = X - f^{-1}(G)$ is simply fuzzy open set in X . Thus $f^{-1}(G)$ is simply fuzzy closed set in X.

Theorem 3.5 If $f : X \to Y$ is a simply fuzzy irresolute map and $g : Y \to Z$ is a simply fuzzy irresolute map. Then $g \circ f : X \to Z$ is a simply fuzzy irresolute map .

Proof: Let G is simply fuzzy open set in Z , since $g :: Y \to Z$ is simply fuzzy irresolute map. Then $g^{-1}(G)$ is simply fuzzy open set in Y and since $f : X \to Y$ is simply fuzzy irresolute map hence $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is simply fuzzy open set in X . Thus $g \circ f$ is simply fuzzy irresolute map.

Theorem 3.6 If $f : X \to Y$ is a simply fuzzy irresolute and $g :: Y \to Z$ is a simply fuzzy continuous map. Then $g \circ f : X \to Z$ is a simply fuzzy continuous map.

Proof: Since g is imply fuzzy continuous map, then for every fuzzy open set G in Z . Then $g^{-1}(G)$ is simply open set in Y and since f is simply fuzzy irresolute map. Then $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is simply fuzzy open set in X. Thus $g \circ f$ is simply fuzzy continuous map.

Theorem 3.7 If $f : X \to Y$ is a simply fuzzy irresolute map. Then it is simply fuzzy continuous map.

Proof: Obvious.

Theorem 3.8 $f : X \to Y$ be a map. Then the following statement are equivalent

1) f is simply fuzzy irresolute map.

2) The inverse image of each simply fuzzy open set in Y is simply open set in X.

Proof: Obvious from the definition.

Theorem 3.9 Every fuzzy $\alpha -$ continuous map is simply fuzzy continuous map.

Proof: Obvious.

Theorem 3.10 Every fuzzy continuous map is simply fuzzy continuous map.

Proof: obvious.

Remark 3.1 The following implication shows that the above theorem

Fuzzy continuous map $\to$ fuzzy $\alpha -$ continuous map $\to$ simply fuzzy continuous map .

Theorem 3.11 The composition of two simply fuzzy continuous maps need not be simply fuzzy continuous.

Proof: Let $f : X \to Y$ and Let $g : Y \to Z$ be simply fuzzy continuous map, since $g : Y \to Z$ is simply fuzzy continuous, then for every fuzzy open set G in Z . Then $g^{-1}(G)$ is simply fuzzy open set in Y and since $f : X \to Y$ is also simply fuzzy continuous map but not all simply fuzzy open sets is open set. Thus $g \circ f$ is simply fuzzy continuous map.

Proposition 3.2 Every fuzzy open map is simply fuzzy open map .

Proof: Obvious.

Theorem 3.12 Let $f :(X,\tau) \to (Y,\sigma)$ and $g :(Y,\sigma) \to (Z,\eta)$ be two maps. Then the following are hold:

1) If f is surjective fuzz open (resp. fuzzy closed) map and g is simply fuzzy open map. Then $g \circ f$ is simply fuzzy continuous map.
2) If $g \circ f$ is simply fuzzy continuous map. Then g is simply fuzzy open map .
3) If $g \circ f$ is fuzzy open map and g is surjective simply fuzzy continuous map. Then f is simply fuzzy open map.

Proof:

1) Let G fuzzy open set in $(X,\tau),$ since f is fuzzy open open map. Then $f(G)$ is fuzzy open set in $(Y,\sigma),$ since g is simply fuzzy open map, thus $(g \circ f)(G)$ is simply fuzzy open set in Z . Hence $g \circ f$ is simply fuzzy open map.
2) Since f is simply fuzzy continuous map. Then for every fuzzy open set G in Y , then $f^{-1}(G)$ is fuzzy open set in X, since $g \circ f$ is simply fuzzy open map, thus $(g \circ f\ )\ f^{-1}(G) = g(G)$ is simply fuzzy open set in Z hence g is simply fuzzy open map.
3) Since $g \circ f$ is fuzzy open map, then for every fuzzy open set G in X , consequently ($g \circ f$)(G) is fuzzy open set in Y since g is simply fuzzy continuous map and injection, thus $g^{-1}(g \circ f)(G) = f(G)$ is simply fuzzy open set in Y hence f is simply fuzzy open map.

4. Simply fuzzy compactness and simply connectedness

Definition A collection $\{A_\alpha : \alpha \in S\}$ of simply fuzzy open set in fuzzy topological space X is called simply fuzzy open cover of subset B if $B \subset \cup\{A_\alpha : \alpha \in S\}$ holds.

Definition 4.1 A fuzzy topological space X is simply fuzzy compact if every simply fuzzy open cover of X has a finite sub cover .

Definition 4.2 A subset B of a fuzzy topological space X is said to be simply relative to X if for every collection $\{A_\alpha : \alpha \in S\}$ of simply fuzzy open subsets of X such that $B \subset \cup\{A_\alpha : \alpha \in S\},$ there exist a finite subset S_0 of S such that $B \subset \cup\{A_\alpha : \alpha \in S_0\}.$

Theorem 4.1 A subset B of a fuzzy topological space X is said to be simply compact if B is simply compact as subset of $X.$

1) A simply fuzzy continuous image of simply compact.
2) If a map $f : X \to Y$ is simply fuzzy irresolute and subset B is simply fuzzy compact relative to X , then the $f(B)$ is simply compact relative $Y.$

Proof:

1) Let $f : X \to Y$ A simply fuzzy continuous map from simply compact space X onto a topological space Y , let $\{A_\alpha : \alpha \in S\}$ be an simply fuzzy open cover of $Y.$ Then $\{f^{-1}(A_\alpha): \alpha \in S\}$ is a simply fuzzy open cover

of X . Since X is simply fuzzy compact, it has a finite sub cover, say $\{f^{-1}(A_1),....., f^{-1}(A_n)\}$. since f is onto, { $A_1,......A_n$ } a cover of Y and so Y is simply compact.

2) Let $\{A_\alpha \ : \ \alpha \in S\}$ be any collection of simply fuzzy open subset of Y such that $f(B) \subset \cup\{A_\alpha \ : \ \alpha \in S\}$. Then $B \subset \cup\{f^{-1}(A_\alpha): \ \alpha \in S\}$ holds by hypothesis there exist a finite subset S_0 of S, $B \subset \cup\{f^{-1}(A_\alpha): \ \alpha \in S_0\}$. Therefore we have $f(B) \subset \cup\{A_\alpha \ : \ \alpha \in S_0\}$ which shows that is simply fuzzy relative to Y.

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