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On a fractional order nonlinear dynamic model of a triadic relationship

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ABSTRACT

In this paper, a fractional order nonlinear dynamic model of involving three person is introduced. The proposed model describes the dynamic behavior of an interrelation of three person under different structures. Stability analysis of the fractional-order nonlinear dynamic model of involving three person is studied using the fractional Routh-Hurwitz criteria. By using stability analysis on fractional order system, we obtain sufficient condition on the parameters for the locally asymptotically stable of equilibrium points. Finally, our results are validated by numerical simulations.

Keywords: Fractional derivative; initial value problem; fractional model; numerical example.

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1 Introduction

Mathematical models are used not only in the natural sciences (e.g. astronomy, biology, physics), but also in the social sciences (e.g. education, history, sociology, life sciences). Since experiments in this area are difficult to design, mathematical models play an important role. The mathematical models capturing the dynamics of people have recently gained attention among many researchers [2,4,6,7,8,9,11,13,14,15,16,17,18] who have provided an extension to Strogatz's seminal model. Also most of models have been restricted to integer order differential equations. In this paper, as an extension, we consider a system of nonlinear fractional differential equations by adding a third person (a secret lover) to the fractional order model reported in [11]. The fractional order model in [11] is given as

$$D^{\alpha}x_1(t) = -\alpha_1 x_1 + \beta_1 x_2(1-\varepsilon x_2^2) + A_1$$
$$D^{\alpha}x_2(t) = -\alpha_2 x_2 + \beta_2 x_1(1-\varepsilon x_1^2) + A_2$$

with initial conditions

$$x_1(0) = x_{01}\,,\; x_2(0) = x_{02}.$$

Also, different from [11], a model with the order 2α is discussed. We are expecting an acceleration in feelings, that is why we increase the order of the derivative between $1 < 2\alpha \le 2.$ If fractional order models have been considered instead of its integer order counterpart, it can be more appropriate for this such dynamics. Because they are influenced by memory [2].

We begin by giving the definitions of fractional order integrals and derivatives [12].

Definition 1. The gamma function $\Gamma(z)$ is defined by the integral

$$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt$$

which converges in the right half of the complex plane $Re(z) > 0.$ One of the basic properties of the gamma function is that it satisfies the following functional equation :

$$\Gamma(z+1) = z.\Gamma(z) = z.(z-1)! = z!$$

Definition 2. The fractional integral of order $\alpha > 0$ of a function $f : R^{+} \to R$ is given by

$$I_a^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau \quad , \qquad \alpha > 0$$

Definition 3. The fractional derivative of order $\alpha > 0$ of a function $f : R^{+} \to R$ is given by

$$\mathbf{D}^{\alpha} f(t) := \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \dfrac{d^n}{dt^n} \displaystyle\int_a^t (t-\tau)^{n-\alpha-1} f(\tau) d\tau, & n-1 \le \alpha < n, \\ \dfrac{d^n}{dt^n} f(t), & \alpha = n \end{cases}.$$

In this work, we use the operator known as "Caputo differential operator" given by

$${}_a^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau) d\tau}{(t-\tau)^{\alpha+1-n}}, \quad (n-1 < \alpha < n).$$

where $n = [\alpha] + 1$ and $[\alpha]$ is the integer part of α . Under natural conditions on the function $f(t),$ for $\alpha \to n$ the Caputo derivative becomes conventional derivative. The main advantage of Caputo's definition is that the initial conditions for fractional differential equations with Caputo derivatives take on the same form as for integer-order differential equations. Also Caputo differential operator is convenient for modeling since the derivative of a constant is zero which means that this kind of derivative can be used to model the rate of change.

The other difference between the Riemann-Liouville and the Caputo approaches, which is important for applications is given below [12]. For the Caputo derivative we have

$$ {}_a^C D_t^{\alpha} ({}_a^C D_t^m f(t)) = {}_a^C D_t^{\alpha+m} f(t), \quad (m = 0,1,2,...;\ n-1 < \alpha < n) $$

while for the Riemann-Liouville (fractional order derivative) derivative

$$ {}_a\mathbf{D}_t^m ({}_a\mathbf{D}_t^{\alpha} f(t)) = {}_a\mathbf{D}_t^{\alpha+m} f(t), \quad (m = 0,1,2,...;\ n-1 < \alpha < n). $$

The interchange of the differentiation operators in Caputo derivative and Riemann-Liouville derivative formulas is allowed under different conditions below:

$$ {}_a^C D_t^{\alpha} \left({}_a^C D_t^m f(t)\right) = {}_a^C D_t^m \left({}_a^C D_t^{\alpha} f(t)\right) = {}_a^C D_t^{\alpha+m} f(t), $$
$$ f^{(s)}(0) = 0, \qquad s = n, n+1, ..., m $$
$$ m = 0,1,2,...; n-1 < \alpha < n $$

and

$$ {}_a\mathbf{D}_t^m \left({}_a\mathbf{D}_t^{\alpha} f(t)\right) = {}_a\mathbf{D}_t^{\alpha} \left({}_a\mathbf{D}_t^m f(t)\right) = {}_a\mathbf{D}_t^{\alpha+m} f(t), $$
$$ f^{(s)}(0) = 0, \qquad s = 0,1,2,...,m $$
$$ m = 0,1,2,...; n-1 < \alpha < n. $$

We see that contrary to the Riemann-Liouville approach, in the case of Caputo derivative there are no restrictions on the values $f^{(s)}(0), \quad (s = 0, 1..., n-1)$.

2 A fractional order nonlinear triadic model

The model proposed in this paper is a fractional order nonlinear dynamic realistic model. Model describes a triadic relationship, in which a person A is involved in romantic relationships with the persons B and C. Here, it is assumed that B and C (both in relation with A) would not know about one another and A would exhibit the similar romantic style towards them both. In this model three cases have been considered (the forgetting process, the pleasure of being loved, and the reaction to the appeal of the partner). These three factors are assumed to be independent and are modeled by nonlinear functions (e.g.: $\beta_i x_i (1-\varepsilon x_i^2)$). The model parameters specify the romantic styles of the couples. The signs of the parameters determine the type of romantic style that the particular individual can exhibit. Using the typology of Strogatz [15,16] and Sprott [18], four romantic styles can be given as:

- **Eager Beaver:** individual 1 is encouraged by his own feelings as well as that of individual 2 (and that of individual 3) ($\alpha_i > 0$ and $\beta_i > 0$).

- **Secure or Cautious lover:** individual 1 retreats from his own feelings but is encouraged by that of individual 2 (and that of individual 3) ($\alpha_i < 0$ and $\beta_i > 0$)

- **Narcissistic Nerd:** individual 1 wants more of what he feels but retreat from the feelings of individual 2 (and that of individual 3) ($\alpha_i > 0$ and $\beta_i < 0$).

- **Hermit:** individual 1 retreats from his own feelings and that of individual 2 (and that of individual 3) ($\alpha_i < 0$ and $\beta_i < 0$).

This classification allows us to characterize the dynamics exhibited by various combinations of the romantic styles.

In this paper, we focus on triadic relational dynamics characterized by secure or cautious lover interactions. Here in this model, we are expecting an acceleration in feelings, that is why, we increase the order of the derivative between $1 < 2\alpha \le 2.$ To model the behavioral features of the dynamics of a triadic relation, the following model is considered:

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$$\begin{cases} D^{2\alpha}x_1(t) = -\alpha_1 x_1 + \beta_1(x_2 - x_3)(1-\varepsilon(x_2 - x_3)^2) + \gamma_1 \\ D^{2\alpha}x_2(t) = -\alpha_2 x_2 + \beta_2 x_1(1-\varepsilon x_1^2) + \gamma_2 \\ D^{2\alpha}x_3(t) = -\alpha_2 x_3 + \beta_3 x_4(1-\varepsilon x_4^2) + \gamma_3 \\ D^{2\alpha}x_4(t) = -\alpha_1 x_4 + \beta_1(x_3 - x_2)(1-\varepsilon(x_3 - x_2)^2) + \gamma_4 \end{cases} \quad (2.1)$$

with initial conditions

$$x_1(0) = 0,\ x_2(0) = 0,\ x_3(0) = 0,\ x_4(0) = 0 \quad (2.2)$$

where $1 < 2\alpha \le 2,$ $\alpha_i > 0,$ $\alpha_i,$ $\beta_i,$ and γ_i $(i = 1,2,3,4)$ are real constants. These parameters are oblivion, reaction, and attraction constants, respectively.

We would like to highlight that this model can be applicable in reality. The functional structure of return factors are motivated by dynamics often portrayed in romance novels and tragic outcomes illustrated in them (Shakespeare's Romeo and Juliet, Patriarch's Canzoniere and Posteritati, Tolstoy's Anna Karenia, Flaubert's Madame Bovary, more recently the tragic suicide of Megan Meier). The constant ε in return function (e.g.: $\beta_2 x_1(1-\varepsilon x_1^2)$) can be interpreted as the compensatory constant. For example, the romantic dynamics between Laura Winslow and Steve Urkel in the sitcom *Family Matter* can serve as a popular media example. When Steve Urkel despairs, Laura Winslow feels sorry for him and her antagonism is overcome by feeling of pity .As a result, she reverses her reaction to passion [3]. This behavioral characteristic is captured by the function of reaction or return function (e.g.: $\beta_3 x_4(1-\varepsilon x_4^2)$). This expression captures the compensation for antagonism with flattery, or pity, for positive and negative values of x_4 in $\beta_3 x_4(1-\varepsilon x_4^2),$ respectively. Triadic relationship of Rainer Maria Rilke, Lou Andreas Salome and Nietzsche can be given for triadic relationship example in real world.

In contrast to the other models such as in [11, and the models stated in [11]], we expand the order of the derivative between 1 and 2, since we are expecting a strong time dependency (an acceleration in time) and memory effect in the dynamic of the relation. The equations of the system (2.1), respectively, govern the feelings (x_1) of A to $B,$ the feelings (x_2) of B to $A,$ the feelings (x_3) of C to $A,$ and the feelings (x_4) of A to $C.$ We note that with zero initial conditions, the following equation is valid [12]:

$$D^{\alpha}(D^{\alpha}x(t)) = D^{2\alpha}(x(t))$$

Let us make the following changes of variables:

$$\begin{aligned} x_1 &= y_1, & D^{\alpha}x_1 &= y_2 \\ x_2 &= y_3, & D^{\alpha}x_2 &= y_4 \\ x_3 &= y_5, & D^{\alpha}x_3 &= y_6 \\ x_4 &= y_7, & D^{\alpha}x_4 &= y_8 \end{aligned}$$

Then (2.1)-(2.2) is transformed to the system

$$\begin{cases} D^{\alpha}y_1(t) = y_2 \\ D^{\alpha}y_2(t) = -\alpha_1 y_1 + \beta_1(y_3 - y_5)(1-\varepsilon(y_3 - y_5)^2) + \gamma_1 \\ D^{\alpha}y_3(t) = y_4 \\ D^{\alpha}y_4(t) = -\alpha_2 y_3 + \beta_2 y_1(1-\varepsilon y_1^2) + \gamma_2 \\ D^{\alpha}y_5(t) = y_6 \\ D^{\alpha}y_6(t) = -\alpha_2 y_5 + \beta_3 y_7(1-\varepsilon y_7^2) + \gamma_3 \\ D^{\alpha}y_7(t) = y_8 \\ D^{\alpha}y_8(t) = -\alpha_1 y_7 + \beta_1(y_5 - y_3)(1-\varepsilon(y_5 - y_3)^2) + \gamma_4 \end{cases} \quad (2.3)$$

with initial conditions

$$y_1(0) = 0,\ y_2(0) = 0,\ y_3(0) = 0,\ y_4(0) = 0$$

$$y_5(0)=0\,,\ y_6(0)=0\,,\ y_7(0)=0\,,\ y_8(0)=0$$

where $0.5<\alpha\le 1,\ \ \alpha_i>0,\ \ \alpha_i,\ \ \beta_i,$ and $\gamma_i\ \ (i=1,2,3,4)$ are real constants.

3 Equilibrium points and their locally asymptotic stability

Let $\alpha\in(0.5,1]$ and consider the system

$$\begin{cases} D^{\alpha}y_1(t)= f_1(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8),\\ D^{\alpha}y_2(t)= f_2(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8),\\ D^{\alpha}y_3(t)= f_3(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8),\\ D^{\alpha}y_4(t)= f_4(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8),\\ D^{\alpha}y_5(t)= f_5(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8),\\ D^{\alpha}y_6(t)= f_6(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8),\\ D^{\alpha}y_7(t)= f_7(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8),\\ D^{\alpha}y_8(t)= f_8(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8), \end{cases} \tag{3.1}$$

with the initial values

$$y_2(0)=0\,,\ y_4(0)=0\,,\ y_6(0)=0\,,\ y_8(0)=0$$

$$y_1(0)=0\,,\ y_3(0)=0\,,\ y_5(0)=0\,,\ y_7(0)=0$$

Here

$$f_1(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8)=y_2,\ f_3(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8)=y_4,$$

$$f_2(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8)=-\alpha_1 y_1+\beta_1(y_3-y_5)(1-\varepsilon(y_3-y_5)^2)+\gamma_1,$$

$$f_4(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8)=-\alpha_2 y_3+\beta_2 y_1(1-\varepsilon y_1^2)+\gamma_2,$$

$$f_5(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8)=y_6,\ f_7(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8)=y_8,$$

$$f_6(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8)=-\alpha_2 y_5+\beta_3 y_7(1-\varepsilon y_7^2)+\gamma_3,$$

$$f_8(y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8)=-\alpha_1 y_7+\beta_1(y_5-y_3)(1-\varepsilon(y_5-y_3)^2)+\gamma_4.$$

To evaluate the equilibrium points, let

$$D^{\alpha}y_i(t)=0\,f_i(y_1^*,y_2^*,y_3^*,y_4^*,y_5^*,y_6^*,y_7^*,y_8^*)=0,i=1,2,3,4,5,6,7,8$$

from which we can get the equilibrium points $K_0=(0,0,0,0,0,0,0,0)$ for $\gamma_1=\gamma_2=\gamma_3=\gamma_4=0$ and $K_1=(y_1^*,y_2^*,y_3^*,y_4^*,y_5^*,y_6^*,y_7^*,y_8^*)$.

The Jacobian matrix $J(K_1)$ for the system given in (2.3) is

$$J(K_1)=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha_1 & 0 & a & 0 & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ b & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha_2 & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -a & 0 & a & 0 & 0 & 0 \end{bmatrix}$$

where

$$a=\beta_1(1-3\varepsilon(y_3^*-y_5^*)^2), b=\beta_2(1-3\varepsilon y_1^{*2}), c=\beta_3(1-3\varepsilon y_7^{*2}).$$

To discuss the local stability of the equilibrium $K_1=(y_1^*,y_2^*,y_3^*,y_4^*,y_5^*,y_6^*,y_7^*,y_8^*)$ of the system given by (2.3), we consider the linearized system at K_1. The characteristic equation of the linearized system is of the form

$$\begin{aligned} P(\lambda) &= \lambda^8+\lambda^6(2\alpha_2+\alpha_1)+\lambda^4(2\alpha_1\alpha_2+\alpha_2^2-ab-ac) \\ &+\lambda^2(\alpha_1\alpha_2^2-a\alpha_2 b-a\alpha_1 c-a\alpha_2 c)-a\alpha_1\alpha_2 c \\ &=0 \end{aligned} \quad (3.2)$$

If λ^2 is taken as $k,$ we have the following reduced equation:

$$P(\lambda)=k^4+a_1k^3+a_2k^2+a_3k+a_4=0 \quad (3.3)$$

where

$$\begin{aligned} a_1 &= (2\alpha_2+\alpha_1) \\ a_2 &= (2\alpha_1\alpha_2+\alpha_2^2-ab-ac) \\ a_3 &= (\alpha_1\alpha_2^2-a\alpha_2 b-a\alpha_1 c-a\alpha_2 c) \\ a_4 &= (-a\alpha_1\alpha_2 c) \end{aligned} \quad (3.4)$$

According to the fractional Routh–Hurwitz criteria we have the following theorem.

Theorem 1. *If* $a_1>0,\ a_2>0,\ a_3>0,\ a_4>0$ *and* $a_1a_2a_3-a_3^2-a_1^2a_4>0$ *then the equilibrium point* $K_1=(y_1^*,y_2^*,y_3^*,y_4^*,y_5^*,y_6^*,y_7^*,y_8^*)$ *is locally asymptotically stable for all* $\alpha\in(0.5,1]$.

Proof. $K_1=(y_1^*,y_2^*,y_3^*,y_4^*,y_5^*,y_6^*,y_7^*,y_8^*)$ equilibrium of the system given by (2.3) is asymptotically stable if all of the eigenvalues, $k_i,\ i=1,2,3,4$ of $J(K_1)$ satisfy the following condition (negative real part) [1,10] :

$$|\arg\lambda_i|>\frac{\alpha\pi}{2}.$$

For $n=4$, the Routh-Hurwitz criteria are $a_1>0,\ a_2>0,\ a_3>0,\ a_4>0$ and $a_1a_2a_3-a_3^2-a_1^2a_4>0.$ It is a necessary and sufficient condition for the negativity of the real parts of all the roots of the polynomial $P(\lambda)$.

If one of the conditions above does not hold, model gives rise to unbounded feeling, which is obviously unrealistic.

4 Numerical simulation of model

By selecting the appropriate parameters, we can make predictions of what might happen to couples at future. The model parameters can be derived from the couple's interaction in the special observation rooms. But this way is not always possible. It is not easy to find suitable couples.

In nonlinear dynamic systems, predictability is possible during periods of stability. Also relationship development would be completely predictable given the right parameters. In this model, parameters provide the condition for the locally asymptotically stable of equilibrium points by using stability analysis on fractional order system.

In this paper we focus on triadic relational dynamics characterized by cautious lover interactions (individual 1 retreats from his own feelings but is encouraged by that of individual 2 (and that of individual 3) ($\alpha_i < 0$ and $\beta_i > 0$)). Depending on the parameters, many scenarios are possible.

Letting

$$\alpha_1 = 0.01 \quad \alpha_2 = 0.02 \quad \beta_1 = 0.0003 \quad \beta_2 = 0.0006 \quad \beta_3 = 0.0005$$
$$\varepsilon = 0.001 \quad \gamma_1 = 0.1 \quad \gamma_2 = 0.03 \quad \gamma_3 = 0.2 \quad \gamma_4 = 0.3$$

and we consider the system with $2\alpha = 1.2$

$$\begin{cases} D^{2\alpha} x_1(t) = -0.01x_1 + 0.0003(x_2 - x_3)(1 - 0.001(x_2 - x_3)^2) + 0.1 \\ D^{2\alpha} x_2(t) = -0.02x_2 + 0.0006x_1(1 - 0.001x_1^2) + 0.03 \\ D^{2\alpha} x_3(t) = -0.02x_3 + 0.0005x_4(1 - 0.001x_4^2) + 0.2 \\ D^{2\alpha} x_4(t) = -0.01x_4 + 0.0003(x_3 - x_2)(1 - 0.001(x_3 - x_2)^2) + 0.3 \end{cases}$$

Let the initial conditions be

$$x_1(0) = 0,\ x_2(0) = 0,\ x_3(0) = 0,\ x_4(0) = 0$$

After system is transformed, following system is obatained with the order $\alpha = 0.6$.

$$\begin{aligned} D^\alpha y_1(t) &= y_2 \\ D^\alpha y_2(t) &= -0.01y_1 + 0.0003(y_3 - y_5)(1 - 0.001(y_3 - y_5)^2) + 0.1 \\ D^\alpha y_3(t) &= y_4 \\ D^\alpha y_4(t) &= -0.02y_3 + 0.0006y_1(1 - 0.001y_1^2) + 0.03 \\ D^\alpha y_5(t) &= y_6 \\ D^\alpha y_6(t) &= -0.02y_5 + 0.0005y_7(1 - 0.001y_7^2) + 0.2 \\ D^\alpha y_7(t) &= y_8 \\ D^\alpha y_8(t) &= -0.01y_7 + 0.0003(y_5 - y_3)(1 - 0.001(y_5 - y_3)^2) + 0.3 \end{aligned} \tag{4.1}$$

Let the initial conditions be

$$\begin{aligned} & y_2(0) = 0,\ y_4(0) = 0,\ y_6(0) = 0,\ y_8(0) = 0 \\ & y_1(0) = 0,\ y_3(0) = 0,\ y_5(0) = 0,\ y_7(0) = 0 \end{aligned} \tag{4.2}$$

Positive equilibrium point for the problem (4.1)-(4.2) is calculated as:

$$y_1^* = 9.76815,\ y_2^* = 0,\ y_3^* = 1.76508,\ y_4^* = 0,$$

$$y_5^* = 10.065,\ y_6^* = 0\ \ y_7^* = 30.2318,\ y_8^* = 0$$

For the numerical solution of (4.1)-(4.2), the predictor corrector method have been used for $\alpha = 0.6$ [5]. The solutions y_1, y_3, y_5, y_7 of the initial value problem (4.1)-(4.2) correspond, respectively to the solutions x_1, x_2,

x_3, x_4 of the problem (2.1) - (2.2).

We not that followings are the some results of our investigations of appropriate model parameters. Parameters provide the condition for the locally asymptotically stable of equilibrium points by using stability analysis on fractional order system.

We have demonstrated via numerical simulations that the fractional order nonlinear triadic model (2.1) - (2.2) can exhibit asymptotic behavior in the presence of nonlinearity for an appropriate set of model parameters. The solutions x_1, x_2, x_3, x_4 (respectively, govern the feelings (x_1) of A to B, the feelings (x_2) of B to A, the feelings (x_3) of C to A, and the feelings (x_4) of A to C) are displayed in the Figure 1. We have observed that the model approaches the equilibrium points asymptotically.

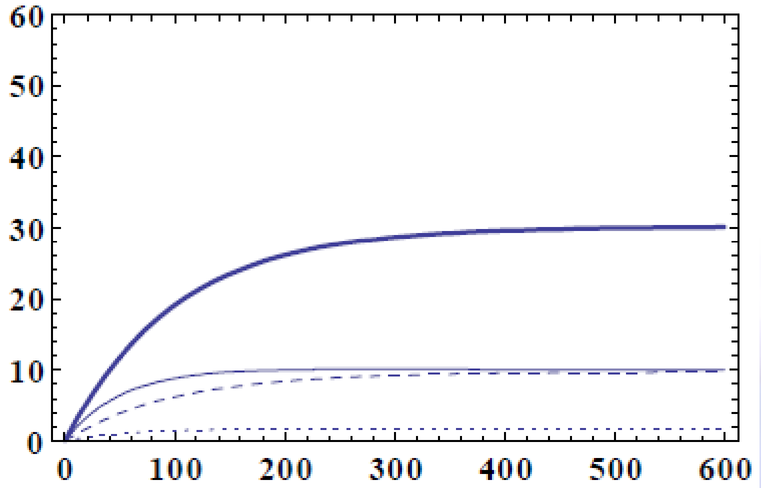

Figure 1. Solutions x_1 (dashed), x_2 (dotted), x_3 (solid), x_4 (thick)

5 Concluding Remarks

Why we have been considered this kind of mathematical model ? The answer is that it can provide two new things. First, we will see that the fractional order model provides a new language for thinking about couples relationship and, second, once we have the equations for a triadic relationships, we can simulate the couple's behavior in circumstances.

So in this paper a fractional order nonlinear dynamic model, involving three person, have been presented and discussed. The resulting model is a fractional order nonlinear dynamic system, which turns out to be positive if the appeals of the two individuals (A and B or A and C) are positive. The model predicts, that the feelings of the partners (A and B) or (A and C) go to an equilibrium level starting from initial conditions. The value of this level, in other words, the quality of the romantic relationship at equilibrium, is higher if the reactiveness to love and appeal are higher. We can say that for the model under consideration, the dynamics of the couples A and B are more robust than the dynamics of the couples A and C.

Finally, we have demonstrated via numerical simulations that a fractional order nonlinear model of triadic relationship can exhibit asymptotic behavior in the presence of an appropriate set of model parameters.

In summary, the paper provides a new viewpoint on fractional order models. With expecting on acceleration in feelings, different fractional order models can be considered.

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