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Direct Rotation β - Numbers

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ABSTRACT

For any partition $\mu = (\mu_1, \mu_2, \ldots, \mu_n)$ of a non - negative integer number r there exist a diagram (A) of β - numbers for each e where e is a positive integer number greater than or equal to two; which introduced by James in 1978.These diagrams (A) play an enormous role in Iwahori-Hecke algebras and q-Schur algebras; as presented by Fayers in 2007. In this paper, we introduced some new diagrams (A^{90}), (A^{180}) and (A^{270}) by employing the "direct rotation application of three diffrent degrees namely 90^o,180^o and 270^o " on the main diagram (A). We concluded that we can find the successive main diagrams (A^{90}), (A^{180}) and (A^{270}) for the guides $b_2, b_3, \ldots$ and b_e depending on the main diagrams (A^{90}), (A^{180}) and (A^{270}) for b_1 and set these facts as rules named Rule (3.1.2), Rule (3.2.2) and Rule (3.3.2) respectively. We depended in our work on the idea of the intersection of the main diagrams (A) given by Mahmood in 2011 and the "upside-down β-numbers" again given by Mahmood in 2013.

Keywords:

β - numbers; Diagram (A); Intersection; Partition; Rotation

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(1) INTRODUCTION

Let r be a non- negative integer. A partition $\mu = (\mu_1, \mu_2, \ldots, \mu_n)$ of r is a sequence of non - negative integers such that $|\mu| = \sum_{i=1}^{n} \mu_i = r$ and $\mu_i \geq \mu_{i+1}$, $\forall i \geq 1$; [1]. For example, μ= (5, 4, 4, 2, 2, 2,1) is a partition of r =20. β - numbers was defined by; see James in [2]: "Fix μ is a partition of r, choose an integer b greater than or equal to the number of parts of μ and define $\beta_i = \mu_i + b - i$, $1 \leq i \leq b$. The set $\{\beta_1, \beta_2, \ldots, \beta_b\}$ is said to be the set of β-numbers for μ." For the above example, if we take b =7, then the set of β - numbers is {11, 9, 8, 5, 4, 3,1}.
Now, let e be a positive integer number greater than or equal to 2, we can represent β - numbers by a diagram called diagram (A).

run. 1	run. 2	. . .	run. e
0	1	. . .	e-1
e	e+1	. . .	2e-1
2e	2e+1	. . .	3e-1
.	.	.	.
.	.	.	.
.	.	.	.

diag.(A)

Where every β will be represented by a bead (●) which takes its location in diag.(A). Returning to above example, diagram (A) of β-numbers for e =2 and e =3 is as shown below in diagram 1 and 2 respectively:

e = 2		b =7	
0	1	—	●
2	3	—	●
4	5	●	●
6	7	—	—
8	9	●	●
10	11	—	●

Dig.1

e =3			b =7		
0	1	2	—	●	—
3	4	5	●	●	●
6	7	8	—	—	●
9	10	11	●	—	●

Dig.2

Note: Along this paper, we mean by diagram(A); diagram (A) of β-numbers.
This subject has a connection with representation theory of Iwahori - Hecke algebras and q-Schur algebras [3].
Also any partition μ of r is called w-regular; w ≥ 2, if there does not exist i ≥1 such that $\mu_i = \mu_{i+w-1} > 0$, and μ is called w-restricted if $\mu_i - \mu_{i+1} < w$, ∀i ≥1.

(2) THE INTERSECTION OF β - NUMBERS IN THE MAIN DIAGRAMS

Mahmood in [4] introduced the definition of main diagram(s) (A) and the idea of the intersection of these main diagrams. in this section we repeat the principals results, as follows: Since the value of b ≥ n; [5], then we deal with an infinite numbers of values of b. Here we want to mention that for each value of b there is a special diagram (A) of β-numbers for it, but there is a repeated part of one's diagram with the other values of b where a "Down –shifted" or "Up-shifted", occurs when we take the following : (b_1 if b = n), (b_2 if b = n+1), . . . and (b_e if b = n+(e-1)).

Definition (2.1): [4] The values of b_1 , b_2, . . . and b_e are called the guides of any diagram (A) of β-numbers .
For the above example, the guides values are b_1 =7 and b_2 = 8 if e =2 where $\mu = (5,4,4,2,2,2,1)$, then:

e = 2		b =7		b_1+1(e)		b_1+2(e)	
0	1	—	●	●	●	●	●
2	3	—	●	—	●	●	●
4	5	●	●	—	●	—	●
6	7	—	—	●	●	—	●
8	9	●	●	—	—	●	●
10	11	—	●	●	●	—	—
12	13			—	●	●	●
14	15					—	●
16	17						

and

e = 2		b_2 = 8		b_2+1(e)		b_2+2(e)	
0	1	●	—	●	●	●	●
2	3	●	—	●	—	●	●
4	5	●	●	●	—	●	—
6	7	●	—	●	●	●	—
8	9	—	●	●	—	●	●
10	11	●	—	—	●	●	—
12	13	●	—	●	—	—	●
14	15			●	—	●	—
16	17					●	—

Dig.3 Illustrates the idea of "Down- shifted"

We define any diagram (A) that corresponds any b guides as a "main diagram" or "guide diagram".

Theorem (2.2): [4] There is e of main diagrams for any partition μ of r.■

The idea of the intersection of any main diagrams is defined by the following:

1- Let τ be the number of redundant part of the partition μ of r, then we have $\mu = (\mu_1, \mu_2, \dots, \mu_n) = (\lambda_1^{\tau_1}, \lambda_2^{\tau_2}, \dots, \lambda_m^{\tau_m})$ such that $r = \sum_{i=1}^{n} \mu_i = \sum_{j=1}^{m} \lambda_j^{\tau_j}$.

2- We denote the intersection of main diagrams by $\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}$.

3 - The intersection result as a numerical value will be denoted by $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s})$, and it is equal to ϕ in the case of no existence of any bead, or γ in the case that γ common beads exist in the main diagrams.

For the above example where $\mu = (5,4,4,2,2,2,1) = (5, 4^2, 2^3, 1)$, r = 20, if e = 2 then there are two guides, the first is $b_1 = 7$ since n=7 and the second is $b_2 = 8$, the β- numbers are given in table 1:

Table 1 β- Numbers

b_s \ β_i	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
$b_1 = 7$	11	9	8	5	4	3	1	
$b_2 = 8$	12	10	9	6	5	4	2	0

Hence,the main diagrams and their intersection will be as shown in diagram 4:

$b_1 = 7$		$b_2 = 8$		$\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s}$	
—	•	•	—	—	—
—	•	•	—	—	—
•	•	•	•	•	•
—	—	•	—	—	—
•	•	—	•	—	•
—	•	•	—	—	—
		•	—	—	—

Dig.4 The intersection of the main diagrams (A) for e=2

Notice that, $\#(\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s})$ =3.

The two principle theorems about the idea of the intersection of any main diagrams are:

Theorem (2.3): [4] For any e ≥ 2, the following holds:

1- $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}) = \phi$ if $\tau_k = 1, \forall k$ where, $1 \le k \le m$.

2- Let Ω be the number of parts of λ which satisfies the condition $\tau_k \ge e$ for some k, then $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}) = [\sum_{t=1}^{\Omega} \tau_t - \Omega\,(e-1)]$.■

Theorem (2.4): [4]

1- Let μ be a partition of r and μ is w-regular, then: $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}) = \begin{cases} \text{value} & \text{if } e < w, \\ \phi & \text{if } e \ge w. \end{cases}$

2- Let μ be a partition of r and μ is h-restricted, then: $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}) = \begin{cases} \text{value if } e < h \text{ or } (e = h \text{ and } h < w), \\ \phi \quad \text{if } e > h \text{ or } (e = h \text{ and } h \ge w). \end{cases}$ ■

Also, Sarah M. Mahmood in [6] gave the same subject by using a new technique which supported the results of Mahmood in [4].

(3) DIRECT ROTATION OF DEGREES 90°, 180° AND 270° β - NUMBERS

In this work, we introduce new diagrams depending on the old diagram (A) with application of " direct rotation of degrees 90°, 180° and 270° respectively ". Note that, all the rotations are about the origin and by direct rotation; we mean: counter clockwise rotation ". The new diagrams have another partitions of the origin partition and if we use the idea of the intersection, the partition of the beads will not be the same (or will not be the sum) in $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s})$ in the normal main diagrams. In order to understand the subject, we'll study this application on our example above, where $\mu = (5, 4^2, 2^3, 1)$ and e = 2, respectively**:**

(3.1) Direct Rotation of degree 90° β - Numbers:

We' ll have a group of diagram (A^{90}) as shown in diagram 6 below:

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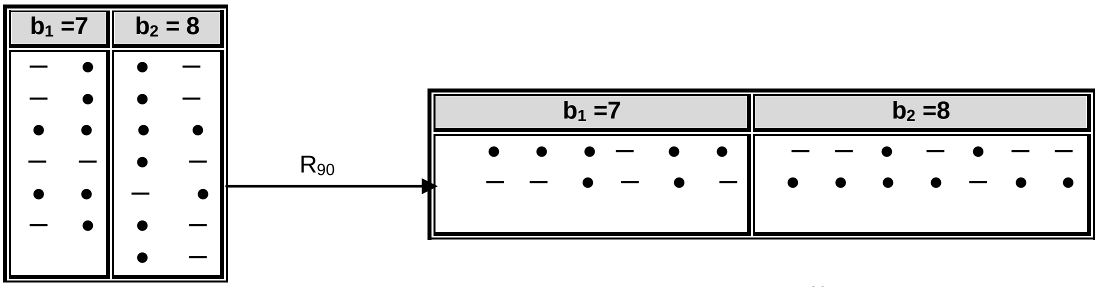
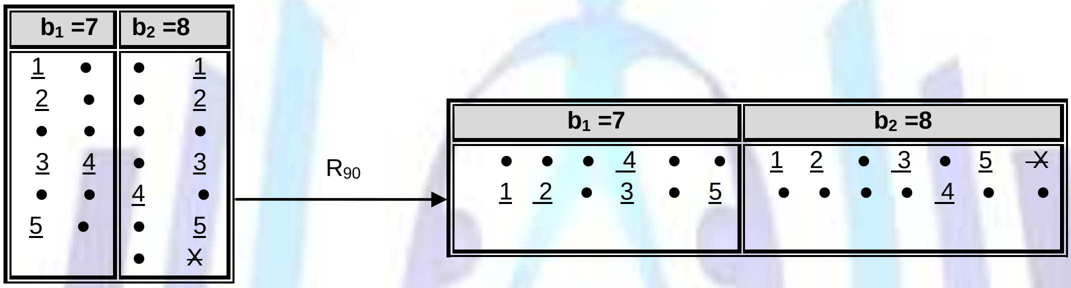


Dig.5 (A) Dig.6 (A^{90})

Now, if we use the old technique for finding any partition of any diagram (A^{90}), the value of the partition will not be equal to the origin partition? so, we delete any effect of (-) in (A) after the position of β_1, and we start with number 1 for the first (-) a (left to right) in any row exist in (A), and with number 2 for the second (-) and ...,etc, and we stop with last (-) before the position β_1 in (A) as shown in diagram 7. Now, to apply "direct rotation of degree 90^o" on (A), the new version (A^{90}) has the same partition of (A), see diagram 8.



Dig.7 (A) Dig.8 (A^{90})

Remark (3.1.1): The main diagram (A^{90}) in case $b_1 = n$, plays a main role to design all the main diagrams (A^{90}) for ($b_2 = n+1$), ... and ($b_e = n+(e-1)$), as follows:

Rule (3.1.2): Since the main diagram (A^{90}) in the case b_1, we can find the successive main diagrams (A^{90}) for $b_2, b_3, \ldots$ and b_e, as follows:

1) 1^{st} row in the case $b_1 = n \to$ last row in the case b_2 and to add one (●) in left $\to$ (e-1) row in the case b_3 and to add one (●) in left $\to \cdots \to 2^{nd}$ row in the case b_e and to add one (●) in left of main diagram (A^{90}).

2) 2^{nd} row in the case $b_1 \to 1^{st}$ row in the case b_2 and to add one (-) in right $\to$ last row in the cas b_3 and to add one (●) in left $\to \cdots \to 3^{rd}$ row in the case b_e and to add one (●) in left.

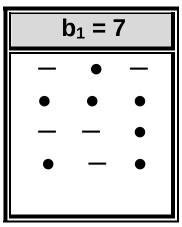
. . . .

. . . .

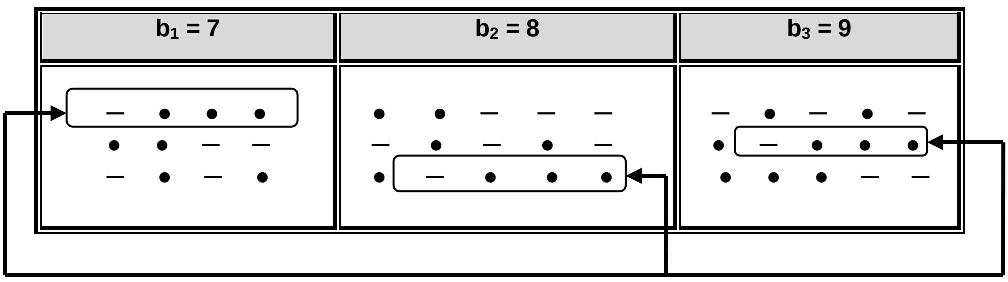
. . . .

e) last row in the case $b_1 \to$ (e-1) row in the case b_2 and to add one (-) in right $\to$ ... $\to 1^{st}$ row in the case b_e and to add one (-) in right. ■

This rule is clarified in diagram 9 For the above example, where $\mu = (5, 4^2, 2^3, 1)$ and e = 3:



Dig.2



Dig.9

Theorem (3.1.3): All the results in [4] about the main diagram (A) is the same of the diagram (A^{90}) but in direct rotation of degree 90^0 position. ■

One of these results is the intersection of the main diagrams. so, the fact mentioned in theorem(3.1.2) is clear in diagram 10 comparing it with diagram 4, for our example when $\mu = (5, 4^2, 2^3, 1)$ and e = 2 and for e=3, see the two diagrams 11 and 12:

$b_1 = 7$	$b_2 = 8$	$\bigcap_{s=1}^{2} \mathbf{m.d.}_{b_s}$
• • • — • • — — • — • —	— — • — • — — • • • • — • •	— — • — • — — — — • — — — —

Dig.10 The intersection of the main diagrams (A^{90}) for e=2

Notice that, $\#(\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s}) = 3$, in both cases.

$b_1 = 7$	$b_2 = 8$	$b_3 = 9$	$\bigcap_{s=1}^{3} \mathbf{m.d.}_{b_s}$
— • — • • • — — • • — •	• — • — • • • — — • • — • — —	• • — • — • • • — — • • — • —	— — — — — • — — — — — — — — —

Dig.11 The intersection of the main diagrams (A) for e=3

$b_1 = 7$	$b_2 = 8$	$b_3 = 9$	$\bigcap_{s=1}^{3} \mathbf{m.d.}_{b_s}$
— • • • • • — — — • — •	• • — — — — • — • — • — • • •	• • — — — — • — • — • — • • •	— • — — — — — — — — — — — — —

Dig.12 The intersection of the main diagrams (A^{90}) for e=3

Notice that, $\#(\bigcap_{s=1}^{3} \mathrm{m.d.}_{b_s}) = 1$, in both cases.

(3.2). Direct Rotation of degree 180^o β - Numbers: We' ll have a group of diagram (A^{180}) as shown in diagram 13 below:

$b_1 = 7$	$b_2 = 8$
— • — • • • — — • • — •	• — • — • • • — — • • — • —

Dig.5 (A)

$\xrightarrow{R_{180}}$

$b_1 = 7$	$b_2 = 8$
 • — • • — — • • • — • —	— • — • • — — • • • — • — •

Dig.13 (A^{180})

Again, if we use the old technique for finding any partition of any diagram (A^{180}), the value of the partition will not be equal to the origin partition? so, we delete any effect of (-) in (A) after the position of β_1, and we start with number 1 for the first (-) a (left to right) in any row exist in (A), and with number 2 for the second (-) and ...,etc, and we stop with last (-) before the position β_1 in (A) as shown in diagram 7. Now, to apply "direct rotation of degree 180^o" on (A), the new version (A^{180}) has the same partition of (A), see diagram 14.

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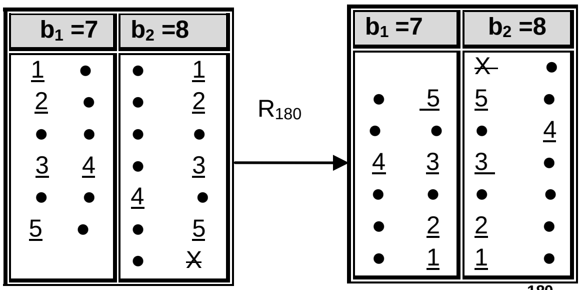


Dig.7 (A) Dig.14 (A^{180})

Remark(3.2.1): The main diagram (A^{180}) in case $b_1 = n$, plays a main role to design all the main diagrams (A^{180}) for ($b_2 = n+1$), ... and ($b_e = n+(e-1)$), as follows:

Rule (3.2.2): Since the main diagram (A^{180}) in the case b_1, we can find the successive main diagrams (A^{180}) for b_2, b_3, ... and b_e , as follows:

1) 1^{st} column in the case $b_1 = n \rightarrow$ last column in the case b_2 and to add one (•) in down → (e-1) column in the case b_3 and to add one (•)in down → ⋯ → 2^{nd} column in the case b_e and to add one (•) in down of main diagram (A^{180}).

2) 2^{nd} column in the case $b_1 \rightarrow 1^{st}$ column in the case b_2 and to add one (-) in up → last column in the case b_3 and to add one (•) in down → ⋯ →3^{rd} column in the case b_e and to add one (•) in down of main diagram (A^{180}).

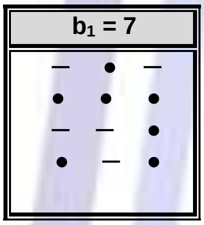
. . . .

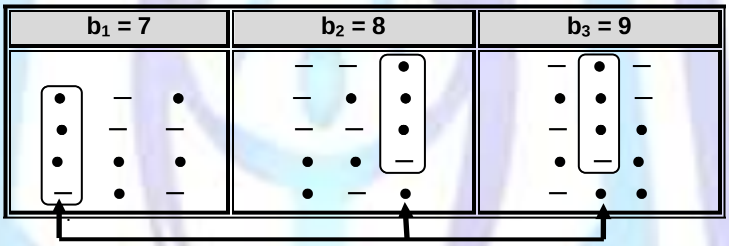
. . . .

. . . .

e) last column in the case $b_1 \rightarrow$ (e-1) column in the case b_2 and to add one (-) in up→ ⋯ → 1^{st} column in the case b_e and to add one of (-) in up. ■

To check this rule For our example, where $\mu = (5, 4^2, 2^3, 1)$ and e = 3, see diagram 15 below:

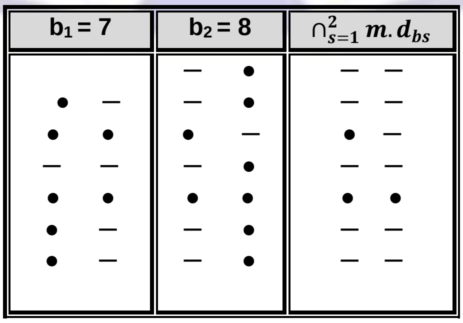




Dig.2 Dig.15

Theorem (3.2.3): All the results in [4] about the main diagram (A) is the same of the diagram (A^{180}) but in direct rotation of degree 180^0 position. ■

Now, as we said before, the intersection of the main diagrams is one of these results, hence see diagram 16 and compare it with diagram 4 for our example when e=2 and for e=3, see diagram 17 compairing it with diagram 11 above:



Dig.16 The intersection of the main diagrams (A^{180}) for e=2

Again, $\#(\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s}) = 3$, in both cases.

$b_1 = 7$			$b_2 = 8$			$b_3 = 9$			$\bigcap_{s=1}^{3} \mathrm{m.d.}_{b_s}$		
			—	—	•	—	•	—	—	—	—
•	—	•	—	•	•	•	•	—	—	—	—
•	—	—	—	—	•	—	•	•	—	—	—
•	•	•	•	•	—	•	—	•	•	—	—
—	•	—	•	—	•	—	•	•	—	—	—

Dig.17 The intersection of the main diagrams (A^{180}) for e=3

Also, $\#(\bigcap_{s=1}^{3} \mathrm{m.d.}_{b_s}) = 1$, in both cases.

(3.3) Rotation 270^0 counter clockwise about the origin: We' ll have a group of diagram (A^{270}) as shown in diagram 18 below:

$b_1 = 7$		$b_2 = 8$	
—	•	•	—
—	•	•	—
•	•	•	•
—	—	•	—
•	•	—	•
—	•	•	—
		•	—

Dig.5 (A)

$\xrightarrow{R_{270}}$

$b_1 = 7$						$b_2 = 8$						
—	•	—	•	—	—	•	•	—	•	•	•	•
•	•	—	•	•	•	—	—	•	—	•	—	—

Dig.18 (A^{270})

Again, if we use the old technique for finding any partition of any diagram (A^{270}), the value of the partition will not be equal to the origin partition? so, we delete any effect of (-) in (A) after the position of β_1, and we start with number 1 for the first (-) a (left to right) in any row exist in (A), and with number 2 for the second (-) and ...,etc, and we stop with last (-) before the position β_1 in (A) as shown in diagram 7. Now, to apply "direct rotation of degree 270^{o}" on (A), the new version (A^{270}) has the same partition of (A), see diagram 19.

$b_1 = 7$		$b_2 = 8$	
1	•	•	1
2	•	•	2
•	•	•	•
3	4	•	3
•	•	4	•
5	•	•	5
		•	✗

Dig.7 (A)

$\xrightarrow{R_{270}}$

$b_1 = 7$						$b_2 = 8$						
•	•	•	4	•	•	1	2	•	3	•	5	✗
1	2	•	3	•	5	•	•	•	•	4	•	•

Dig.19 (A^{270})

Remark(3.3.1): The main diagram (A^{270}) in case $b_1 = n$, plays a main role to design all the main diagrams (A^{270}) for ($b_2 = n+1$), ... and ($b_e = n+(e-1)$), as follows:

Rule (3.3.2): Since the main diagram (A^{270}) in the case b_1, we can find the successive main diagrams (A^{270}) for b_2, b_3, ..., and b_e, as follows:

1) 1^{st} row in the case $b_1 = n \rightarrow 2^{nd}$ row in the case b_2 and to add one (-) in left $\rightarrow 3^{rd}$ row in b_3 and to add one (-) in left $\rightarrow \cdots \rightarrow$ last row in the case b_e and to add one (-) in left of main diagram (A^{270}).
2) 2^{nd} row in the case $b_1 \rightarrow 3^{rd}$ row in the case b_2 and to add one (-) in left $\rightarrow \cdots \rightarrow$ last row in the case $b_{(e-1)}$ and to add one (-) in left $\rightarrow 1^{st}$ row in the case b_e and to add one (●) in right.

. ...

. ...

. ...

e) last row in the case $b_1 \rightarrow 1^{st}$ row in the case b_2 and to add one (●) in right $\rightarrow 2^{nd}$ row in the case b_3 and to add one (●) in right $\rightarrow \cdots \rightarrow$ (e-1) row in the case b_e and to add one (●) in right.■

To materialize rule (3.3.2) For the above example, where $\mu = (5, 4^2, 2^3, 1)$ and e = 3,see diagram20:

$b_1 = 7$
— • — • • • — — • • — •

Dig.2

$b_1 = 7$	$b_2 = 8$	$b_3 = 9$
• — • — — — • • • • • —	• • • — • — • — • — — — — • •	— — • • • • • • — • — • — • —

Dig.20

Theorem (3.3.3): All the results in [4] about the main diagram (A) is the same of the diagram (A^{270}) but in direct rotation of degree 270^0 position. ■

To perceive theorem (3.3.3) for this type of rotation , return back to our example where $\mu = (5, 4^2, 2^3, 1)$ and observe digrams 21 and compare it withdiagram 4 for e=2 and diagram 22 to be compared with diagram 11 for e=3:

$b_1 = 7$	$b_2 = 8$	$\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s}$
— • — • — — • • — • • •	• • — • • • • — — • — • — —	— — — — • — — — — • — • — —

Dig.21 The intersection of the main diagrams (A^{270}) for e=2

Notice that, $\#(\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s}) = 3$, in both cases.

$b_1 = 7$	$b_2 = 8$	$b_3 = 9$	$\bigcap_{s=1}^{3} \mathbf{m.d.}_{b_s}$
• — • — — — • • • • • —	• • • — • — • — • — — — — • •	— — • • • • • • — • — • — • —	— — — — — — — — — — — — — • —

Dig.22 The intersection of the main diagrams (A^{270}) for e=3

Also, $\#(\bigcap_{s=1}^{3} \mathrm{m.d.}_{b_s}) = 1$, in both cases.

(4) CONCLUSION

In this paper, a procedure is suggested for the diagrams (A^{90}), (A^{180}) and (A^{270}) of β-numbers which they represent the direct rotation of degrees 90^o, 180^o and 270^o of diagram (A) of β-numbers respectively, to have the same partition of diagram (A) of β-numbers. Furthermore, for each degree of rotation, a rule for designing all the main diagrams of the direct rotation for $b_2, b_3, \ldots,$ and b_e is setted depending on the main diagram of the direct rotation for b_1. And finally, we foundout that the intersection of the new main diagrams of the direct rotation is the same of the old main diagrams (A) but in direct rotation position.

REFERENCES

[1] Mathas, A. 1999. Iwahori-Hecke Algebras and Schur Algebras of the Symmetric Groups. Amer. Math. Soc.University Lecture Series, 15.

[2] James, G. 1978. Some combinatorial results involving Young diagrams. Math. Proc. Cambridge Philos. Soc.,83,1-10,

[3] Fayers, M. 2007. Another runner removal theorem for r- decomposition number of Iwahori – Hecke algebras and q- Schur algebras. J. algebra, 310, 396- 404.

[4] Mahmood, A. S. 2011. On the intersection of Young's diagrams core. J. Education and Science (Mosul University), 24, no. 3, 143- 159,

[5] Mohammad , H. S. 2008. Algorithms of The Core of Algebraic Young's Tableaux. M.Sc.Thesis, College of Education, University of Mosul,

[6] Mahmood, S. M. 2011. On e- Regular and the Intersection of Young's Diagrams Core. M. Sc. Thesis, College of Education, University of Mosul.

[7] Mahmood, A. S. 2013. Upside - down β- numbers. Australian J. of Basic and Applied Sciences 7, no 7 , 986- 990.

[8] Mahmood, A. S. and Ali,Sh. S. 2013.Rightside - Left β- numbers.(To appear).