



## On Soft $\pi$ gr-Closed sets in Soft Topological Spaces

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### ABSTRACT

The aim of this paper is to introduce soft  $\pi$ gr-closed sets in soft topological space which is defined over the universe of the given set with a fixed set of parameters. Further, we investigate its properties and its relationship with other soft closed sets.

**Key Words:** Soft sets; Soft Topology; Soft  $\pi$ gr-closed sets; Soft  $\pi$ gr-  $T_{1/2}$ -space.

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## 1. INTRODUCTION

Molodtsov [10] initiated the concept of soft set theory in 1999 as a general mathematical tool for dealing with uncertainties. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc., Maji, Biswas and Roy [9] made a theoretical study of soft set theory in more detail. Maji et al[9] proposed several operations on soft sets and some basic properties. Shabir and Naz [13] introduced the notion of soft topological spaces. Tridev Jyoti Naog[14] studies a new approach to the theory of soft sets. Kannan.K [7] studied soft g-closed sets in soft topological spaces along with its properties. In topological space, the concept of generalized (g-closed) set was introduced by Levine.N[8]. The concept of  $\pi$ -closed sets in topological spaces was initiated by Zaitsev[16] and the concept of  $\pi$ g-closed set was introduced by Noiri and Dontchev.[3]. N.Palaniappan[11] studied and introduced regular closed sets in topological spaces. Jeyanthi.V and Janaki.C [6] introduced  $\pi$ gr-closed sets in topological spaces. The concept of  $\pi$ g $\alpha$ -closed,  $\pi$ gp-closed,  $\pi$ gs-closed,  $\pi^*$ g-closed,  $\pi$ gb-closed sets in topological spaces was introduced by C.Janaki[5], Park. J.H[12], Aslim .G et al[1], Ganes M Pandya [4] and Sreeja and Janaki[14].

In this paper, we introduce and study soft  $\pi$ gr-closed sets in topological spaces and obtain its relationship with other soft closed sets. Further, we obtain the basic results and properties.

## 2. Preliminaries

**Definition 2.1** ([2],[10],[13],[14])

Let  $U$  be the initial universe and  $P(U)$  denote the power set of  $U$ . Let  $E$  denote the set of all parameters. Let  $A$  be a non-empty subset of  $E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For  $\epsilon \in A$ ,  $F(\epsilon)$  may be considered as the set  $\epsilon$ - approximate elements of the soft set  $(F, A)$ . Clearly, a soft set is not a set.

For two soft sets  $(F, A)$  and  $(G, B)$  over the common universe  $U$ , we say that  $(F, A)$  is a soft subset of  $(G, B)$  if (i)  $A \subseteq B$  and (ii) for all  $e \in A$ ,  $F(e)$  and  $G(e)$  are identical approximations. We write  $(F, A) \tilde{\subseteq} (G, B)$ .  $(F, A)$  is said to be a soft superset of  $(G, B)$ , if  $(G, B)$  is a soft subset of  $(F, A)$ . Two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  are said to be soft equal if  $(F, A)$  is a soft subset of  $(G, B)$  and  $(G, B)$  is a soft subset of  $(F, A)$ .

**Definition :2.2** ([2],[10],[13],[14])

The union of two soft sets of  $(F, A)$  and  $(G, B)$  over the common universe  $U$  is soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,  $H(e) = F(e)$  if  $e \in A - B$ ,  $H(e) = G(e)$  if  $e \in B - A$  and  $H(e) = F(e) \cup G(e)$  if  $e \in A \cap B$ . We write  $(F, A) \cup (G, B) = (H, C)$ .

**Definition :2.3** ([2],[10],[13],[14])

The Intersection  $(H, C)$  of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  denoted  $(F, A) \cap (G, B)$  is defined as  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$  for all  $e \in C$ .

**Definition:2.4** ([2],[10],[13],[14])

For a soft set  $(F, A)$  over the universe  $U$ , the relative complement of  $(F, A)$  is denoted by  $(F, A)'$  and is defined by  $(F, A)' = (F', A)$ , where  $F': A \rightarrow P(U)$  is a mapping defined by  $F'(e) = U - F(e)$  for all  $e \in A$ .

**Definition:2.5** ([2],[7])

Let  $\tau$  be the collection of soft sets over  $X$ , then  $\tau$  is called a soft topology on  $X$  if  $\tau$  satisfies the following axioms:

- 1)  $\phi, \tilde{X}$  belong to  $\tau$ .
- 2) The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
- 3) The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ . For simplicity, we can take the soft topological space  $(X, \tau, E)$  as  $X$  throughout the work.

**Definition:2.6** ([2],[7])

Let  $(X, \tau, E)$  be soft space over  $X$ . A soft set  $(F, E)$  over  $X$  is said to be soft closed in  $X$ , if its relative complement  $(F, E)'$  belongs to  $\tau$ . The relative complement is a mapping  $F': E \rightarrow P(X)$  defined by  $F'(e) = X - F(e)$  for all  $e \in E$ .

**Definition:2.7** ([2],[7])



Let  $X$  be an initial universe set,  $E$  be the set of parameters and  $\tau = \{\varphi, \tilde{X}\}$ . Then  $\tau$  is called the soft indiscrete topology on  $X$  and  $(X, \tau, E)$  is said to be a soft indiscrete space over  $X$ . If  $\tau$  is the collection of all soft sets which can be defined over  $X$ , then  $\tau$  is called the soft discrete topology on  $X$  and  $(X, \tau, E)$  is said to be a soft discrete space over  $X$ .

**Definition:2.8** ([2],[7])

Let  $(X, \tau, E)$  be a soft topological space over  $X$  and the soft interior of  $(F, E)$  denoted by  $\text{Int}(F, E)$  is the union of all soft open subsets of  $(F, E)$ . Clearly,  $(F, E)$  is the largest soft open set over  $X$  which is contained in  $(F, E)$ . The soft closure of  $(F, E)$  denoted by  $\text{Cl}(F, E)$  is the intersection of all closed sets containing  $(F, E)$ . Clearly,  $(F, E)$  is smallest soft closed set containing  $(F, E)$ .

$$\text{Int}(F, E) = \bigcup \{ (O, E) : (O, E) \text{ is soft open and } (O, E) \tilde{\subset} (F, E) \}.$$

$$\text{Cl}(F, E) = \bigcap \{ (O, E) : (O, E) \text{ is soft closed and } (F, E) \tilde{\subset} (O, E) \}.$$

**Definition:2.9** ([2],[7],[9])

Let  $U$  be the common universe set and  $E$  be the set of all parameters. Let  $(F, A)$  and  $(G, B)$  be soft sets over a common universe set  $U$  and  $A, B \subset E$ . Then  $(F, A)$  is a subset of  $(G, B)$ , denoted by  $(F, A) \tilde{\subset} (G, B)$ .  $(F, A)$  equals  $(G, B)$ , denoted by  $(F, A) = (G, B)$  if  $(F, A) \tilde{\subset} (G, B)$  and  $(G, B) \tilde{\subset} (F, A)$ .

**Definition:2.10**

A soft subset  $(A, E)$  of  $X$  is called

- (i) a soft generalized closed (Soft g-closed) [6] in a soft topological space  $(X, \tau, E)$  if  $\text{Cl}(A, E) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$ .
- (ii) a soft semi open [2] if  $(A, E) \tilde{\subset} \text{Int}(\text{Cl}(A, E))$
- (iii) a soft regular open if  $(A, E) = \text{Int}(\text{Cl}(A, E))$ .
- (iv) a soft  $\alpha$ -open if  $(A, E) \tilde{\subset} \text{Int}(\text{Cl}(\text{Int}(A, E)))$
- (v) a soft b-open if  $(A, E) \tilde{\subset} \text{Cl}(\text{Int}(A, E)) \cup \text{Int}(\text{Cl}(A, E))$
- (vi) a soft pre-open set if  $(A, E) \tilde{\subset} \text{Int}(\text{Cl}(A, E))$ .
- (vii) a soft clopen is  $(A, E)$  is both soft open and soft closed.

The complement of the soft semi open, soft regular open, soft  $\alpha$ -open, soft b-open, soft pre-open sets are their respective soft semi closed, soft regular closed, soft  $\alpha$ -closed, soft b-closed and soft pre-closed sets.

The finite union of soft regular open sets is called soft  $\pi$ -open set and its complement is soft  $\pi$ -closed set.

The soft regular open set of  $X$  is denoted by  $\text{SRO}(X)$  or  $\text{SRO}(X, \tau, E)$ .

**Definition:2.11** [7]

A soft topological space  $X$  is called a soft  $T_{1/2}$ -space if every soft g-closed set is soft closed in  $X$ .

**Definition:2.12**

The soft regular closure of  $(A, E)$  is the intersection of all soft regular closed sets containing  $(A, E)$ . (i.e) The smallest soft regular closed set containing  $(A, E)$  and is denoted by  $\text{srcl}(A, E)$ .

The soft regular interior of  $(A, E)$  is the union of all soft regular open sets contained in  $(A, E)$  and is denoted by  $\text{srint}(A, E)$ .

Similarly, we define soft  $\alpha$ -closure, soft pre-closure, soft semi closure and soft b-closure of the soft set  $(A, E)$  of a topological space  $X$  and are denoted by  $\text{s}\alpha\text{cl}(A, E)$ ,  $\text{spcl}(A, E)$ ,  $\text{sscl}(A, E)$  and  $\text{sbcl}(A, E)$  respectively.

### 3.SOFT $\pi$ GR-CLOSED SETS.

Let us introduce the following definitions.

**Definition:3.1**

A subset  $(A, E)$  of a soft topological space  $X$  is called



- (i) a soft rg-closed set if  $Cl(A,E) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft regular open.
- (ii) a soft  $\pi$ g-closed set if  $Cl(A,E) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi$ - open.
- (iii) a soft  $\pi^*$ g-closed if  $Cl(Int(A,E)) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi$ - open.
- (iv) a soft  $\pi$ g $\alpha$ -closed if  $sacl(A,E) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi$ - open.
- (v) a soft  $\pi$ gp-closed if  $spcl(A,E) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi$ - open.
- (vi) a soft  $\pi$ gb-closed if  $sbcl(A,E) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi$ - open.
- (vii) a soft  $\pi$ gs-closed if  $sscl(A,E) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi$ - open.

**Definition:3.2**

A soft subset  $(A,E)$  of a soft topological space  $X$  is called a soft  $\pi$ gr-closed set in  $X$  if  $srcl(A,E) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$ , where  $(U,E)$  is soft  $\pi$ - open in  $X$ . We denote the soft  $\pi$ gr-closed set of  $X$  by  $S\pi GRC(X)$ .

**Result :3.3**

Every soft regular closed set is soft  $\pi$ gr-closed but not conversely.

**Example:3.4**

Let  $X=\{a,b,c,d\}$ ,  $E=\{e_1,e_2\}$ . Let  $F_1,F_2,\dots, F_6$  are functions from  $E$  to  $P(X)$  and are defined as follows:

$$\begin{aligned} F_1(e_1) &= \{c\}, F_1(e_2) = \{a\}, \\ F_2(e_1) &= \{d\}, F_2(e_2) = \{b\}, \\ F_3(e_1) &= \{c,d\}, F_3(e_2) = \{a,b\}, \\ F_4(e_1) &= \{a,d\}, F_4(e_2) = \{b,d\}, \\ F_5(e_1) &= \{b,c,d\}, F_5(e_2) = \{a,b,c\}, \\ F_6(e_1) &= \{a,c,d\}, F_6(e_2) = \{a,b,d\}. \end{aligned}$$

Then  $\tau = \{\emptyset, \tilde{X}, (F_1,E), \dots, (F_6,E)\}$  is a soft topology and elements in  $\tau$  are soft open sets.

The soft closed sets are its relative complements.

Here the soft set  $(H,E) = \{\{b,c,d\}, \{a,b,c\}\}$  is soft  $\pi$ gr-closed but not soft regular closed.

**Remark:3.5**

The concept of soft closed and soft  $\pi$ gr-closed are independent.

**Example:3.6**

In Example 3.4, (i) the soft set  $(A,E) = \{\{a\}, \{d\}\}$  of a soft topological space  $X$  is soft closed but not soft  $\pi$ gr-closed in  $X$ .

(ii) the soft subset  $(H,E) = \{\{b,c,d\}, \{a,b,c\}\}$  is soft  $\pi$ gr-closed but not soft closed in  $X$ .

**Remark: 3.7**

The concept of soft g-closed and soft  $\pi$ gr-closed are independent.

**Example:3.8**

In Example 3.4, (i) the soft set  $(A,E) = \{\{a\}, \{d\}\}$  of a soft topological space  $X$  is soft g-closed but not soft  $\pi$ gr-closed in  $X$ .

(ii) the soft subset  $(H,E) = \{\{b,c,d\}, \{a,b,c\}\}$  is soft  $\pi$ gr-closed but not soft g-closed in  $X$ .

**Theorem : 3.9**

Every soft  $\pi$ gr-closed set is soft  $\pi$ g $\alpha$ -closed, soft  $\pi$ gp-closed, soft  $\pi$ gb-closed, soft  $\pi$ gs-closed, soft  $\pi$ g-closed and soft  $\pi^*$ g-closed but not conversely.

**Proof:** Straight forward.



**Example : 3.10**

In example 3.4, i) The soft set  $(A,E) = \{\{a\},\{d\}\}$  of a soft topological space  $X$  is soft  $\pi g\alpha$ -closed and soft  $\pi g$ -closed but not soft  $\pi gr$ -closed.

ii) The soft set  $(F,E) = \{\{a\},\{b\}\}$  of a soft topological space  $X$  is soft  $\pi gb$ -closed, soft  $\pi gp$ -closed and soft  $\pi gs$ -closed but not soft  $\pi gr$ -closed.

iii) The soft subset  $(G,E) = \{\{c\},\{d\}\}$  of topological space  $X$  is soft  $\pi^*g$ -closed but not soft  $\pi gr$ -closed.

**Remark:3.11**

Every soft  $\pi gr$ -closed set is soft  $rg$ -closed but not conversely.

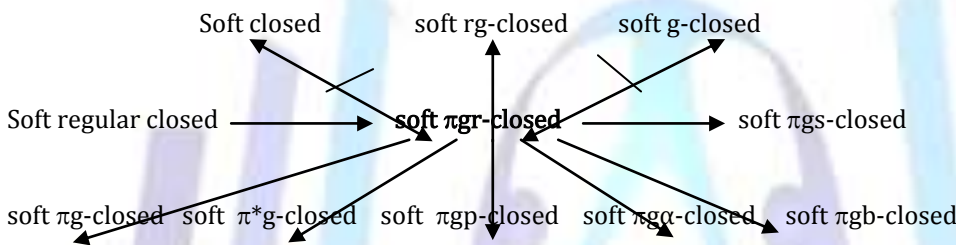
**Proof:** Straight forward.

**Example:3.12**

The soft subset  $(A,E) = \{\{a\},\{d\}\}$  of a soft topological space  $X$  is soft  $rg$ -closed but not soft  $\pi gr$ -closed in  $X$ .

**Remark:3.13**

The relationship between soft  $\pi gr$ -closed sets and soft sets are represented diagrammatically as follows:



**Remark:3.14**

The union of two soft  $\pi gr$ -closed sets is again a soft  $\pi gr$ -closed set.

**Proof:** Straight Forward.

**Remark:3.15**

The intersection of two soft  $\pi gr$ -closed sets need not be soft  $\pi gr$ -closed and is shown in the following example.

**Example:3.16**

In example 3.4, The soft sets  $(I,E) = \{\{b,d\},\{b,d\}\}$  and  $(J,E) = \{\{a,c,d\},\{a,b,c\}\}$  are soft  $\pi gr$ -closed sets in  $X$  but their intersection  $(K,E) = \{\{d\},\{b\}\}$  is not soft  $\pi gr$ -closed in  $X$ .

**Theorem:3.17**

If the soft subset  $(A,E)$  is soft regular closed and soft  $\pi g$ -closed, then  $(A,E)$  is soft  $\pi gr$ -closed in  $X$ .

**Proof:** Let  $(A,E)$  be a soft regular closed set and soft  $\pi g$ -closed set in  $X$ . Since  $(A,E)$  is soft  $\pi g$ -closed in  $X$ ,  $Cl(A,E) \tilde{\subset} (U,E)$  whenever  $(A,E) \tilde{\subset} (U,E)$  and  $(U,E)$  is soft  $\pi$ -open. Since  $srcl(A,E) = (A,E)$ , that is  $Cl(Int(A,E)) = (A,E)$ . But  $Cl(Int(A,E)) \tilde{\subset} Cl(A,E)$ . Hence  $(A,E) \tilde{\subset} Cl(A,E) \tilde{\subset} (U,E)$ .

$\Rightarrow (A,E) \tilde{\subset} (U,E)$ . Hence  $srcl(A,E) \tilde{\subset} (U,E)$ , whenever  $(A,E) \tilde{\subset} (U,E)$  and  $U$  is soft  $\pi$ -open. Hence  $(A,E)$  is soft  $\pi gr$ -closed in  $X$ .

**Theorem:3.18**

If  $(A,E)$  is soft  $\pi$ -open and soft  $\pi gr$ -closed, then it is soft closed.

**Proof:** Suppose  $(A,E)$  is soft  $\pi$ -open and soft  $\pi gr$ -closed. Then  $srcl(A,E) \tilde{\subset} (A,E)$ . But  $(A,E) \tilde{\subset} srcl(A,E)$ . Hence  $srcl(A,E) = (A,E)$ . The above implies  $(A,E)$  is soft regular closed and hence soft closed in  $X$ .

**Theorem:3.19**

If a soft subset  $(A,E)$  of a soft topological space  $X$  is soft  $\pi gr$ -closed set and  $(A,E) \tilde{\subset} (B,E) \tilde{\subset} srcl(A,E)$ . Then  $(B,E)$  is also soft  $\pi gr$ -closed subset of  $X$ .



**Proof:** Let  $(A,E)$  be a soft  $\pi$ gr-closed set in  $X$  and  $(B,E) \tilde{\subset} (U,E)$ , where  $(U,E)$  is soft  $\pi$ -open. Since  $(A,E) \tilde{\subset} (B,E)$ ,  $(A,E) \tilde{\subset} (U,E)$ . Since  $(A,E)$  is soft  $\pi$ gr-closed, soft  $\text{srcl}(A,E) \tilde{\subset} (U,E)$ . Given  $(B,E) \tilde{\subset} \text{srcl}(A,E)$ . Then  $\text{srcl}(B,E) \tilde{\subset} \text{srcl}(A,E) \tilde{\subset} U$ . Hence  $\text{srcl}(B,E) \tilde{\subset} (U,E)$  and hence  $(B,E)$  is soft  $\pi$ gr-closed.

**Theorem:3.20**

If  $(A,E)$  is soft  $\pi$ gr-closed, then  $\text{srcl}(A,E) - (A,E)$  does not contain any non-empty soft  $\pi$ -closed set.

**Proof:** Let  $(F,E)$  be a non-empty soft  $\pi$ -closed set such that  $(F,E) \tilde{\subset} \text{srcl}(A,E) - (A,E)$ . The above implies  $(F,E) = X - (A,E)$ . Since  $(A,E)$  is soft  $\pi$ gr-closed,  $X - (A,E)$  is soft  $\pi$ gr-open. Since  $(F,E)$  is soft  $\pi$ -closed set,  $X - (F,E)$  is soft  $\pi$ -open. Since  $\text{srcl}(A,E) \tilde{\subset} X - (F,E)$ ,  $(F,E) \tilde{\subset} X - \text{srcl}(A,E)$ . Thus  $(F,E) \tilde{\subset} \emptyset$ , which is a contradiction. The above implies  $(F,E) = \emptyset$  and hence  $\text{srcl}(A,E) - (A,E)$  does not contain non-empty soft  $\pi$ -closed set.

**Corollary:3.21**

Let  $(A,E)$  be a soft  $\pi$ gr-closed set. Then  $(A,E)$  is soft regular closed iff  $\text{srcl}(A,E) - (A,E)$  is soft  $\pi$ -closed.

**Proof:** Let  $(A,E)$  be soft regular closed. Then  $\text{srcl}(A,E) = (A,E)$  and  $\text{srcl}(A,E) - (A,E) = \emptyset$ , which is soft  $\pi$ -closed. On the other hand, let us suppose that  $\text{srcl}(A,E) - (A,E)$  is soft  $\pi$ -closed. Then by theorem 3.20,  $\text{srcl}(A,E) - (A,E) = \emptyset$ . The above implies  $\text{srcl}(A,E) = (A,E)$ . Hence  $(A,E)$  is soft regular closed.

## 4. SOFT $\pi$ GR-OPEN SETS.

**Definition:4.1**

A soft set  $(A,E)$  is called a soft  $\pi$ gr-open set in a soft topological space  $X$  if the relative complement  $(A,E)'$  is soft  $\pi$ gr-closed in  $X$  and the soft  $\pi$ gr-open set of  $X$  is denoted by  $S\pi RGO(X)$ .

**Remark:4.2**

For a soft subset  $(A,E)$  of  $X$ ,  $\text{srcl}(X - (A,E)) = X - \text{srint}(A,E)$ .

**Theorem:4.3**

The soft subset  $(A,E)$  of  $X$  is soft  $\pi$ gr-open iff  $(F,E) \tilde{\subset} \text{srint}(A,E)$  whenever  $(A,E)$  is soft  $\pi$ -closed and  $(F,E) \tilde{\subset} (A,E)$ .

**Proof:** Let  $(A,E)$  be soft  $\pi$ gr-open. Let  $(F,E)$  be soft  $\pi$ -closed set and  $(F,E) \tilde{\subset} (A,E)$ . Then  $X - (A,E) \tilde{\subset} X - (F,E)$ , where  $X - (F,E)$  is soft  $\pi$ -open. Since  $(A,E)$  is soft  $\pi$ gr-open,  $X - (A,E)$  is soft  $\pi$ gr-closed. Then  $\text{srcl}(X - (A,E)) \tilde{\subset} X - (F,E)$ . Since  $\text{srcl}(X - (A,E)) = X - \text{srint}(A,E)$ .

$\Rightarrow X - \text{srint}(A,E) \tilde{\subset} X - (F,E)$ . Hence  $(F,E) \tilde{\subset} \text{srint}(A,E)$ .

On the other hand, let  $(F,E)$  is soft  $\pi$ -closed and  $(F,E) \tilde{\subset} (A,E)$  implies  $(F,E) \tilde{\subset} \text{srint}(A,E)$ . Let  $X - (A,E) \tilde{\subset} (U,E)$ , where  $X - (U,E)$  is soft  $\pi$ -closed. By hypothesis,  $X - (U,E) \tilde{\subset} \text{srint}(A,E)$ . Hence  $X - \text{srint}(A,E) \tilde{\subset} (U,E)$ . since  $\text{srcl}(X - (A,E)) = X - \text{srint}(A,E)$ . The above implies  $\text{srcl}(X - (A,E)) \tilde{\subset} (U,E)$ , whenever  $X - (A,E)$  is soft  $\pi$ -open. Hence  $X - (A,E)$  is soft  $\pi$ gr-open in  $X$ .

**Theorem:4.4**

If  $\text{srint}(A,E) \tilde{\subset} (B,E) \tilde{\subset} (A,E)$ , and  $(A,E)$  is soft  $\pi$ gr-open, then  $(B,E)$  is soft  $\pi$ gr-open.

**Proof:** Given  $\text{srint}(A,E) \tilde{\subset} (B,E) \tilde{\subset} (A,E)$ . Then  $X - (A,E) \tilde{\subset} X - (B,E) \tilde{\subset} \text{srcl}(X - (A,E))$ . Since  $(A,E)$  is soft  $\pi$ gr-open,  $X - (A,E)$  is soft  $\pi$ gr-closed. Then  $X - (B,E)$  is also soft  $\pi$ gr-closed. Hence  $(B,E)$  is soft  $\pi$ gr-open.

**Remark:4.5**

For any set  $(A,E) \tilde{\subset} A$ ,  $\text{srint}(\text{srcl}(A,E) - (A,E)) = \emptyset$ .

**Theorem:4.6**

If  $(A,E) \tilde{\subset} X$  is soft  $\pi$ gr-closed, then  $\text{srcl}(A,E) - (A,E)$  is soft  $\pi$ gr-open.

**Proof:** Let  $(A,E)$  be soft  $\pi$ gr-closed and let  $(F,E)$  be a soft  $\pi$ -closed set such that  $(F,E) \tilde{\subset} \text{srcl}(A,E) - (A,E)$ . Then  $(F,E) = \emptyset$ . So,  $(F,E) \tilde{\subset} \text{srint}(\text{srcl}(A,E) - (A,E))$ . Hence  $\text{srcl}(A,E) - (A,E)$  is soft  $\pi$ gr-open.

**Theorem:4.7**

The intersection of two soft  $\pi$ gr- open sets is again a soft  $\pi$ gr-open set.

**Proof:** Straight forward.

**Remark:4.8**

The union of two soft  $\pi$ gr-open sets need not be a soft  $\pi$ gr-open set and is shown in the following example.

**Example:4.9**

Let  $(B,E)=\{\{a,c\},\{a,c\}\}$  and  $(C,E)=\{\{b\},\{d\}\}$  are two soft  $\pi$ gr-open sets. Then their union  $(D,E)=\{\{a,b,c\},\{a,c,d\}\}$  is not soft  $\pi$ gr-open in  $X$ .

**5. SOFT  $\pi$ GR-T<sub>1/2</sub>-SPACES.****Definition:5.1**

A soft topological space  $X$  is a soft  $\pi$ gr- T<sub>1/2</sub>-space if every soft  $\pi$ gr-closed set is soft regular closed.

**Theorem:5.2**

For a soft topological space  $(X,\tau,E)$ , the following conditions are equivalent.

- (i) The soft topological space  $X$  is soft  $\pi$ gr-T<sub>1/2</sub>-space.
- (ii) Every singleton of  $X$  is either soft  $\pi$ -closed or soft regular open.

**Proof:**

(i)  $\Rightarrow$  (ii): Let  $(A,E)$  be a soft singleton set in  $X$  and let us assume that  $(A,E)$  is not soft  $\pi$ -closed. Then  $X-(A,E)$  is not soft  $\pi$ -open and hence  $X-(A,E)$  is trivially soft  $\pi$ gr-closed. Since in a soft  $\pi$ gr-T<sub>1/2</sub>-space, every soft  $\pi$ gr-closed set soft regular closed. Then  $X-(A,E)$  is soft regular closed. Hence  $(A,E)$  is soft regular open.

(ii)  $\Rightarrow$  (i) : Assume that every singleton of a soft topological space  $X$  is either soft  $\pi$ -closed or soft regular open.

Let  $(A,E)$  be a soft  $\pi$ gr-closed set in  $X$ . Obviously,  $(A,E) \tilde{\subset} \text{srcl}(A,E)$ .

To Prove  $\text{srcl}(A,E) \tilde{\subset} (A,E)$

Case (i): Let the singleton set  $(F,E)$  be soft  $\pi$ -closed. Suppose  $(F,E)$  does not belong to  $(A,E)$ . Then  $(F,E) \tilde{\subset} \text{srcl}(A,E) - (A,E)$ , which is a contradiction to the fact that  $\text{srcl}(A,E) - (A,E)$  does not contain any non-empty soft  $\pi$ -closed set. Therefore,  $(F,E) \tilde{\subset} (A,E)$ . Hence  $\text{srcl}(A,E) \tilde{\subset} (A,E)$ . The above is soft regular closed and hence every  $S\pi\text{GRC}(A,E)$  is soft regular closed. Hence the soft topological space  $X$  is soft  $\pi$ gr-T<sub>1/2</sub>-space.

Case(ii): Let  $(F,E)$  be soft regular open in  $X$ . Since  $(F,E) \in \text{srcl}(A,E)$ , we have  $(F,E) \cap (A,E) \neq \emptyset$ . Hence  $(F,E) \tilde{\subset} (A,E)$ . Therefore,  $\text{srcl}(A,E) \tilde{\subset} (A,E)$ . Hence  $\text{srcl}(A,E) = (A,E)$ .

$\Rightarrow (A,E)$  is soft regular closed and hence  $S\pi\text{RG}$ -closed in  $X$ .

**Theorem:5.3**

- (i) Soft  $\text{RO}(X,\tau,E) \tilde{\subset} \text{Soft } \pi\text{RGO}(X,\tau,E)$
- (ii) A soft topological space  $(X,\tau,E)$  is  $\pi$ gr-T<sub>1/2</sub>-space iff soft  $\text{RO}(x,\tau,E) = \pi\text{GRO}(X,\tau,E)$

**Proof:**

(i) Let  $(A,E)$  be soft regular open. Then  $X-(A,E)$  is soft regular closed and so soft  $\pi$ gr-closed. Hence  $(A,E)$  is soft  $\pi$ gr-open and soft  $\text{RO}(X,\tau,E) \tilde{\subset} \pi\text{GRO}(X, \tau,E)$

(ii) Necessity: Let  $(X,\tau,E)$  be  $\pi$ gr-T<sub>1/2</sub>-space. Let  $(A,E) \in \pi\text{GRO}(X,\tau,E)$ . Then  $X-(A,E)$  is soft  $\pi$ gr-closed. Since the space soft  $\pi$ gr-T<sub>1/2</sub>-space,  $X-(A,E)$  is soft regular closed. The above implies  $(A,E)$  is soft regular open in  $X$ . Hence  $S\pi\text{GRO}(X,\tau,E) = \text{SRO}(X, \tau,E)$ .

Sufficiency: Let  $S\pi\text{GRO}(X,\tau,E) = \text{SRO}(X, \tau,E)$ . Let  $(A,E)$  be soft  $\pi$ gr-closed. Then  $X-(A,E)$  is soft  $\pi$ gr-open. Thus  $X-(A,E) \in \text{SRO}(X,\tau,E)$  and hence  $(A,E)$  is soft regular closed.

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