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Linear System in general form for Hyperbolic type

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ABSTRACT

In this paper, we consider the linear system of partial differential equations hyperbolic type Which is solved by T.V. Chekmarev [1] and solve the general form for this Hyperbolic type.

Indexing terms/Keywords

Hyperbolic; partial differential equations; linear system.

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INTRODUCTION

we consider the system of linear system of partial differential equations hyperbolic type

$$u_x = a(x,y)u + b(x,y)v + f(x,y)$$
$$v_y = c(x,y)u + d(x,y)v + g(x,y) \qquad (1)$$

with initial condition

$$u(x_0, y) = \varphi(y)$$
$$v(x, y_0) = \psi(x) \qquad (2)$$

T.V.CHEKMARIOV [1] solve this system when a=d=0

$$u_x = b(x,y)v + f(x,y)$$
$$v_y = c(x,y)u + g(x,y)$$

we can reduce this system to one differential equation of second order and we can solve it by Riemann method, but the solution is exist in case existence for b(x,y), c(x,y), f(x,y)and g(x,y)and continuity for derivatives.

The Main Result

If we take the system (1) with initial condition (2) , we want to solve it by using the result theory of integral Voltera.

Rewrite the system (1)

$$u_x - a(x,y)u = b(x,y)v + f(x,y)$$
$$v_y - d(x,y)v = c(x,y)u + g(x,y)$$

Or

$$\left(e^{-\int_{x0}^{x} a(\xi,y)d\xi} u \right)_x = \left(b(x,y)v + f(x,y)\right) e^{-\int_{x0}^{x} a(\xi,y)d\xi}$$

$$\left(e^{-\int_{y0}^{y} d(x,\eta)d\eta} \right)_y = \left(c(x,y)u + g(x,y)\right) e^{-\int_{y0}^{y} d(x,\eta)d\eta}$$

Integrate by x and then by y , we get

$$e^{-\int_{x0}^{x} a(\xi,y)d\xi} u(x,y) - u(x_0,y) = \int_{x_o}^{x} [b(\xi,y)v(\xi,y) + f(\xi,y)] e^{-\int_{x0}^{x} a(\xi_1,y)d\xi_1} d\xi$$

$$e^{-\int_{y0}^{y} d(x,\eta)d\eta} v(x,y) - v(x,y_0) = \int_{y_0}^{y} [c(x,\eta)u(x,\eta) + g(x,\eta)] e^{-\int_{y0}^{y} d(x,\eta_1)d\eta_1} d\eta \qquad (3)$$

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Rewrite (3) in form

$$u(x,y)=\varphi(y)e^{\int_{x0}^{x}a(\xi,y)d\xi}+\int_{x_0}^{x}b(\xi,y)v(\xi,y)e^{\int_{\xi}^{x}a(\xi_1,y)d\xi_1}d\xi+\int_{x_0}^{x}f(\xi,y)e^{\int_{\xi}^{x}a(\xi_1,y)d\xi_1}d\xi \qquad (4)$$

$$v(x,y)=\psi(x)e^{\int_{y0}^{y}d(x,\eta)d\eta}+\int_{y_0}^{y}c(x,\eta)u(x,\eta)e^{\int_{\eta}^{y}d(x,\eta_1)d\eta_1}d\eta+\int_{y_0}^{y}g(x,\eta)e^{\int_{\eta}^{y}d(x,\eta_1)d\eta_1}d\eta$$

Let

$$\omega(x,y)=\int_{x_0}^{x}f(\xi,y)e^{\int_{\xi}^{x}a(\xi_1,y)d\xi_1}d\xi+\varphi(y)e^{\int_{x0}^{x}a(\xi,y)d\xi} \qquad (5)$$

$$\omega_1(x,y)=\int_{y_0}^{y}g(x,\eta)e^{\int_{\eta}^{y}d(x,\eta_1)d\eta_1}d\eta+\psi(x)e^{\int_{y0}^{y}d(x,\eta)d\eta}$$

In (4),If we put *u(x,y)* in *v(x,y)* we get

$$v(x,y)=\omega_1+\int_{y_0}^{y}c(x,\eta)e^{\int_{\eta}^{y}d(x,\eta_1)d\eta_1}[\omega(x,\eta)+\int_{x_0}^{x}b(\xi,\eta)v(\xi,\eta)e^{\int_{\xi}^{x}a(\xi_1,\eta)d\xi_1}d\xi]d\eta$$

Or

$$v(x,y)=\Omega_1(x,y)+\int_{x_0}^{x}\int_{y_0}^{y}k_1(x,y,\xi,\eta)v(\xi,\eta)d\eta d\xi \qquad (6)$$

where

$$\Omega_1(x,y)=\omega_1(x,y)+\int_{y_0}^{y}c(x,\eta)\omega(x,\eta)e^{\int_{\eta}^{y}d(x,\eta_1)d\eta_1}d\eta$$

$$k_1(x,y)=c(x,\eta)b(\xi,\eta)e^{\int_{\xi}^{x}a(\xi_1,\eta)d\xi_1+\int_{\eta}^{y}d(x,\eta_1)d\eta_1}$$

In (4), If we put *v(x,y)* in *u(x,y)* we get

$$u(x,y)=\omega(x,y)+\int_{x_0}^{x}b(\xi,y)e^{\int_{\xi}^{x}a(\xi_1,y)d\xi_1}[\omega_1(\xi,y)+\int_{y_0}^{y}c(\xi,\eta)u(\xi,\eta)e^{\int_{\eta}^{y}d(\xi,\eta_1)d\eta_1}d\eta]d\xi$$ Or

$$u(x,y)=\Omega(x,y)+\int_{x_0}^{x}\int_{y_0}^{y}k(x,y,\xi,\eta)u(\xi,\eta)d\eta d\xi \qquad (7)$$

where

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$$\Omega(x,y)=\omega(x,y)+\int_{x_0}^{x} b(\xi,y)\,\omega_1(\xi,y)\,e^{\int_{\xi}^{x} a(\xi_1,y)d\xi_1}d\xi$$

$$k(x,y,\xi,\eta)=b(\xi,y)c(\xi,\eta)e^{\int_{\xi}^{x} a(\xi_1,y)d\xi_1+\int_{\eta}^{y} d(\xi,\eta_1)d\eta_1}$$

The problem (1),(2) reduces to double integral (6),(7).

Let R and R1 Rezolvent kernel k and k1 then the solution of (6),(7) is

$$u(x,y)=\Omega(x,y)+\int_{x_0}^{x}\int_{y_0}^{y} R(x,y,\xi,\eta)\Omega(\xi,\eta)d\eta d\xi$$

$$v(x,y)=\Omega_1(x,y)+\int_{x_0}^{x}\int_{y_0}^{y} R_1(x,y,\xi,\eta)\Omega_1(\xi,\eta)d\eta d\xi \qquad (8)$$

Then, (8) give us the solution for the problem (1) ,(2).

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