ISSN 2347-1921

Random Fixed Point Theorem in Fuzzy Metric Spaces

[1]Arvind Gupta, Anil Rajput[2] and [3]Anupama Gupta

[1]Professor, Department of Mathematics, Govt. M.V.M., Bhopal (M. P.)

[2] Professor, Department of Mathematics, CSA Govt. P. G. College, Sehore

[3]Assitt Prof., Professor, Department of Mathematics, Barkatullah University IT, Bhopal (M.P.)

Abstract.

In the present paper, we prove a fixed point theorem in fuzzy metric spaces through weak Compatibility.

Keywords: Fuzzy metric space; common fixed point; t-norm; compatible map; weak compatible map.

Council for Innovative Research

Peer Review Research Publishing System

Journal: Journal of Advances in Mathematics

Vol 5, No. 1

editor@cirworld.com

www.cirworld.com, member.cirworld.com

Introduction

The concept of Fuzzy sets was introduced by zadeh [4], following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michalek [5] and George and Veeramani [1] modified the notion of fuzzy metric spaces with the help of continuous t-norm. Vasuki [6] investigated some fixed point theorem in fuzzy metric spaces for R- weakly commuting mappings. In this paper we prove a common fixed point theorem for six maps under the condition of weak compatibility and compatibility in fuzzy metric spaces.

Preliminaries

Definition 2.1: A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if * is satisfying the following conditions:

(a) * is commutative and associative;

(b) * is continuous;

(c) a * b = a for all a ∈ [0, 1];

(d) a* b ≤ c * d whenever a ≤ c and b ≤ d and a, b, c, d ∈ [0, 1].

Definition 2.2: A 3-tuple (X , M , *) is said to be a fuzzy metric space if X is an arbitrary set , * is a continuous t-norm and M is a fuzzy set on X^2 × (0, ∞) satisfying the following conditions ; for all x, y, z ∈ X and s, t > 0.

(FM-1)M (x, y, t) > 0;

(FM-2) M(x, y, t) = 1 if and only if x = y;

(FM-3) M(x, y, t) = M(y, x, t);

(FM-4) M(x, y, t)* M(y, z, s) ≤ M(x, z, t+s);

(FM-5) M(x, y, .): (0, ∞)→[0, 1] is continuous.

Then M is called a fuzzy metric on X. The function M(x, y, t) denote the degree of nearness between x and y with respect to t.

Example 2.3: Let (X, d) be a metric space. Denote a *b =ab for a, b ∈ [0, 1] and let M_d be a fuzzy set on X^2 × (0, ∞) defined as follows:

$$M_d(x, y, t) = \frac{t}{t+d(x,y)}$$

Then (X, M_d , *) is a fuzzy metric space, we call this fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Definition 2.4: Let (X, M,*) be a fuzzy metric space, then

(a) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for each ∈ > 0 and each t > 0, there exists n_0 ∈ N such that M (x_n, x, t) > 1 - ∈ for all n ≥ n_0

(b)A sequence $\{x_n\}$ in X is said to be Cauchy if for each ∈ > 0 and each t > 0, there exist n_0 ∈ N such that M (x_n,x_m, t)> 1 - ∈ for all n, m ≥ n_0 .

(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Proposition2.5: In a fuzzy metric space (X, M,*) , if a * a ≥ a for a ∈ [0, 1]then a* b = min {a, b} for all a, b ∈ [0,1].

Definition 2.6: Two self mappings A and S of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty}$ M (ASx_n, SAx_n, t) = 1 whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$ = x for some x in X .

Definition2.6: Two self maps A and B of a fuzzy metric space (X, M,*) are called weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if Ax = Bx for some x ∈ X then ABx = BAx.

Remark 2.7: If self maps A and B of a fuzzy metric space (X, M,*) are compatible then they are weakly compatible. Let (X, M,*) be a fuzzy metric space with the following condition:

(FM-6) $\lim_{t\to\infty}$ M (x, y, t) =1 for all x, y ∈ X.

Lemma 2.8: Let (X, M, *) be a fuzzy metric space. If there exists k ∈ [0, 1] such that M (x, y, kt) ≥ M(x, y, t) then x = y .

Lemma 2.9: Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, *) with the condition (FM-6). If there exists k ∈ [0, 1] such that M (y_n, y_{n+1}, kt) ≥ M (y_{n-1}, y_n, t) for all t > 0 and n ∈ N then $\{y_n\}$ is a Cauchy sequence in X.

Main Results

Theorem3.1: Let A, B, S, T, L and N be self maps on a complete fuzzy metric space (X, M, *) with t * t ≥ t for all t ∈ [0, 1], satisfying:

(a) L (X) ⊆ ST(X) , N (X) ⊆ AB(X);

(b) There exists a constant k ∈ [0, 1] such that

$$M^2 (Lx, Ny, kt)* [M (ABx, Lx, kt)* M (STy, Ny, kt)] \geq M(STx, Lx, t) * M(ABy, Ny, t) * M(ABy, Lx, t)*M (STx, Ny, t) * M (STx, ABy, t)$$

(c) AB =BA, ST =TS, LB =BL, NT =TN.

(d) Either AB or L is continuous.

(e) The pair (L, AB) is compatible and (N, ST) is weakly compatible.

Then A, B, S, T, L, N have a unique common fixed point.

Proof: Let x_0 be an arbitrary point of X. By (a) there exists x_1, x_2 ∈ X such that Lx_0 = STx_1 = y_0 and Nx_1 = ABx_2 = y_1. Inductively we can construct sequence $\{x_n\}$ and $\{y_n\}$ such that

Lx_{2n} = STx_{2n+1} = y_{2n} and Nx_{2n+1} =ABx_{2n+2} = y_{2n+1} for n= 0, 1, 2.....

Step -1 By taking x=x_{2n} and Y=x_{2n+1} in (b) we have

$$M^2 (Lx_{2n}, Nx_{2n+1}, Kt)*[M (ABx_{2n}, Lx_{2n}, Kt) *M (STx_{2n+1}, Nx_{2n+1}, kt)] \geq M(STx_{2n},Lx_{2n},t) * M(ABx_{2n+1},Nx_{2n+1},t) *M(ABx_{2n+1},Lx_{2n},T) *M(STx_{2n}, Nx_{2n+1}, t) * M (STx_{2n}, ABx_{2n+1}, t)$$

$$M^2(y_{2n}, y_{2n+1}, kt) *[M(y_{2n-1}, y_{2n}, kt) * M(y_{2n}, y_{2n+1}, kt)] \geq M(y_{2n-1}, y_{2n},t) * M(y_{2n}, y_{2n+1}, t) * M(y_{2n} , y_{2n},t) * M(y_{2n-1} , y_{2n+1},t) *M(y_{2n-1} , y_{2n},t)$$

$$\Rightarrow M^2 (y_{2n}, y_{2n+1}, kt) *[M(y_{2n-1}, y_{2n}, kt) ,M(y_{2n}, y_{2n+1}, kt)] \geq M(y_{2n-1}, y_{2n}, t) * M(y_{2n}, y_{2n+1}, t) * 1 * M (y_{2n-1}, y_{2n+1}, t) * M(y_{2n-1}, y_{2n}, t)$$

$$\Rightarrow M^2(y_{2n}, y_{2n+1},kt)*[M(y_{2n-1}, y_{2n} ,kt) *M (y_{2n}, y_{2n+1}, kt)] \geq M(y_{2n-1}, y_{2n}, t)* M(y_{2n}, y_{2n+1},t) * M(y_{2n-1}, y_{2n+1}, t)* M(y_{2n-1} , y_{2n} ,t)$$

$$\Rightarrow M^2 (y_{2n} , y_{2n+1},kt) \geq M(y_{2n-1} , y_{2n},t)* M(y_{2n-1}, y_{2n+1},t) \geq M(y_{2n-1} , y_{2n} ,t) * M(y_{2n-1}, y_{2n},t)* M(y_{2n} , y_{2n+1} , t)$$

$$\Rightarrow M (y_{2n} , y_{2n+1} , kt) \geq M(y_{2n-1}, y_{2n},t)$$

In general

$$M (y_{2n+1},y_{2n+2}, kt) \geq M (y_{2n} , y_{2n+1},t)$$

In general for all n even or odd we have

M(y_n,y_{n+1}, kt) ≥ M(y_{n-1}, y_n, t) for k ∈ (0, 1) and all t > 0. Thus by Lemma 2, { y_n} is a Cauchy sequence in X. Since (X, M,*) is complete, it converges to a point z in X, and also its subsequences converges as follows.

$$\{Lx_{2n}\} \to z, \quad \{ABx_{2n}\} \to z$$

$\{Nx_{2n+1}\} \to z$, and $\{STx_{2n+1}\} \to z$.

Case-1: AB is continuous.

Since AB is continuous. AB (AB) $x_{2n} \to ABz$ and (AB) $Lx_{2n} \to ABz$.

Since (L, AB) is complete. L (AB) $x_{2n} \to ABz$.

Step 2: By taking x =ABx_{2n} and y =x_{2n+1} in (b) we have

$$M^2\,(L\,(AB)\,x_{2n}\,,\,Nx_{2n+1}\,,kt) * [\,M(AB(AB)\,x_{2n} * L(AB)\,x_{2n}\,,kt)\,,\,M(\,STx_{2n+1}\,,Nx_{2n+1}\,,kt)]$$
$$\geq M(ST(ABx_{2n})\,,L(AB)\,x_{2n},t) * M(ABx_{2n+1}\,,\,Nx_{2n+1}\,,t)$$
$$* M(ABx_{2n+1},\,L(AB)\,x_{2n}\,,t) * M(ST(AB)\,x_{2n}\,,Nx_{2n+1},t\,)$$
$$* M(\,ST(AB)\,x_{2n}\,,ABx_{2n+1}\,,t),$$

This implies that as $n \to \infty$

$$M^2\,(ABz,\,z,\,kt) * [M\,(ABz,\,ABz,\,kt) * M(z\,,z,kt\,)] \geq M(\,z,\,ABz\,,t) * M(ABz\,,z\,,t) * M(ABz\,,ABz\,,t\,)$$
$$* M\,(z,\,z,\,t) * M\,(z,\,ABz,\,t)$$

$\Rightarrow$ $M^2\,(ABz\,,z\,,kt\,)*[\,1 * 1] \geq M\,(ABz\,,z,t)* M(ABz\,,z\,,t) * 1 * 1 * M(ABz\,,z\,,t)$

$\Rightarrow$ $M^2\,(ABz,\,z,\,kt) \geq M\,(ABz,\,z,\,t)$

$\Rightarrow$ $M\,(ABz,\,z,\,kt) \geq 1$

$\Rightarrow ABz = z.$

Step 3: By taking x =z and y =x_{2n+1} in (b) we have

$$M^2\,(Lz,\,Nx_{2n+1}\,,kt)*[M\,(ABz,\,Lz,\,kt),\,M\,(STx_{2n+1}\,,Nx_{2n+1}\,kt)] \geq M(STz\,,\,Lz\,,t)$$
$$* M(ABx_{2n+1}\,,Nx_{2n+1}\,,\,t)$$
$$* M(ABx_{2n+1}\,,\,Lz,t)$$
$$* M\,(STz\,,Nx_{2n+1}\,,t)$$
$$* M\,(STz,\,ABx_{2n+1}\,,t)$$

This implies that as $n \to \infty$ we have

$$M^2\,(Lz,\,z,\,kt) * [\,M(z,\,Lz,\,kt),M(z,\,z,\,kt)\,] \geq M\,(z,\,Lz\,,\,t) * M\,(z,\,z,\,t) * M(z,\,Lz,\,t\,)*M\,(z,\,z,\,t)$$
$$* M(z,z,t)$$

$\Rightarrow$ $M^2\,(Lz,\,z,\,kt) *[\,M(\,Lz,\,z,\,kt),\,1\,] \geq M(Lz,\,z,\,t) * 1 * M(\,Lz,\,z,\,t) * 1 * 1$

$\Rightarrow$ $M^2\,(Lz,\,z,\,kt) \geq M\,(Lz,\,z,\,t)$

$\Rightarrow$ $M\,(Lz,\,z,\,kt) \geq 1$

Lz = z =ABz

Step -4: By taking x = Bz and y = x_{2n+1} we have

$$M^2\,(L\,(Bz),\,Nx_{2n+1}\,,kt) *[\,M\,(AB(Bz)\,,\,L\,(Bz)\,,kt),\,M(STx_{2n+1}\,,Nx_{2n+1}\,,kt)]$$
$$\geq M\,(\,ST\,(Bz)\,,L(Bz)\,,t) * M(ABx_{2n+1}\,,\,nx_{2n+1}\,,t)$$
$$* M(\,ABx_{2n+1}\,,L(Bz)\,,t) * M(ST(Bz),\,Nx_{2n+1}\,,t\,)$$
$$* M(\,ST(Bz)\,,\,ABx_{2n+1}\,,t\,)$$

$$\Rightarrow M^2\,(Bz\,,\,z\,,kt\,)*[\,M(Bz\,,\,Bz\,,\,kt\,),\,M(\,z\,,\,z\,,\,kt)\,] \geq M(z,\,Bz,\,t) * M(\,z,\,z,\,t) * M(\,z,\,Bz,\,t)$$

$$* M(z, z, t) * M(z, z, t)$$

$\Rightarrow$ $M^2(Bz, z, kt) * 1 * 1 \geq M(Bz, z, t) * 1 * M(Bz, z, t) * 1 * 1$

$\Rightarrow$ $M^2(Bz, z, t) \geq M(Bz, z, t)$

$\Rightarrow$ $M(Bz, z, kt) \geq 1$

Thus we have Bz = z

Since z = ABz we also have z = Az therefore a = Az = Bz = Lz.

Step -5: Since L (X) $\subseteq$ ST (X) there exists v ∈ X such that z = Lz =STv. By taking x =x_{2n} , y = v in (b) we have

$$M^2(Lx_{2n}, Nv, kt) * [M(ABx_{2n}, Lx_{2n}\ kt), M(STv, Nv, kt)] \geq M(STx_{2n}, Lx_{2n}, t) * M(ABv, Nv, t)$$
$$* M(ABv, Lx_{2n}, t) * M(STx_{2n}, Nv, t)$$
$$* M(STx_{2n}, ABv, t)$$

Which implies that as n → ∞

$$M^2(z, Nv, kt) * [M(z, z, kt), M(z, Nv, kt)] \geq M(z, z, t) * M(z, Nv, t) * M(z, z, t)$$
$$* M(z, Nv, t) * M(z, z, t)$$

$\Rightarrow$ $M^2(z, Nv, kt) * 1 * M(z, Nv, kt) \geq 1 * M(z, Nv, t) * 1 * M(z, Nv, t) * 1$

$\Rightarrow$ $M^2(z, Nv, kt) \geq M(z, Nv, t)$

$\Rightarrow M(z, Nv, kt) \geq 1$

Thus we have z =Nv and so z = Nv = STv.

Since (N, ST) is weakly compatible we have ST (Nv) = N (STv). Thus STz =Nz.

Step -6: By taking x =x_{2n} and y = z in (b) ans using step -5 we have

$$M^2(Lx_{2n}, Nz, kt) * [M(ABx_{2n}, Lx_{2n}, kt), M(STz, Nz, kt)] \geq M(STx_{2n}, Lx_{2n}, t)$$
$$* M(ABz, Nz, t) * M(ABz, Lx_{2n}, t)$$
$$* M(STx_{2n}, Nz, t) * M(STx_{2n}, ABz, t)$$

$\Rightarrow M^2(z, Nz, kt) * [M(z, z, kt), M(z, Nz, kt)] \geq M(z, z, t) * M(z, Nz, t) * M(z, z, t)$

$$* M(z, Nz, t) * M(z, z, t)$$

$\Rightarrow$ $M^2(z, Nz, kt) * 1 * M(z, Nz, kt) \geq 1 * M(z, Nz, t) * 1 * M(z, Nz, t) * 1$

$\Rightarrow$ $M^2(z, Nz, kt) \geq M(z, Nz, t)$

$\Rightarrow$ $M(z, Nz, kt) \geq 1$

Thus we have z = Nz and therefore

$$Z = Az = Bz = Lz = Nz = STz$$

Step -7: By taking x = x_{2n} and y = Tz in (b) we have

$$M^2(Lx_{2n}, N(Tz), kt) * [M(ABx_{2n}, Lx_{2n}, kt), M(ST(Tz), N(Tz), kt)] \geq M(STx_{2n}, Lx_{2n}, t)$$
$$* M(AB(Tz), N(Tz), t)$$
$$* M(AB(Tz), Lx_{2n}, t)$$
$$* M(STx_{2n}, N(Tz), t)$$
$$* M(STx_{2n}, AB(Tz),$$

Since NT =TN and ST =TS, we have NTz =TNz =Tz and ST (Tz) =T (STz) =Tz letting n→∞ we have

$$M^2(z, Tz, kt) * [M(z, z, kt), M(Tz, Tz, kt)] \geq M(z, z, t) * M(z, Tz, t) * M(z, z, t)$$

$$* \text{ M (z, Tz, t)} * \text{ M (z, z, t)}$$

$\Rightarrow$ M^2 (z ,Tz ,kt)∗ 1∗ 1 ≥ 1∗ M(z ,Tz ,t) ∗ 1∗ M(z ,Tz,t) ∗ 1

$\Rightarrow$ M^2 (z, Tz, kt) ≥ M (z, Tz, t)

$\Rightarrow$ M (z, Tz, kt) ≥ 1

Thus z =Tz . Since Tz =STz we also have z = Sz.

Therefore z = Az= Bz = Lz = Nz = Sz = Tz, that is z is the common fixed point of the six maps.

Case -3: L is continuous. Since L is continuous LLx_{2n} → Lz and L (AB) x_{2n} → Lz. Since (L, AB) is compatible, (AB) Lx_{2n} →Lz.

Step-8:— By taking x = Lx_{2n} and y = x_{2n+1} in (b) we have

M^2 (LLx_{2n} , Nx_{2n+1} ,kt) ∗[M (ABLx_{2n}, LLx_{2n} ,Kt), M (STx_{2n+1} ,Nx_{2n+1} ,kt)]

≥ M(STLx_{2n} ,LLx_{2n} ,t) ∗ M(ABx_{2n+1} ,Nx_{2n+1} ,t)

∗M(ABx_{2n+1}, LLx_{2n} ,t)

∗M (STLx_{2n}, Nx_{2n+1},t)∗M(STLx_{2n} ,ABx_{2n+1} ,t)

$\Rightarrow$ M^2 (Lz ,z ,kt)∗[M(Lz ,Lz, kt) , M(z , z ,kt)] ≥ M(z ,z ,t) ∗ M(z , z, t)∗ M(z , Lz ,t)

∗M (Lz, z, t)∗ M (Lz, z, t)

$\Rightarrow$ M^2 (Lz , z ,kt) ∗ 1∗ 1 ≥ M (z , Lz ,t) ∗ 1 ∗ M(Lz ,z ,t)∗ M(Lz ,z ,t)

∗ M(Lz , z ,t)

$\Rightarrow$ M^2 (Lz, z, kt) ≥ M (Lz, z, t)

$\Rightarrow$ M (Lz, z, kt) ≥ 1

Thus we have z = Lz and using steps 5-7 we have z = Lz = Nz =Sz =Tz.

Step -9: Since N(X) ⊆ AB(X) there exists v ∈ X such that z = Nz =Sz = Tz. By taking x = v, y =x_{2n+1} in (b) we have

M^2 (Lv, Nx_{2n+1} ,kt) ∗ [M((ABv , Lv ,kt) , M(STx_{2n+1} , Nx_{2n+1} ,kt)] ≥ M(STv , Lv ,t)

∗ M (ABx_{2n+1} . Nx_{2n+1} ,t) ∗ M (ABx_{2n+1} ,Lv, t) ∗ M (STv, Lv, t)

∗ M (STv, ABx_{2n+1} ,t)

Which implies that as n→∞

M^2 (Lv, v, t)∗ [M(z , Lv , kt) , M (z , z ,kt)] ≥ M (z , Lv ,t) ∗ M (z , z ,t) ∗ M (z , Lv ,t) ∗

M (z, z, t) ∗ M (z, z, t)

M^2 (Lv, z kt) ∗ [M (z, Lv, kt), 1] ≥ M (z Lv, t) ∗ 1∗ M (z, Lv, t) ∗ 1 ∗ 1

M^2 (Lv, z, kt) ≥ M (z, Lv, t)

M (Lv, z, kt) ≥ 1

Thus we have z = Lv = ABv

Since (L, AB) is weakly compatible, we have Lz = ABz and using step-4, we have z = Bz. therefore

z = Az = Bz =Sz = Tz = Lz = Nz

That is z is the common random fixed point of the six maps in this case also.

Step -10 : — for uniqueness, let (w ≠ z) be another common fixed point of A , B , S , T , L and N taking x = z , y = w in (b) we have

M^2 (Lz, Nw, kt) ∗ [M (ABz, Lz, kt), M (STw, Nw, kt)] ≥ M (STz, Lz, t) ∗M (ABw, Nw, t)

∗M(ABw , Lz ,t) ∗M(STz ,Nw , t)

∗M(STz , ABw ,t)

⇒ M^2 (Lz ,Nw ,kt) ∗[M(z , z ,kt) , M(w , w ,kt)] ≥M(z , z ,t)∗ M(w , w ,t) ∗M(w , z ,t)

∗ M (z, w, t)∗M(z , w , t)

⇒ M^2 (z , w ,kt)∗ 1∗1 ≥ 1∗ 1 ∗ M (w , z , t) ∗ M(w , z ,t)

∗ M(z , w ,t)

⇒ M^2 (z, w, kt) ≥ M (z, w, t)

⇒ M (z, w, kt) ≥ 1

Thus we have z = w This completes the proof of the theorem. If we take B =T = Ix (the identity map on X) in the main theorm we have the following

Corollary : Let A, S, L and N be self maps on a complete fuzzy metric space (X, M,∗) with t ∗t ≥t for all t ∈ [0,1] satisfying

(a) L (X) ⊆ ST(X), N(X) ⊆ A(X)

(b) There exists a constant k ∈ (0, 1) such that

M^2 (Lx, Ny, Kt) ∗ [M(Ax , Lz ,kt) , M(Sy ,Ny ,kt)] ≥ M(Sx ,Lx ,t) ∗ M(Ay ,Ny ,t) ∗M(Ay ,Lx ,t)

∗M (Sx, Ny, t) ∗M (Sx, Ay, t)

For all x, y ∈ X and t >0

(c) Either A or L is continuous.

(d) The pair (L, A) is compatible and (N, S) is weakly compatible. Then A, S If we take A = S, L = N and B =T = Lx is the main theorem, we have the following:

Corollary : Let (X ,M ,∗) be a compatible fuzzy metric space with t ∗t ≥t for all t ∈ [0 ,1]and let A and L be compatible maps on X such that L (X) ⊂ A (X) , if A is continuous and there exists a constant k ∈ (0 ,1) such that

M^2 (Lx, Ly, kt) ∗ [M (Ax ,Lx ,kt) , M(Ay ,Ly ,kt)] ≥M(Ax , Lx ,t) ∗ M(Ay ,Ly , t) ∗M(Ay,Ly,t)

∗M (Ax, Ly, t)∗M(Ax , Ay ,t)

For all x, y ∈ X and t>0 then A and L have a unique fixed point.

References

[1] A. George, P. Veeramani, On some results in Fuzzy metric spaces, Fuzzy set and systems, 64 (1994), 395-399

[2] B. Singh and S. Jain, Fixed point theorem using semi compatibility and weak compatibility in Fuzzy metric space, VJMS, 1 (2007), 139-147

[3] K. Singh and M. S .Rathore A common Fixed point theorem in Fuzzy metric spaces, IJMA, vol, 2011, no 17, 819-826

[4] L .A.Zadeh, Fuzzy sets Inform and control, 8 (1965), 338-353.

[5] O. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetika 11(1975), 326-334

[6] R. Vasuki, Common Fixed point for R- weakly commuting maps in Fuzzy metric spaces, Indian J. Pure Appl.Math, 30(1999),419-423

[7] S .Sharma, Common fixed point theorem in Fuzzy metric spaces, Fuzzy sets and systems, 127 (2002), 345-352

[8] Y. J. Cho, H. K. Pathak, S .M. Kang, J .S. Jung , Common Fixed points of compatible maps of type (β) on fuzzy metric spaces, Fuzzy sets and systems , 93 (1998) ,99-111.