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(1,2) - Domination in the Total Graphs of C_n , P_n and $K_{1,n}$

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Abstract:

In this paper, we discuss the (1,2) - domination in the total graphs of C_n , P_n and $K_{1,n}$

Key words: Dominating set; domination number; (1,2)-dominating set; (1,2)-domination number; total graph.

Mathematics Subject Classification: 05C69

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1. Introduction

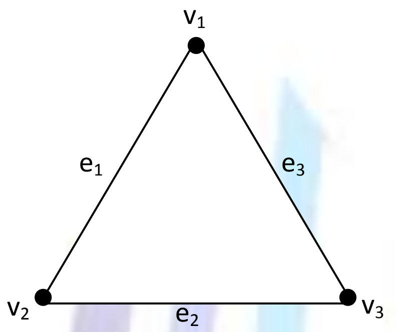
By a graph $G = (V,E)$, we mean a finite, undirected graph without loops or multiple edges. A subset D of V is a dominating set of G if every vertex of V-D is adjacent to a vertex of D. The domination number of G, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set. A *(1,2) dominating set* in a graph G is a set S having the property that for every vertex v in V-S there is at least one vertex in S at distance 1 from v and a second vertex in S within distance 2 of v. The order of the smallest (1,2) - dominating set of G is called the *(1,2) - domination number* of G denoted by $\gamma_{(1,2)}(G)$.

For a given graph G,let $T(G)$ be a graph with vertex set $V(G)\cup E(G)$. Two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one of the following holds (i) x,y are in $V(G)$ and x is adjacent to y in G. (ii) x,y are in $E(G)$ are x,y are adjacent in G (iii) x is in $V(G)$, y is in $E(G)$ and x,y are incident in G. That is, the total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.

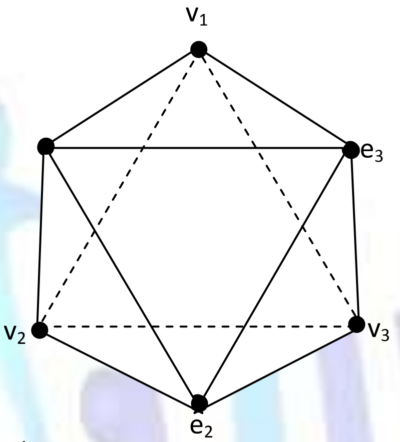
2. (1,2)- domination in Total graph of C_n

Consider the following examples

C_3 :



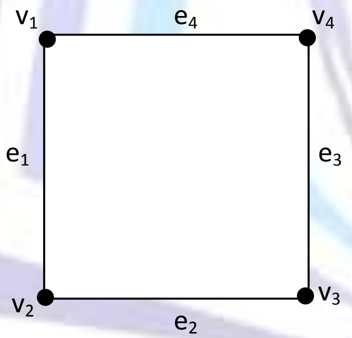
$T(C_3)$:



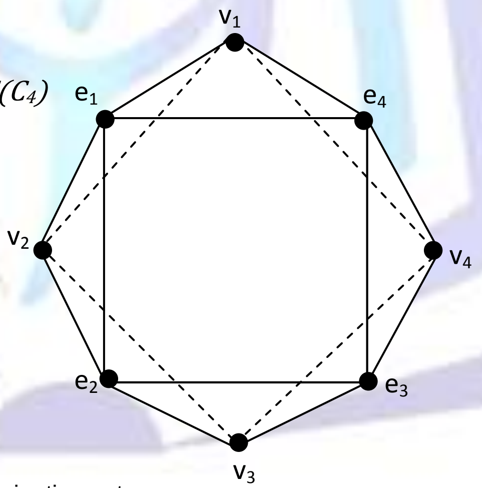
Hence in $T(C_3)$, $\{e_1, v_3\}$ is a dominating set and also (1,2) dominating set.

$\gamma[T(C_3)] = 2$ and $\gamma_{(1,2)}[T(C_3)] = 2$

C_4 :



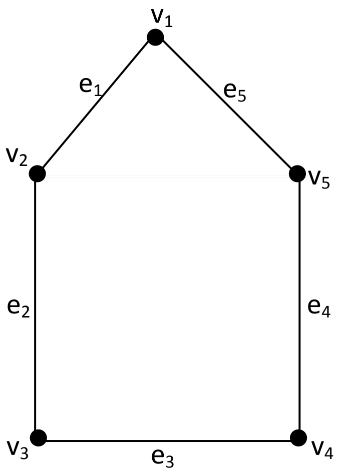
$T(C_4)$



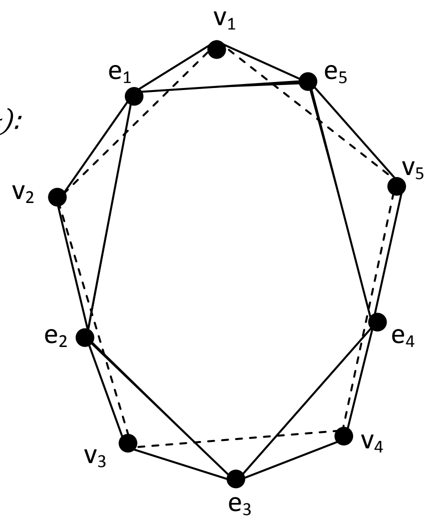
In $T(C_4)$, $\{e_1, v_3\}$ is a dominating set and also (1,2) dominating set.

$\gamma[T(C_4)] = 2$ and $\gamma_{(1,2)}[T(C_3)] = 2$.

C_5:



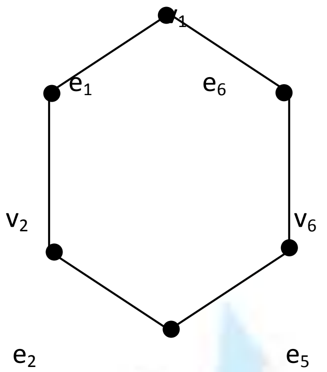
$T(C_5)$:



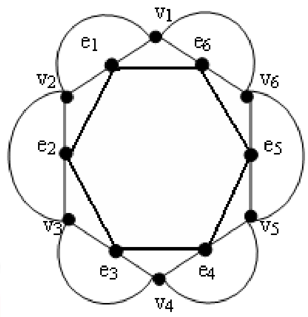
In $T(C_5)$, $\{e_1, v_4\}$ is a dominating set and also (1,2) dominating set.

$\gamma\,[T(C_5)] = 2$ and $\gamma_{(1,2)}\,[T(C_5)] = 2$

C_6 :

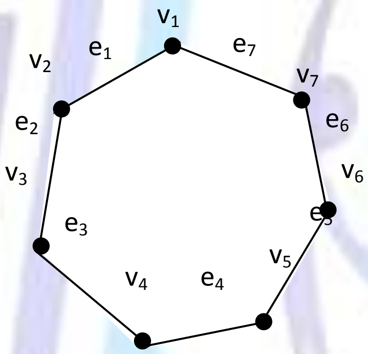


$T(C_6)$:

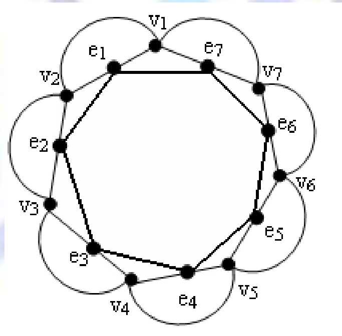


$\{e_1, v_4, v_6\}$ is a dominating set and also (1,2) dominating set.

C_7 :

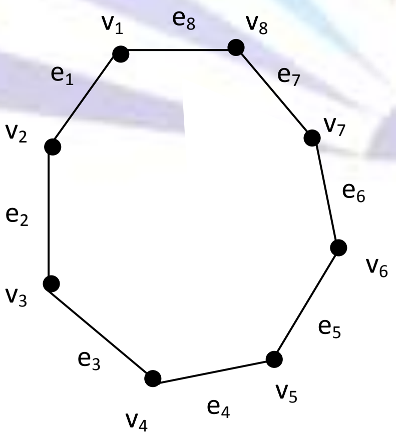


$T(C_7)$:

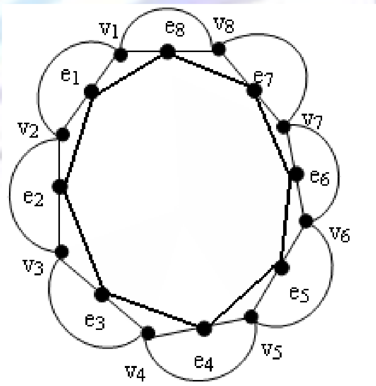


$\{e_1, v_4, v_6\}$ is a dominating set and also a (1,2) dominating set.

C_8 :

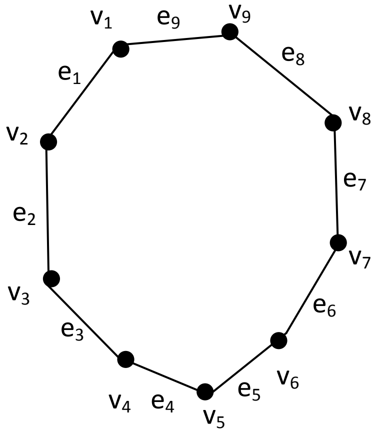


$T(C_8)$:

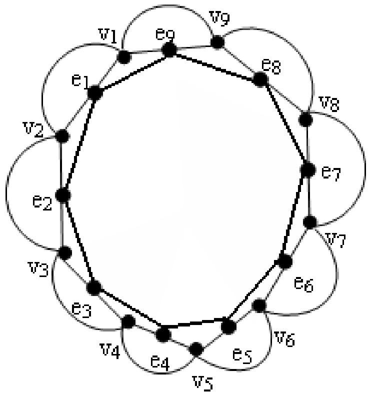


$\{e_1, v_4, v_6, v_8\}$ is a dominating set and also a (1,2) dominating set.

C_9 :

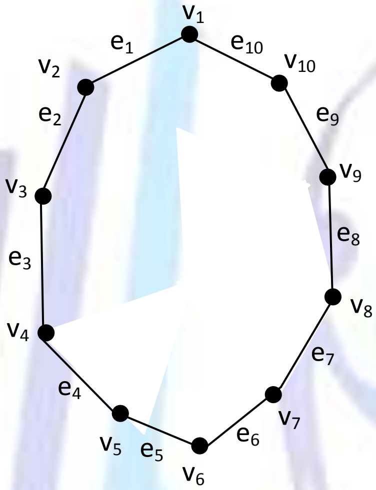


$T(C_9)$:

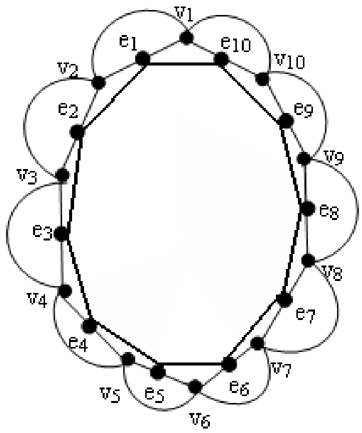


$\{e_1, v_4, v_6, v_8\}$ is a dominating set and also a (1,2) dominating set.

C_{10} :



$T(C_{10})$:



$\{e_1, v_4, v_6, v_8, v_{10}\}$ is a dominating set and also a (1,2) dominating set.

From the above examples, we have the following theorems.

Theorem 2.1

$$\gamma[T(C_n)] = \left\lfloor \frac{n}{2} \right\rfloor.$$

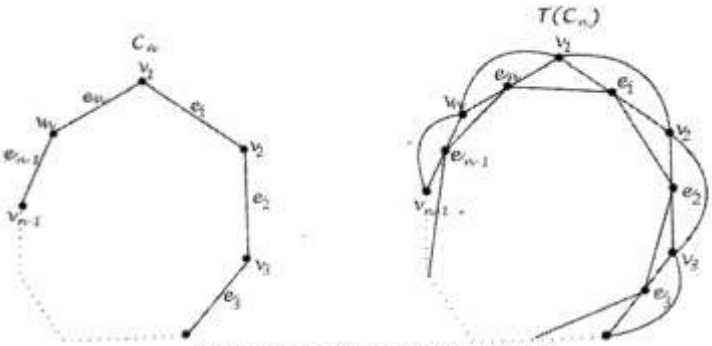


Figure 1. n-cycle and its Total Graph

Proof:

It can be easily observed that $\gamma[T(C_3)]=2$, $\gamma[T(C_4)]=2$, $\gamma[T(C_5)]=2$ and $\gamma[T(C_6)]=3$.

Let $V(C_n) = \{v_1, v_2,..., v_n\}$ and $E(C_n) = \{e_1, e_2,..., e_n\}$ in which $e_i = v_i\ v_{i+1}$. By the definition of total graph $V[T(C_n)] = \{v_1, v_2,..., v_n\} \cup \{e_1, e_2,..., e_n\}$ and

$E[T(C_n)] = \{e_i\ e_{i+1}/1\leq i \leq n\} \cup e_n e_1 \cup \{v_i\ v_{i+1}/1\leq i \leq n\} \cup v_n v_1 \cup \{e_i\ v_{i+1}/1\leq i \leq n\text{-}1\} \cup \{e_n v_1\} \cup \{v_i\ e_i/1\leq i\leq n\}$. The total number of vertices in $T(C_n)$ is $2n$ and each vertex is of degree 4. The cycles of $T(C_n)$ are $C_i = e_i v_{i+1} e_{i+1}$ $(1\leq i \leq n\text{-}1)$, $C_n = e_n e_1 v_1$, $C'_i = v_i v_{i+1} e_i (1\leq i \leq n\text{-}1)$, $C'_n = v_n v_1 e_n$. In $T(C_n)$, each e_i is adjacent to e_{i+1} and e_1 is adjacent to e_n. Also v_i is adjacent to v_{i+1} and v_1 is adjacent to v_n. Now we can find the dominating set of $T(C_n)$.

The minimum dominating set

$$D = \begin{cases} D_1 = \{e_1, v_4, v_6, ..., v_n\} \text{ if } n = 2k \\ D_2 = \{e_1, v_4, v_6, ..., v_{n-1}\} \text{ if } n = 2k+1 \quad \text{where } k = 3,4,5,.... \end{cases}$$

$$|D_1| = \frac{n-4}{2} + 1 + 1 = \frac{n}{2}$$

$$|D_2| = \frac{n-1-4}{2} + 1 + 1 = \frac{n-1}{2}$$

$$|D| = \begin{cases} \dfrac{n}{2} \quad \text{if } n = 2k \\ \dfrac{n-1}{2} \text{if } n = 2k+1 \end{cases}$$

$$= \begin{cases} \dfrac{n}{2} \quad \text{if } n = 2k \\ \dfrac{n}{2} - \dfrac{1}{2} \text{if } n = 2k+1 \end{cases}$$

Hence $\gamma[T(C_n)] = \left\lfloor \dfrac{n}{2} \right\rfloor$ for all values of C_n.

Theorem 2.2

$$\gamma_{(1,2)}[T(C_n)] = \left\lfloor \frac{n}{2} \right\rfloor$$

Proof:

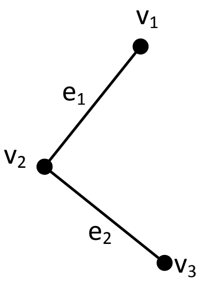
It can be easily observed that $\gamma_{(1,2)}[T(C_3)]=2$, $\gamma_{(1,2)}[T(C_4)]=2$, $\gamma_{(1,2)}[T(C_5)]=2$ and $\gamma_{(1,2)}[T(C_6)]=3$. Let $V(C_n) = \{v_1, v_2,..., v_n\}$ and $E(C_n) = \{e_1, e_2,..., e_n\}$ in which $e_i = v_i\ v_{i+1}$. By the definition of total graph $V[T(C_n)] = \{v_1, v_2,..., v_n\} \cup \{e_1, e_2,..., e_n\}$ and

$E[T(C_n)] = \{e_i\ e_{i+1}/1\leq i \leq n\} \cup e_n e_1 \cup \{v_i\ v_{i+1}/1\leq i \leq n\} \cup v_n v_1 \cup \{e_i\ v_{i+1}/1\leq i \leq n\text{-}1\} \cup \{e_n v_1\} \cup \{v_i\ e_i/1\leq i\leq n\}$. The total number of vertices in $T(C_n)$ is $2n$ and each vertex is of degree 4. The cycles of $T(C_n)$ are $C_i = e_i v_{i+1} e_{i+1}$ $(1\leq i \leq n\text{-}1)$, $C_n = e_n e_1 v_1$, $C'_i = v_i v_{i+1} e_i (1\leq i \leq n\text{-}1)$, $C'_n = v_n v_1 e_n$. Every minimum cardinality dominating set is also a (1,2) - dominating set in $T(C_n)$.Hence $\gamma_{(1,2)}[T(C_n)] = \left\lfloor \dfrac{n}{2} \right\rfloor$.

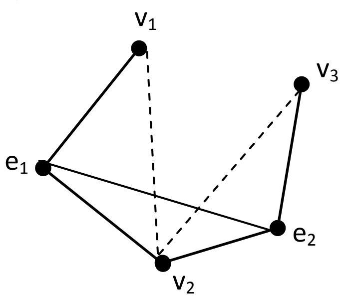
3. (1,2) domination in Total graph of P_n

Consider the following examples

P_3 :



$T(P_3)$:



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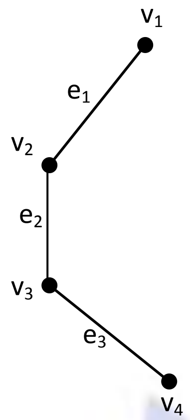
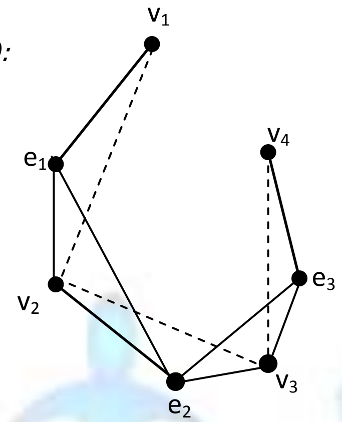
In $T(P_3)$, $\{v_2\}$ is a dominating set and $\{e_1, v_3\}$ is a (1,2) dominating set.

Therefore $\gamma[T(P_3)] = 1$ and $\gamma_{(1,2)}[T(P_3)] = 2$

P_4 :



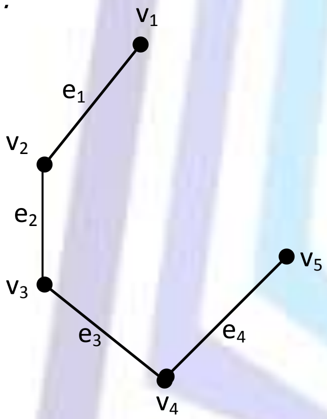
$T(P_4)$:



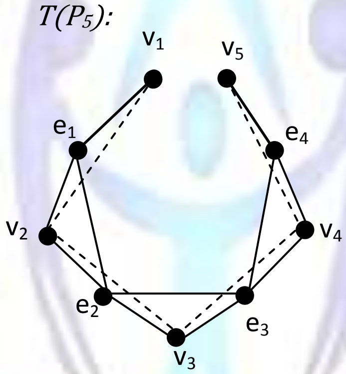
In $T(P_4)$, $\{v_2, e_3\}$ is a dominating set and $\{v_1, v_2, v_4\}$ is a (1,2) dominating set.

Therefore $\gamma[T(P_4)] = 2$ and $\gamma_{(1,2)}[T(P_4)] = 3$

P_5 :



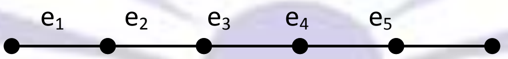
$T(P_5)$:



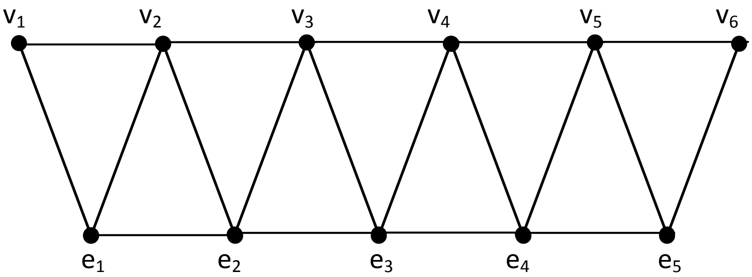
In $T(P_5)$, $\{v_2, v_4\}$ is a dominating set and $\{v_1, v_3, v_5\}$ is a (1,2) dominating set.

Therefore $\gamma[T(P_5)] = 2$ and $\gamma_{(1,2)}[T(P_5)] = 3$

P_6 :

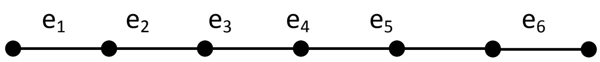


$T(P_6)$:



$\{v_2, v_4, v_6\}$ is a dominating set and $\{v_1, v_3, v_5\}$ is a (1,2) dominating set.

P_7 :



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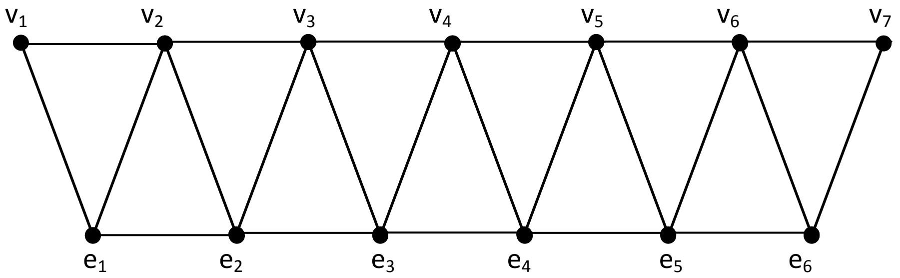
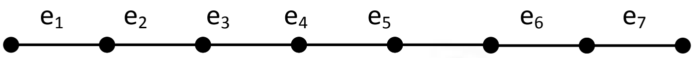
T(P₇):

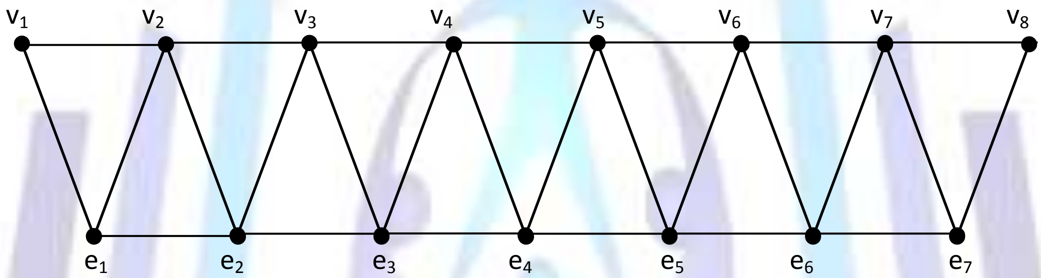


{v_2, v_4, v_6} is a dominating set and { v_1, v_3,v_5,v_7} is a (1,2) dominating set.

P_8 :

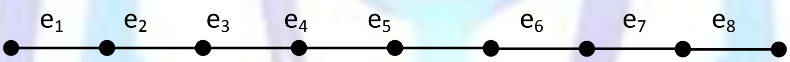


$T(P_8)$:

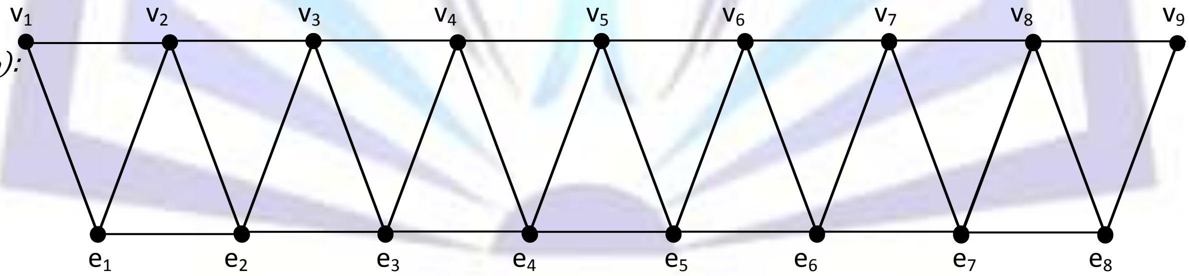


{v_2, v_4, v_6, v_8} is a dominating set and { v_1, v_3,v_5,v_7} is a (1,2) dominating set.

P_9 :



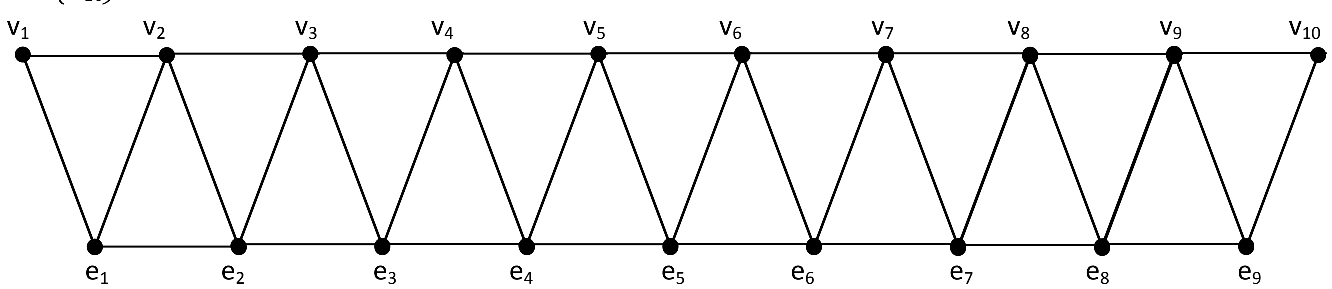
$T(P_9)$:



{v_2, v_4, v_6, v_8} is a dominating set and { v_1, v_3,v_5,v_7,v_9} is a (1,2) dominating set.

P_{10} :

$T(P_{10})$:



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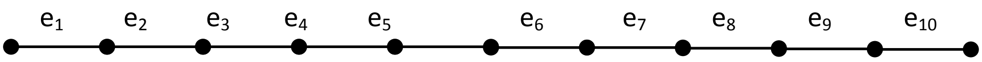
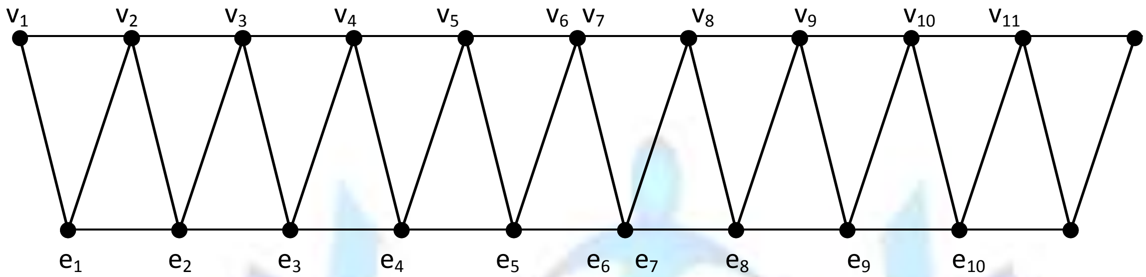
$\{v_2, v_4, v_6, v_8, v_{10}\}$ is a dominating set and $\{v_1, v_3, v_5, v_7, v_9\}$ is a (1,2) dominating set.

P_{11} :



$T(P_{11})$:



$\{v_2, v_4, v_6, v_8, v_{10}\}$ is a dominating set and $\{v_1, v_3, v_5, v_7, v_9, v_{11}\}$ is a (1,2) dominating set.

From the above examples, we have the following theorems.

Theorem 3.1

$$\gamma[T(P_n)] = \left\lfloor \frac{n}{2} \right\rfloor$$

Proof : It is easy to observe that $\gamma[T(P3)] = 1$, $\gamma[T(P_4) = 2$, $\gamma[T(P_5)] = 3$

Let P_n be a path on n vertices where $n > 5$.

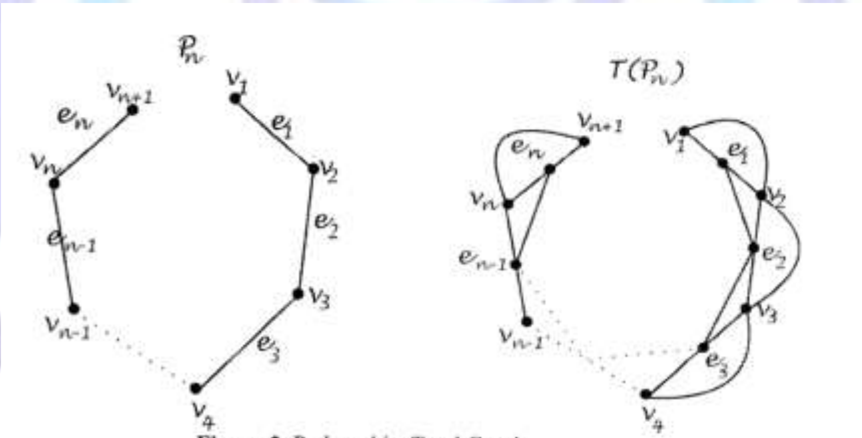


Figure 2. Path and its Total Graph

Let $V(P_n) = \{v_1, v_2,..., v_n\}$ and $E(P_n) = \{e_1, e_2,..., e_n\}$. By the definition of total graph, $V[T(P_n)] = V(P_n) \cup E(P_n)$, $E[T(P_n)] = \{v_i e_i / 1 \le i \le n\} \cup \{e_i v_{i+1} / 1 \le i \le n-1) \cup \{v_i v_{i+1} / 1 \le i \le n\} \cup \{e_i e_{i+1} / 1 \le i \le n-1\}$. The cycles of $T(P_n)$ are $C_i = v_i v_{i+1} e_i$ $(1 \le i \le n)$, and $C'_i = e_i e_{i+1} v_{i+1} (1 \le i \le n-1)$. In $T(P_n)$ v_1 and v_n are non- adjacent. Also e_1 and e_{n-1} are non adjacent. Each vertex v_i, $2 \le i \le n-1$, and e_i, $2 \le i \le n-2$ are of degree 4. Now we can find the minimum dominating set. For $n = 2k$, $\{v_2, v_4, ..., v_n\}$ is a dominating set and for $n = 2k+1$, $\{v_2, v_4, ..., v_{n-1}\}$ is a dominating set. Therefore the minimum dominating set

$$D = \begin{cases} D_1 = \{v_2, v_4, ..., v_n\} & if \quad n = 2k \\ D_2 = \{v_2, v_4, ..., v_{n-1}\} & if \quad n = 2k+1 \end{cases}$$

$$| D_1| = \frac{n-2}{2} + 1 = \frac{n}{2}$$

$| D_2| = \frac{n-1-2}{2} + 1 = \frac{n-1}{2}$

Hence $|D| = \begin{cases} \frac{n}{2} & if\ n = 2k \\ \frac{n}{2} - \frac{1}{2} & if\ n = 2k+1 \end{cases}$

Hence $\gamma[T(P_n)] = \left\lfloor \frac{n}{2} \right\rfloor$ for all values of P_n.

Theorem 3.2

$\gamma_{(1,2)}[T(P_n) = \left\lceil \frac{n}{2} \right\rceil$

Proof

It is easy to observe that $\gamma_{(1,2)}[T(P_3)] = 2$, $\gamma_{(1,2)}[T(P_4)] = 3$, $\gamma_{(1,2)}[T(P_5)] = 3$.

Let P_n be a path on n vertices where n >5.

Let $V(P_n) = \{v_1, v_2,..., v_n\}$ and $E(P_n) = \{e_1, e_2,..., e_n\}$. By the definition of total graph, $V[T(P_n)] = V(P_n) \cup E(P_n)$, $E[T(P_n)] = \{v_i e_i / 1\le i\le n\} \cup \{e_i v_{i+1} / 1\le i \le n-1)$

$\cup\{v_i v_{i+1} / 1\le i \le n\} \cup \{e_i e_{i+1} / 1\le i \le n-1\}$. The cycles of $T(P_n)$ are $C_i = v_i v_{i+1} e_i$ $(1\le i \le n)$, and $C'_i = e_i e_{i+1} v_{i+1} (1\le i \le n-1)$. In $T(P_n)$ v_1 and v_n are non- adjacent. Also e_1 and e_{n-1} are non adjacent.

Now we can find the (1,2)- dominating set.

For $n= 2k$, $\{v_1, v_3, ..., v_{n-1}\}$ will form a (1,2)- dominating set and for $n = 2k+1$, $\{v_1, v_3, ..., v_n\}$ will form a (1,2) dominating set for $T(P_n)$.

Therefore the minimum dominating set

$S = \begin{cases} S_1 = \{v_1, v_3, ..., v_{n-1}\} & if \quad n = 2k \\ S_2 = \{v_1, v_3, ..., v_n\} & if \quad n = 2k+1 \end{cases}$

$| S| = \begin{cases} | S_1 | & if\ n = 2k \\ | S_2 | & if\ n = 2k+1 \end{cases}$

$| S_1| = \frac{n-1-1}{2} + 1 = \frac{n}{2}$

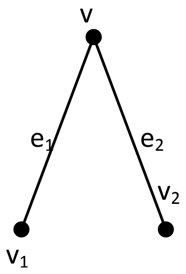
$| S_2| = \frac{n-1}{2} + 1 = \frac{n-1+2}{2} = \frac{n+1}{2}$

$| S| = \begin{cases} \frac{n}{2} & if\ n=2k \\ \frac{n+1}{2} & if\ n=2k+1 \end{cases}$

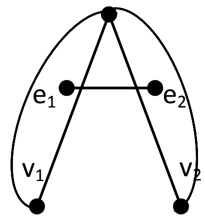
$\gamma_{(1,2)}[T(P_n)] = \left\lceil \frac{n}{2} \right\rceil$

4. (1,2)- domination in Total graph of $K_{1,n}$

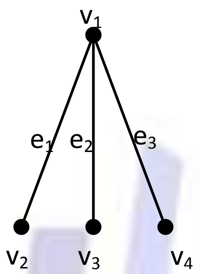
$K_{1,2}$:



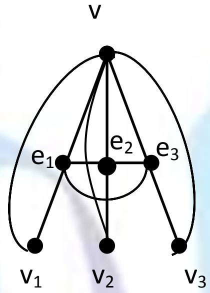
$T(K_{1,2})$:



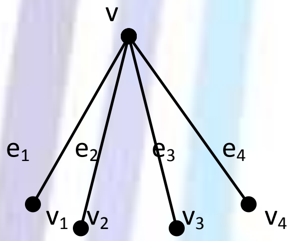
$K_{1,3}$



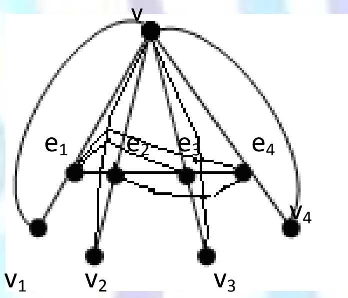
$T(K_{1,3})$:



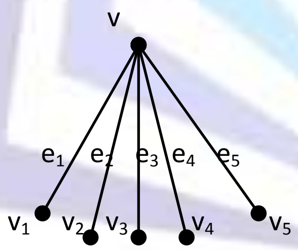
$K_{1,4}$



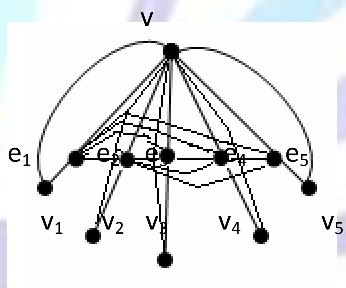
$T(K_{1,4})$:



$K_{1,5}$



$T(K_{1,5})$:



In the above examples the central vertex {*v*} is a dominating set and {*v*, e_1} is a(1,2) - dominating set.

Theorem 4.1

$$\gamma\,[T(K_{1,n})] = 1$$

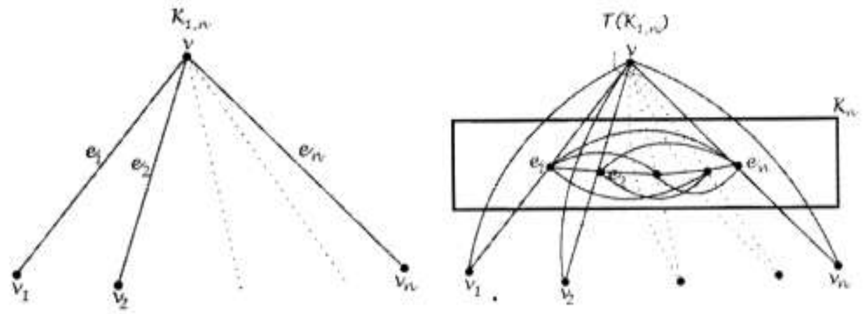


Figure 3. Star graph and its Total Graph

Proof :

Let $V(K_{1,n}) = \{v, v_1, v_2,..., v_n\}$ and $E(K_{1,n}) = \{e_1, e_2,..., e_n\}$. By the definition of total graph $V[T(K_{1,n})] = \{v\} \cup \{e_i / 1 \le i \le n\} \cup \{v_i / 1 \le i \le n\}$in which the vertices. $e_1, e_2, ..., e_n$ induces a clique of order n. Also the vertex v is adjacent with v_i $(1 \le i \le n)$. In $T(K_{1,n})$, the central certex $\{v\}$ dominates all other vertices. Hence $\gamma[T(K_{1,n})] = 1$

Theorem 4.2

$\gamma_{(1,2)}[T(K_{1,n})] = 2$

Proof:

Let $V(K_{1,n}) = \{v, v_1, v_2,..., v_n\}$ and $E(K_{1,n}) = \{e_1, e_2,..., e_n\}$. By the definition of total graph $V[T(K_{1,n})] = \{v\} \cup \{e_i / 1 \le i \le n\} \cup \{v_i / 1 \le i \le n\}$in which the vertices. $e_1, e_2, ..., e_n$ induces a clique of order n. Also the vertex v is adjacent with v_i $(1 \le i \le n)$. In $T(K_{1,n})$, the central certex $\{v\}$ dominates all other vertices. Hence $\gamma[T(K_{1,n})] = 1$. So we can form a (1,2) - dominating set by selecting the central vertex v and any one of $e_i, (1 \le i \le n)$. Hence $\gamma_{(1,2)}[T(K_{1,n})] = 2$.

5. Relation between domination number and (1,2) domination number in the total graph of C_n, P_n and $K_{1,n}$.

Theorem 5.1

In $T(C_n)$, domination number equals (1,2) - domination number.

Proof :

By theorem 2.1 and 2.2, we have this result.

Theorem 5.2

In $T(P_n)$, domination number is less than or equal to (1,2) - domination number.

Proof:

This result is obvious from theorem 3.1 and 3.2.

Theorem 5.3

In $T(K_{1,n})$, domination number is less than (1,2) - domination number.

Proof :

This is clear from theorem 4.1 and 4.2.

6. Conclusion

In the paper, we found the domination number and (1,2) domination number of total graphs of C_n, P_n and $K_{1,n}$. It can be seen that domination number is less than or equal to the (1,2)- domination number in all cases which conincides with the result of [5].

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