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Four steps Block Predictor- Block Corrector Method for the solution of $y'' = f(x, y, y')$

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ABSTRACT

A method of collocation and interpolation of the power series approximate solution at some selected grid points is considered to generate a continuous linear multistep method with constant step size.predictor-corrector method was adopted where the predictors and the correctors considered two and three interpolation points implemented in block method respectively.The efficiency of the proposed method was tested on some numerical examples and found to compete favorably with the existing methods.

Keywords: Collocation; interpolation; power series approximation; block method; step size; grid points; efficiency.

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1 INRODUCTION

This paper examines the solution to general second order initial value problem of the form

$$y'' = f(x, y, y'), y(x_0) = y_0, y'(x_0) = y_0' \tag{1}$$

The method of solving higher order ordinary differential equation by method of reduction had been reported to increase the dimension of the resulting differential equations, hence writing the computer code is difficult since it requires a special way to incoporate the subroutine to supply the starting values. As a result, this leads to longer computer time and human effort.(Adesanya *et al.* [1], Awoyemi *and Idowu* [2], Awoyemi *et al.* [3], Jator [4]).

In order to cater for some of the setbacks of the method of reduction, Scholars developed direct method in the form of linear multistep method which can be either implicit or explicit. Implicit linear multistep method which has better stability condition than the explicit are implemented in predictor - corrector method. The major setback of this method is that the predictors are in reducing order of accuracy, which consequently has a great effect on the result generated ,(Adesanya *et al.* [5])

Notable scholars have studied the direct solution to higher order initial value problems.(Awoyemi *et al* [6], Awoyemi [7,8], Kayode and Awoyemi [9], Kayode [10], Adesanya *et al* [11], Yahaya and Badmus [12]);they proposed continuous linear multistep methods which were implimented in predictor-corrector mode. Continuous methods have the advantage of evaluating at all points within the integration interval, thus reducing the computational burden when evaluation is required at more than one point within the integration interval. They developed a separate reducing order of accuracy predictors and used Taylor series expansion to provide the starting values in order to impliment the corrector. Jator [4], Jator and Li [13], Omar and Suleiman [14], Awoyemi [7], Zarima *et al* [15] have proposed direct block methods of the form

$$A^{(0)}Y_m^{(i)} = \sum_{i=0}^{1} e y_n^{(i)} + h^2\left[d_i f(y_n) + b_i f(Y_m)\right] \tag{2}$$

where

$$Y_m = \left[y_n \quad y_{n+1} \quad y_{n+2} \quad . \quad .. \quad y_{n+r}\right]^T$$

$$F(y_m) = \left[f_n \quad f_{n+1} \quad f_{n+2} \quad . \quad .. \quad f_{n+r}\right]^T$$

$$F(y_n) = \left[f_{n-1} \quad f_{n-2} \quad f_{n-3} \quad . \quad .. \quad f_n\right]^T$$

$e_i = r \times r$ matrix, $A^{(0)} = r \times r$ identity martix.

Block method was later proposed to cater for some of the setbacks of predictor - corrector method.Despite the success of this method, the interpolation point cannot exceed the order of the differential equation, hence all the interpolation point cannot be exhausted resultting in a method of lower order being developed.(Adesanya *et al.* [1,5]). In order to cater for the setback of block method, Scholars developed block predictor-corrector method (Milne approach). This method formed a bridge between the predictor - corrector method and block method. (James *et al.* [16], Adesanya *et al.* [1,5]). According to literature the major setback of Block predictor-corrector method is that the results are generated at an overlapping interval, hence this affect the accuracy of the method and the nature of the model cannot be determined at the selected grid points.

In this paper, we developed a method using the milne approach but the corrector was implemented at a non overlapping interval, hence this method cater for some of the setbacks of the block predictor-corrector method as mentioned above. The numerical experiment compared the results generated by block method, block predictor-corrector method and our new method tagged block predictor-block corrector method.

2 METHODOLOGY

2.1 Development of the continuous Linear Multistep Methods

We consider a power series approximate solution in the form

$$y(x) = \sum_{j=0}^{r+s-1} a_j x^j \tag{3}$$

where r and s are the number of interpolation and collocation points respectively.

The second derivative of (3) gives

$$y''(x) = \sum_{j=2}^{r+s-1} j(j-1)a_j x^{j-2} \tag{4}$$

Substituting (4) into (1) gives

$$f(x, y, y') = \sum_{j=2}^{r+s-1} j(j-1)a_j x^{j-2} \tag{5}$$

Interpolating (3) and collocating (5) at some selected grid points gives a system of non linear equations in the form

$$AX = U \tag{6}$$

where

$$A = [a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \quad a_{r+s-1}]^T$$

$$U = [y_n \quad y_{n+1} \quad \dots \quad y_{n+r} \quad f_n \quad f_{n+1} \quad \dots \quad f_{n+s}]^T$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & \dots & x_n^{r+s-1} \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & \dots & x_{n+1}^{r+s-1} \\ . & . & . & . & . & . \\ 1 & x_{n+r} & x_{n+r}^2 & x_{n+r}^3 & \dots & x_{n+r}^{r+s-1} \\ 0 & 0 & 2 & 6x_n & \dots & (s+r-1)(s+r-2)x_n^{r+s-1} \\ 0 & 0 & 2 & 6x_{n+1} & \dots & (s+r-1)(s+r-2)x_{n+1}^{r+s-1} \\ . & . & . & . & . & . \\ 0 & 0 & 2 & 6x_{n+s} & \dots & (s+r-1)(s+r-2)x_{n+s}^{r+s-1} \end{bmatrix}$$

Solving (6) for the unknown constants $a_j's$ using Guassian elimination method and substituting back into (3) gives a continuous linear multistep method in the form

$$y(t) = \sum_{j=0}^{r} \alpha_j(t) y_{n+j} + h^2 \sum_{j=0}^{s} \beta_j(t) f_{n+j} \tag{7}$$

where $\alpha_j(t)$ and $\beta_j(t)$ are polynomials,

$$f_{n+j} = (fx_n + jh, y(x_n + jh), y'(x_n + jh)), t = \frac{x - x_n}{h}$$

2.2 Development of the Block Predictor

Interpolating (3) at $x_{n+r}, r = 0,1$ and collocating (5) at $x_{n+s}, s = 0(1)4,$ the parameters in (6) becomes

$$A = [a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6]^T$$

$$U = [y_n \quad y_{n+1} \quad f_n \quad f_{n+1} \quad f_{n+2} \quad f_{n+3} \quad f_{n+4}]^T$$

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$$X=\begin{bmatrix}1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6\\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6\\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4\\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 & 20x_{n+1}^3 & 30x_{n+1}^4\\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 & 30x_{n+2}^4\\ 0 & 0 & 2 & 6x_{n+3} & 12x_{n+3}^2 & 20x_{n+3}^3 & 30x_{n+3}^4\\ 0 & 0 & 2 & 6x_{n+4} & 12x_{n+4}^2 & 20x_{n+4}^3 & 30x_{n+4}^4\end{bmatrix}$$

Hence (7) reduces to

$$y(t)=\sum_{j=0}^{1}\alpha_j(t)y_{n+j}+h^2\sum_{j=0}^{4}\beta_j(t)f_{n+j} \tag{8}$$

where

$$\alpha_0=1-t$$

$$\alpha_1=t$$

$$\beta_0=\frac{1}{1440}\left(2t^6-30t^5-175t^4-500t^3+720t^2-367t\right)$$

$$\beta_1=-\frac{1}{360}\left(2t^6-27t^5+130t^4-240t^3+135t\right)$$

$$\beta_2=\frac{1}{240}\left(2t^6-24t^5+95t^4-120t^3+47t\right)$$

$$\beta_3=-\frac{1}{360}\left(2t^6-21t^5+70t^4-80t^3+29t\right)$$

$$\beta_4=\frac{1}{1440}\left(2t^6-18t^5+55t^4-60t^3+21t\right)$$

Solving for the independent solution in (8), gives the continuous block formular in the form

$$y_{n+j}=\sum_{i=0}^{1}\frac{(jh)^i}{i!}y_n^{(i)}+h^2\sum_{j=0}^{4}\sigma_j(x)f_{n+j} \tag{9}$$

where

$$\sigma_0=\frac{1}{1440}\left(2t^6-30t^5-175t^4-500t^3+720t^2\right)$$

$$\sigma_1=-\frac{1}{360}\left(2t^6-27t^5+130t^4-240t^3\right)$$

$$\sigma_2=\frac{1}{240}\left(2t^6-24t^5+95t^4-120t^3\right)$$

$$\sigma_3 = -\frac{1}{360}\left(2t^6 - 21t^5 + 70t^4 - 80t^3\right)$$

$$\sigma_4 = \frac{1}{1440}\left(2t^6 - 18t^5 + 55t^4 - 60t^3\right)$$

Evaluating (9) at $t = 1(1)4$ and implementing in block method, the parameters in (2) reduces to:

When $i = 0$

$A^{(0)} = 4 \times 4$ identity matrix

$Y_m^{(0)} = \left[y_{n+1} \quad y_{n+2} \quad y_{n+3} \quad y_{n+4}\right]^T$

$$e_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad e_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$d_0 = \begin{bmatrix} 0 & 0 & 0 & \frac{367}{1440} \\ 0 & 0 & 0 & \frac{53}{90} \\ 0 & 0 & 0 & \frac{147}{160} \\ 0 & 0 & 0 & \frac{56}{45} \end{bmatrix}, \qquad b_0 = \begin{bmatrix} \frac{3}{8} & -\frac{47}{240} & \frac{29}{360} & -\frac{7}{480} \\ \frac{8}{5} & \frac{-1}{3} & \frac{8}{45} & -\frac{1}{30} \\ \frac{117}{40} & \frac{27}{80} & \frac{3}{8} & -\frac{9}{160} \\ \frac{64}{15} & \frac{16}{15} & \frac{64}{45} & 0 \end{bmatrix}$$

When $i = 1$

$Y_m^{(i)} = \left[y'_{n+1} \quad y'_{n+2} \quad y'_{n+3} \quad y'_{n+4}\right]^T$

$$e_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad d_1 = \begin{bmatrix} 0 & 0 & 0 & \frac{251}{720} \\ 0 & 0 & 0 & \frac{29}{90} \\ 0 & 0 & 0 & \frac{27}{80} \\ 0 & 0 & 0 & \frac{14}{45} \end{bmatrix}$$

$$b_1 = \begin{bmatrix} \frac{323}{360} & -\frac{11}{80} & \frac{53}{360} & -\frac{19}{720} \\ \frac{62}{45} & \frac{4}{15} & \frac{2}{45} & -\frac{1}{90} \\ \frac{51}{40} & \frac{9}{10} & \frac{21}{40} & -\frac{3}{80} \\ \frac{64}{45} & \frac{8}{15} & \frac{64}{45} & \frac{14}{45} \end{bmatrix}$$

2.3 Development of the Block Corrector

Interpolating (3) at $x_{n+r}, r = 0(1)2$ and collocating (5) at $x_{n+s}, s = 0(1)4$, (6) reduces to

$$A = [a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7]^T$$

$$U = [y_n \quad y_{n+1} \quad y_{n+2} \quad f_n \quad f_{n+1} \quad f_{n+2} \quad f_{n+3} \quad f_{n+4}]^T$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 & 42x_n^5 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 & 20x_{n+1}^3 & 30x_{n+1}^4 & 42x_{n+1}^5 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 & 30x_{n+2}^4 & 42x_{n+2}^5 \\ 0 & 0 & 2 & 6x_{n+3} & 12x_{n+3}^2 & 20x_{n+3}^3 & 30x_{n+3}^4 & 42x_{n+3}^5 \\ 0 & 0 & 2 & 6x_{n+4} & 12x_{n+4}^2 & 20x_{n+4}^3 & 30x_{n+4}^4 & 42x_{n+4}^5 \end{bmatrix}$$

Hence (7) reduces to

$$y(t) = \sum_{j=0}^{2} \alpha_j(t) y_{n+j} + h^2 \sum_{j=0}^{4} \beta_j(t) f_{n+j} \qquad (10)$$

where

$$\alpha_0 = \frac{1}{42}\left(2t^7 - 28t^6 + 147t^5 - 350t^4 + 336t^3 - 149t + 42\right)$$

$$\alpha_1 = -\frac{1}{21}\left(2t^7 - 28t^6 + 147t^5 - 350t^4 + 336t^3 - 128t\right)$$

$$\alpha_2 = \frac{1}{42}\left(2t^7 - 28t^6 + 147t^5 - 350t^4 + 336t^3 - 107t\right)$$

$$\beta_0 = -\frac{1}{10080}\left(38t^7 - 546t^6 + 3003t^5 - 7875t^4 + 9884t^3 - 5040t^2 + 536t\right)$$

$$\beta_1 = -\frac{1}{1260}\left(51t^7 - 707t^6 + 3654t^5 - 8470t^4 + 7728t^3 - 2256t\right)$$

$$\beta_2 = -\frac{1}{720}\left(2t^7 - 34t^6 + 219t^5 - 635t^4 + 696t^3 - 248t\right)$$

$$\beta_3 = -\frac{1}{1260}\left(t^7 - 7t^6 + 70t^4 - 112t^3 + 48t\right)$$

$$\beta_4 = \frac{1}{10080}\left(2t^7 - 14t^6 + 21t^5 + 35t^4 - 84t^3 + 40t\right)$$

Evaluating (10) at $t = 3,4$ and it first derivative at $t = 0,1$ and implementing in block method gives

$$A^{(0)}Y_m = A^{(i)}Y_{m-1} + A^{(k)}Y_{m-2} + h^2\left[B^{(0)} f_{m-1} + B^{(i)} f_m\right] \qquad (11)$$

where

$A^{(0)} = 4\times 4$ identity matrix

$Y_m = \left[y_{n+1} \quad y_{n+2} \quad y_{n+3} \quad y_{n+4}\right]^T$

$Y_{m-1} = \left[y_{n-1} \quad y_{n-2} \quad y_{n-3} \quad y_n\right]^T$

$Y_{m-2} = \left[y'_{n-1} \quad y_{n-2} \quad y'_n \quad y'_{n+1}\right]^T$

$F_m = \left[f_{n+1} \quad f_{n+2} \quad f_{n+3} \quad f_{n+4}\right]^T$

$F_{m-1} = \left[f_{n-1} \quad f_{n-2} \quad f_{n-3} \quad f_n\right]^T$

$$A^{(i)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad A^{(k)} = \begin{bmatrix} 0 & 0 & \frac{82}{189} & \frac{107}{189} \\ 0 & 0 & \frac{122}{189} & \frac{256}{189} \\ 0 & 0 & \frac{6}{7} & \frac{15}{7} \\ 0 & 0 & \frac{244}{189} & \frac{512}{189} \end{bmatrix}$$

$$B^{(0)} = \begin{bmatrix} 0 & 0 & 0 & \frac{15649}{272160} \\ 0 & 0 & 0 & \frac{397}{3402} \\ 0 & 0 & 0 & \frac{577}{3360} \\ 0 & 0 & 0 & \frac{2552}{8505} \end{bmatrix}, \qquad B^{(i)} = \begin{bmatrix} -\frac{4523}{34020} & \frac{533}{45360} & -\frac{19}{6804} & \frac{97}{272160} \\ \frac{3272}{8505} & \frac{463}{2835} & -\frac{184}{8505} & \frac{41}{17010} \\ \frac{421}{420} & \frac{629}{560} & \frac{5}{84} & \frac{1}{3360} \\ \frac{15616}{8505} & \frac{1168}{567} & \frac{8704}{8505} & \frac{608}{8505} \end{bmatrix}$$

3 ANALYSIS OF THE BASIC PROPERTIES OF THE METHOD

3.1 Order of the method

Let the linear operator $L\{y(x):h\}$ associated with the block method be defined as

$$L\{y(x):h\} = A^{(0)}Y_m - A^{(i)}Y_{m-1} - A^{(k)}Y_{m-2} - h^2\left[B^{(0)}F_{m-1} + B^{(i)}F_m\right] \qquad (12)$$

Expanding (12) in Taylor's series gives

$$L\{y(x):h\} = C_0 y(x) + C_1 h y'(x) + \ldots + C_p h^p y^{(p)}(x) + C_{p+1} h^{p+1} y^{(p+1)}(x+) + \ldots \qquad (13)$$

Definition 1 *Order*

The linear operator and associated method are said to be of order p if $C_0 = C_1 = \ldots = C_{p+1} = 0$ and $C_{p+2} \neq 0, C_{p+2}$ is called the error constant and implies that the local truncation error is given by

$$t_{n+k} = C_{p+2} h^{p+2} y^{p+2} + 0(h^{p+3})$$

Expanding (2) and (11) in Taylor's series expansion and comparing the powers of h,the order of the block corrector is six with error constants

$$\left[-1.4754\times10^{-4} \quad -7.7825\times10^{-4} \quad -8.964\times10^{-4} \quad -3.6729\times10^{-3}\right]^T$$

3.2 Consistency of the Method

A block method is said to be consistent if it has order p ± 1.

From the above, it clearly shows that our method is consistent.

3.3 Zero Stability:-

A block method is said to be zero stable if $h \to 0,$ the root $r_j; j = 1(1)k$ of the first characteristics polynomials $\rho(R) = 0,$ that is $\rho(R) = \det\left[\sum A^{(0)} R^{k-1}\right] = 0$ satisfying $|R| \leq 1$ must have multiplicity equal to unity.For our method

$$\rho(r) = \left| R\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right| = 0$$

and $R = 0,0,0,1$. Hence the method is zero stable

4 NUMERICAL EXPERIMENT

4.1 Numerical Examples

Error I: Block Predictor-Block Corrector method

Error II: Block predictot-corrector method

Error III: Block method.

Test Problem 1

$$y'' - x(y')^2 = 0, y(0) = 1, y'(0) = \frac{1}{2}, h = 0.01$$

Exact Solution: $y(x) = 1 + \frac{1}{2}\ln\left(\frac{2+x}{2-x}\right)$

Table 1:Comparing our method with existing methods

X-value	Error I	Error II	Error III
.1	9.103829e-15	4.862777e-14	9.992007e-15
.2	1.110223e-14	2.160494e-13	8.149037e-14
.3			

	1..576517e-14	5.255796e-13	4.700684e-13
.4	1.798561e-14	1.025402e-12	1.637801e-12
.5	2.775558e-14	1.803224e-12	4.664935e-12
.6	4.352074e-14	3.007816e-12	1.116263e-11
.7	5.595524e-14	4.899192e-12	2.501044e-11
.8	3.397282e-13	7.946088e-12	5.215339e-11
.9	5.551115e-14	1.302736e-11	1.076854e-10
.0	1.461054e-14	2.188583e-11	2.170679e-10

Test Problem II

Consider the initial value problem

$$y'' = \frac{(y')^2}{2y} - 2y; \quad y(\frac{\pi}{6}) = \frac{1}{4}, y'(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}, h = 0.01.$$

Exact solution $y(x) = (\sin(x))^2$;

Table 2: Comparing our method with existing methods

X-value	Error I	Error II	Error III
.003	1.517786e-12	2.138789e-11	4.779190e-10
.103	1.949219e-12	3.059430e-11	5.974645e-10
.203	2.530198e-12	4.070444e-11	6.895838e-10
.303	3.297806e-12	5.124245e-11	7.468689e-10
.403	4.226508e-12	6.169587e-11	7.709915e-10
.503	5.276668e-12	7.154077e-11	7.651488e-10
.603	6.378194e12	8.026635e-11	7.381171e-10
.703	7.273737e-12	8.739887e-11	6.996770e-10
.803	7.936540e-12	9.252354e-11	6.594617e-10
.903	8.164913e-12	9.530710e-11	6.275389e-10
.003	7.862899e-12	9.551315e-11	6.080769e-10

4.2 DISCUSION OF RESULT

We have considered two non-linear second order initial value problems in this paper as shown in Tables 1 annd 2. We compared our nw method with the existing methods; the block and block predictor-corrector.the results re-affirms the claim of $[1]$ that though block predictor-corrector method takes longer time to implement, it gives better approximation than the block method. the results equally agrees with the theory that block predicctor-corrector method could not give maximum ressults due to the overlapping of the result which prompted our new method. the results also shows that our new method has better stabilty properties than the existing methods.

5 CONCLUSION

In this paper we have proposed a four steps block predictor-block corrector method. A block method which has the properties of evaluation at all points within the interval of integration is adopted to give the independent solution at non overlapping intervals as the predictor to an order six corrector.The new method performed better than those of the block predictor-corrector and the Block methods. We therefore, recommend the block predictor-block corrector method for use in the quest for solutions to initial value problems of ordinary differential equations.

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