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On folding of fuzzy dynamical chaotic manifold

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Abstract

In this paper, we introduce some types of fuzzy dynamical chaotic manifold .We will also introduce a new type of folding, which is applied to fuzzy dynamical chaotic manifold and we will study the relation between the geometric folding and the chaotic folding of fuzzy dynamical chaotic manifold into itself. And the fuzzy dynamical chaotic manifold will be an achieved deduced.

Keywords: Fuzzy; dynamical manifold; chaotic; folding

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Introduction:

Folding of a manifold are discussed in [8] and the isometric foldingof manifold we defined by S.A, Robertson [11],also some studies of the folding are discussed in [3-10] Modern discussions of chaos are almost always based on the work of Edward N.Lorenz [1].

In this paper, We will introduce some types of folding of fuzzy dynamical chaotic manifold into it self and we will discuss the relation between the geometric dynamical manifold and fuzzy dynamical chaotic manifold .We will obtain some theorems which describe the relation between the folding and fuzzy dynamical chaotic manifold, and the end of the limit of folding are treated.

Definitions and background:

(1) Map $f : M \to N$, where M, N are ∞ - Riemannian manifolds of dimensions m, n respectively is said to be an isometric folding of M into N, if and only if for any piecewise geodesic path $\gamma : J \to N$, the induced path for $\gamma : J \to M$ is piecewise geodesic and of the same length as $\gamma, \gamma = [0,1]$ [11] .

If f not preserves lengths then f is a topological folding.

(2) Chaotic manifold is a manifold, which has many physical characters. If dimension of the manifold changed with the time then the manifold is called dynamical manifold.

(3) A fuzzy subset $\tilde{A}$ in X has membership function $\mu(x)$ taking its values in the interval $(0,1)$. The membership function $\mu(x)$ indicates the degree of membership of $x \in X$ in $\tilde{A}$.

The Main Results:

Aiming to our study we will introduce some foldings.

Theorem (1):

The folding of the principle manifold M into itself induces a folding to every pure chaotic manifold M_i into it self.

Proof:

Let $(M, M_1, M_2, ..., M_i)$ is chaotic manifold, let $f : (M, M_1, M_2, ..., M_i) \to (N, N_1, N_2, ..., N_i)$ if $f(M) = N$, $f_1(M_1) = N_1$, $f_2(M_2) = N_2$,..., $f_i(M_i) = N_i$, where $M, M_1, M_2, ..., M_i$ are the pure chaotic manifolds $N, N_1, N_2, ..., N_i$ are a pure chaotic folded manifolds of M_j, $j = 1,2,...,i$ If f is topological folding induce a topological folding $f_1(M_1)$ and the set of singularities of f induces a set of singularities of f_1 which are homeomorphism to each others . See Fig. (1)

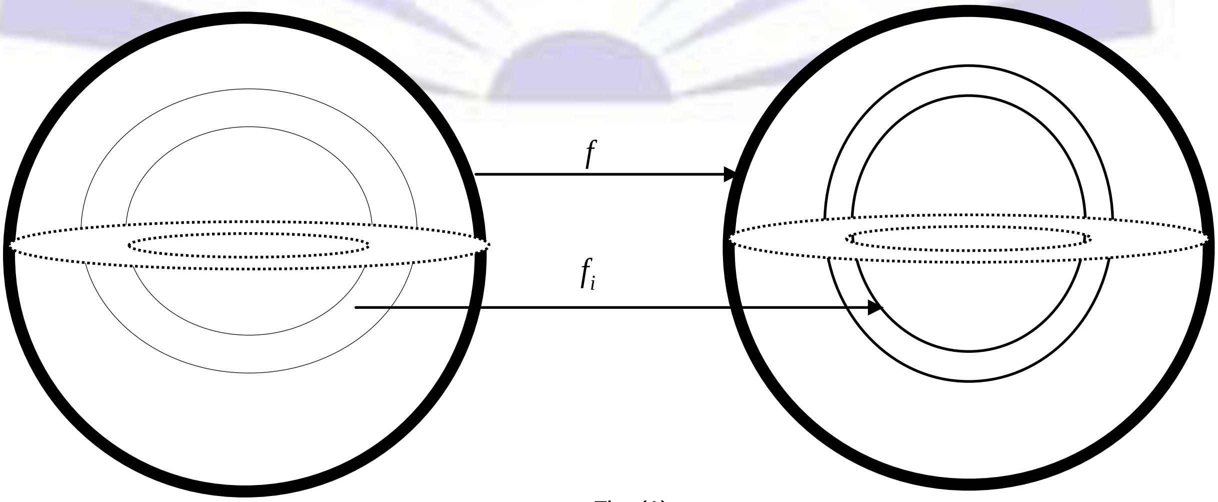

Fig. (1)

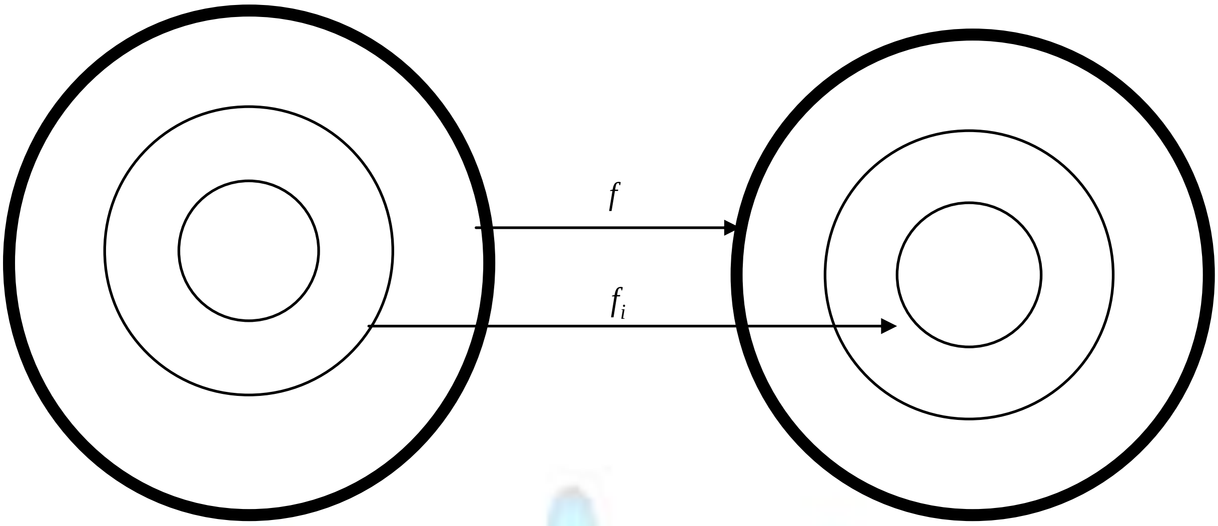

Fig. (2)

And if the set of singularities of $f = \phi \Rightarrow \sum f_i = \phi$ see Fig.(2)

Lemma(1):

The limit of folding of f_j induce a limit of folding of f_j of every pure chaotic manifolds into itself and $\lim_{j\to\infty} f_j(M)$ homeomorphic to $\lim_{j\to\infty} f_j(M_k), j = 1,2,\ldots i$

Proof:

Let $\dim M = m \quad , \dim M_j = m, j = 1,2,\ldots, i$

Assume we have a sequence of foldings

$f_1 : M \to M \quad , f_1(M) = M^1$ induce a folding

$\bar{f}_1 : M_1 \to M_1 \quad , \bar{f}_1(M_1) = M_1^{\ 1}$

$\bar{f}_2 : M_2 \to M_2 \quad , \bar{f}_1(M_2) = M_2^{\ 1}$

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$\bar{f}_i : M_i \to M_i \quad , \bar{f}_i(M_i) = M_i^{\ 1}$

Where $M_1^1, M_2^1, \ldots, M_i^1$ are homeomorphism to M^1 M^1 and of the same dimensions.

Lemma (2):

For any chaotic folding we will arrive to the folding diagrams.

Proof:

Consider $f_1 : M \to M_2^1$, There is a homeomorphism $H_1^1 : M \to M_1$ and we will arrive to the following sequence of commutative diagrams.

$$\begin{array}{ccccccccccc}
M & \xrightarrow{f_1} & M_2^1 & \xrightarrow{f_2} & M_3^2 & \xrightarrow{f_3} & M_4^3 & \xrightarrow{f_4} & \cdots & \xrightarrow{\lim_{j\to\infty} f_j} & M'' \\
\downarrow {\scriptstyle H_1^1} & & \downarrow {\scriptstyle H_1^2} & & \downarrow {\scriptstyle H_1^3} & & \downarrow {\scriptstyle H_1^4} & & \downarrow & & \downarrow \\
M & \xrightarrow{\bar f_1^1} & M_2^1 & \xrightarrow{\bar f_1^2} & M_3^2 & \xrightarrow{\bar f_1^3} & M_4^3 & \xrightarrow{\bar f_1^4} & \cdots & \xrightarrow{\lim_{j\to\infty} \bar f_1^j} & M''
\end{array}$$

,

$$\begin{array}{ccccccccccc}
M & \xrightarrow{f_1} & M_2^1 & \xrightarrow{f_2} & M_3^2 & \xrightarrow{f_3} & M_4^3 & \xrightarrow{f_4} & \cdots & \xrightarrow{\lim_{j\to\infty} f_j} & M'' \\
\downarrow {\scriptstyle H_2^1} & & \downarrow {\scriptstyle H_2^2} & & \downarrow {\scriptstyle H_2^3} & & \downarrow {\scriptstyle H_2^4} & & \downarrow & & \downarrow \\
M & \xrightarrow{\bar f_2^1} & M_2^1 & \xrightarrow{\bar f_2^2} & M_3^2 & \xrightarrow{\bar f_2^3} & M_4^3 & \xrightarrow{\bar f_2^4} & \cdots & \xrightarrow{\lim_{j\to\infty} \bar f_2^j} & M''
\end{array}$$

,

$$\begin{array}{ccccccccccc}
M & \xrightarrow{f_1} & M_2^1 & \xrightarrow{f_2} & M_3^2 & \xrightarrow{f_3} & M_4^3 & \xrightarrow{f_4} & \cdots & \xrightarrow{\lim_{j\to\infty} f_j} & M'' \\
\downarrow {\scriptstyle H_3^1} & & \downarrow {\scriptstyle H_3^2} & & \downarrow {\scriptstyle H_3^3} & & \downarrow {\scriptstyle H_4^4} & & \downarrow & & \downarrow \\
M & \xrightarrow{\bar f_3^1} & M_2^1 & \xrightarrow{\bar f_3^2} & M_3^2 & \xrightarrow{\bar f_1^3} & M_4^3 & \xrightarrow{\bar f_3^4} & \cdots & \xrightarrow{\lim_{j\to\infty} \bar f_3^j} & M''
\end{array}$$

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$$\begin{array}{ccccccccccc}
M & \xrightarrow{f_1} & M_2^1 & \xrightarrow{f_2} & M_3^2 & \xrightarrow{f_3} & M_4^3 & \xrightarrow{f_4} & \cdots & \xrightarrow{\lim\limits_{j\to\infty} f_j} & M'' \\
\downarrow{\scriptstyle H_i^1} & & \downarrow{\scriptstyle H_i^2} & & \downarrow{\scriptstyle H_i^3} & & \downarrow{\scriptstyle H_i^4} & & \downarrow & & \downarrow \\
M & \xrightarrow{\bar{f}_i^1} & M_2^1 & \xrightarrow{\bar{f}_i^2} & M_3^2 & \xrightarrow{\bar{f}_i^3} & M_4^3 & \xrightarrow{\bar{f}_i^4} & \cdots & \xrightarrow{\lim\limits_{j\to\infty} \bar{f}_i^j} & M''
\end{array}$$

Such that, H_i is a homeomorphism

$H_1^2 \circ f_1 = \bar{f}_1 \circ H_1^1,$

$H_1^{K+1} \circ f_K = \bar{f}_1^K \circ H_1^K,$

$H_1^{K+1} \circ f_K = \bar{f}_2^K \circ H_2^K$

Theorem (2):

The converse of the above theorem is not true. Any folding of M_i into itself not induces a folding of M into itself.

Proof:

Let $f(M, M_1, M_2, ..., M_i) \rightarrow (M, M_1, M_2, ..., M_i)$

Such that $f(M, M_1, M_2, ..., M_i) = (M, f(M_1), M_2, ..., M_i)$

$f(M_1)$ is homeomorphic or not homeomorphic to M ,if $f_1, f_2, ..., f_j$ is a sequence of foldings of M into itself , $\lim f_j(M_1) = M_1^1$

Then $\dim M_1^1 = n-1, \dim M = n, \dim M_1 = \dim M_2 = ... = \dim \ M_i = n$.

This means that any type of folding or the limit of these types not induce a folding or a limit folding of the manifold M. Consider the dynamical chaotic manifold $(M(t), (M_1(t), \mu_1), (M_2(t), \mu_2), ..., (M_i(t), \mu_i))$

Theorem (3):

The variation of M under the time t not induce a variation of $((M_1(t), \mu_1), (M_2(t), \mu_2), ..., (M_i(t), \mu_i))$, such that $\dim \ M_i(t) = n+k$.

Proof:

Let $M_1(t)$ at $t = t_0$ is an $n-$ manifold $(M_1(t), \mu_1), (M_2(t), \mu_2), ..., (M_i(t), \mu_i)$ are all of dimensions n, if after time t a manifold $M(t)$ changed into $\overline{M}(t) \dim \ \overline{M}(t) = n+1 \rightarrow \dim(M_1(t), \mu_1) = n = \dim \ (M_j(t), \mu_j) = n,$ $j = 1, 2, ..., i$ if $M(t)$ changed into $n-1$, this induce a variation of dimensions in $(M_1(t), \mu_1)$, $(M_2(t), \mu_2)$ $, ..., (M_i(t), \mu_i)$, $\dim(M_j(t), \mu_j) = n-1$.

Theorem (4):

The variation of the dimensions of $(M_j(t), \mu_j)$ not induces a variations of dimensions of $M(t)$.

Proof:

Let $(M_1(t), \mu_1), (M_2(t), \mu_2), ..., (M_i(t), \mu_i)$, such that $\dim M_i(t) = n$, $\dim M(t) = n$ at $t = t_0$, there after time t , $(M_1(t), \mu_1), (M_2(t), \mu_2), ..., (M_i(t), \mu_i)$ changed into $(\overline{M}_1(t), \mu_1), (\overline{M}_2(t), \mu_2), ..., (\overline{M}_i(t), \mu_i)$, where $\dim \overline{M}_i(t) = n-1$, $M(t)$ changed into $\overline{M}(t)$ i.e. , $\dim M(t) = n$.

i.e. The variation of the dimensions of $(M_i(t), \mu_i)$ not induces a variations of dimensions of $M(t)$.

The Limit of foldings of fuzzy dynamical chaotic manifold:-

Applying folding on a fuzzy dynamical chaotic manifold many time will lead to different types of manifolds depends on the types of folding which have been applied to it. The limit of foldings of fuzzy dynamical chaotic manifold will be treated in the following

Theorem (5):

The folding of fuzzy dynamical chaotic manifold under the variation of time induces the variation of the fuzzyication.

Proof:

When the time varies there are three types of folding of fuzzy dynamical chaotic manifold.

The first type:

If $f_1 : (M(t), (M_1(t), \mu_1), (M_2(t), \mu_2), ..., (M_i(t), \mu_i)) \rightarrow (M(t), (M_1(t), \overline{\mu}_1), (M_2(t), \overline{\mu}_2), ..., (M_i(t), \overline{\mu}_i))$ where $\overline{\mu}_i = \mu_i / n$. This means that the density will be smallest if we consider sequence of the foldings of fuzzy dynamical chaotic manifold then $\lim_{i \to \infty} f_i(\mu_i) = 0$.

The second type :

if $f_2 : (M(t), (M_1(t), \mu_1), (M_2(t), \mu_2), ..., (M_i(t), \mu_i)) \rightarrow (M(t), (M_1(t), \overline{\mu}_1), (M_2(t), \overline{\mu}_2), ..., (M_i(t), \overline{\mu}_i))$where $\overline{\mu}_i = n\mu_i$. It should be noted that $\lim_{i \to \infty} \mu_i = 1$.

The third type :

If $f_3 : (M(t), (M_1(t), \mu_1), (M_2(t), \mu_2), ..., (M_i(t), \mu_i)) \rightarrow (M(t), (M_1(t), \overline{\mu}_1), (M_2(t), \overline{\mu}_2), ..., (M_i(t), \overline{\mu}_i))$ where $\overline{\mu}_i = \mu_i$ Such that $\mu_i \in [0,1]$.

The generalization of the fuzzy dynamical chaotic manifold is of the form $M(t), (M_1(t), \mu_1(t))$ $(M(t), (M_1(t), \mu_1(t)), (M_2(t), \mu_2(t)), ..., (M_i(t), \mu_i(t)))$.

Applications:

(1) A sheet of glass with some fogs concentrated on this glass, an electric field effect on this glass and another magnetic field also effect on this fogs glass the force of the electric and magnetic fields varies by the time. This is an example of chaotic dynamical manifold.

(2) The skeleton of human body represent the geometric manifold, and muscles, blood, mistier, electric field...etc. which represent a dynamical chaotic manifold, The density of every character differed from a body to another one, This variation $= \mu$ which is the fuzzy fiction of this dynamical manifold.

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