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Semi-B_c-Continuous Functions in Topological Spaces

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ABSTRACT

The concept of semi-B_c-continuous functions in topological spaces is introduced and studied. Some of their characteristic properties are considered. Also we investigate the relationships between these classes of functions and other classes of functions.

Keywords:

Bc-open; semi-open; semi-Bc-continuous.

Mathematics Subject Classification: 13F05, 13D 05.

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1 Introduction

Functions and of course continuous functions stand among the most important and most researched points in the whole of the Mathematical Science. Many different forms of continuous functions have been introduced over the years. In 1980, Joseph and Kwack [11] introduced the notion of (θ, s)-continuous functions. In 1999, Jafari [8] introduced the notion of (p, s)-continuous functions. In this paper, we introduce and study the semi-B_c-continuous functions in topological spaces. Moreover we obtain basic properties and preservation theorem of semi-B_c-continuous functions. Relations between these types of functions and other classes of functions are obtained.

2 Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A is said to be regular open (resp., regular closed) if Int(Cl(A)) = A (resp., Cl(Int(A)) = A). A subset A of a space X is called b-open [3] if $A \subset Int(Cl(A)) \cup Cl(Int(A))$. A subset A is said to be semi-open [12] (resp., β-open [2], preopen [13] and α-open [14]) if $A \subset Cl(Int(A))$ (resp., $A \subset Cl(Int(Cl(A)))$, $A \subset Int(Cl(A))$ and $A \subset Int(Cl(Int(A)))$). The family of all semi-open (resp., β-open, preopen, b-open and α-open) sets in (X, τ) is denoted by SO(X, τ) (resp., βO(X, τ), PO(X, τ), BO(X, τ) and τ^α). The complement of a semi-open (resp., preopen, b-open and α-open) set is said to be semi-closed (resp., preclosed, b-closed and α-closed). A subset A of a space X is called θ-semi-open [11] if for each x ∈ A, there exists a semi-open set G such that $x \in G \subset Cl(G) \subset A$. A subset A of a space A is called B_c-open [7] if for each $x \in A \in BO(X)$, there exists a closed set F such that $x \in F \subset A$. The complement of a B_c-open set is called B_c-closed. The intersection of all preclosed (resp., semi-closed and B_c-closed) sets of X containing A is called the preclosure [19] (resp., semi-closure [15] and B_c-closure [7]) of A and is denoted by pCl(A) (resp., sCl(A) and $B_cCl(A)$). The union of all semi-open (resp., preopen and B_c-open) sets of X contained in A is called the semi-interior (resp., preinterior and B_c-interior) of A and is denoted by sInt(A) (resp., pInt(A) and $B_cInt(A)$). A point x ∈ X is said to be in the semi-closure [15] (resp., θ-semi-closure [11]) of a subset A of X, denoted by sCl(A) (resp., θsCl(A)), if $U \cap A \neq \varphi$ (resp., $A \cap Cl(U) \neq \varphi$) for each U ∈ SO(X) containing x. A subset A is said to be semi-closed (resp., θ-semi-closed) if A = sCl(A) (resp., A = θsCl(A)).

Theorem 2.1. Let A be a subset of a topological space (X, τ). Then:

1. If A ∈ SO(X), then pCl(A) = Cl(A) [16].
2. A ∈ PO(X) if and only if sCl(A) = Int(Cl(A)) [17].

Definition 2.2. A function f : X → Y is said to be:

1. b-irresolute [21] if $f^{-1}(V)$ is b-open in X for every b-open set V of Y.
2. almost contra-b-continuous [1] $f^{-1}(V) \in BC(X)$ for every V ∈ RO(X).
3. (θ, s)-continuous [11] if for each point x ∈ X and each semi-open set V of Y with f (x) ∈ V, there exists an open set U of X containing x such that $f(U) \subset Cl(V)$.
4. (p, s)-continuous [8] if for each point x ∈ X and each semi-open set V of Y with f (x) ∈ V, there exists a preopen set U of X containing x such that $f(U) \subset Cl(V)$.
5. α-quasi-irresolute [10] if for each point x ∈ X and each semi-open set V of Y with f (x) ∈ V, there exists an α-open set U of X containing x such that $f(U) \subset Cl(V)$.
6. weakly θ-irresolute [10] if for each point x ∈ X and each semi-open set V of Y with f (x) ∈ V, there exists a semi-open set U of X containing x such that $f(U) \subset Cl(V)$.
7. (b, s)-continuous [18] if for each point x ∈ X and each semi-open set V of Y with f (x) ∈ V, there exists a b-open set U of X containing x such that $f(U) \subset Cl(V)$.
8. quasi-irresolute [20] if for each x ∈ X and each semi-open set V of Y containing *f* (x), there exists a semi-open set U of X containing x such that $f(U) \subset sCl(V)$.
9. β-quasi-irresolute [9] if for each point x ∈ X and each semi-open set V of Y with f (x) ∈ V, there exists a β-open set U of X containing x such that $f(U) \subset Cl(V)$.

3 Semi-B_c-continuous functions

Definition 3.1. A function $f : X \to Y$ is called semi-B_c-continuous at a point x ∈ X if for each semi-open set V of Y containing *f* (x), there exists a B_c-open set U of X containing x such that $f(U) \subset Cl(V)$. If *f* is semi-B_c-continuous at every point x of X, then it is called semi-B_c-continuous.

Theorem 3.2. A function $f : X \to Y$ is semi-B_c-continuous if and only if for each x ∈ X and each regular closed set F of Y containing *f* (x), there exists a B_c-open set U of X containing x such that $f(U) \subset F$.

Proof. Suppose that each regular closed set F of Y containing *f* (x), there exists a B_c-open set U of X containing x such that $f(U) \subset F$. Let V be a semi-open set in Y containing *f* (x), so Cl(V) = F (say) is regular closed, then by hypothesis there exists a B_c-open set U of X containing x such that $f(U) \subset F = Cl(V)$. Hence *f* is semi-B_c-continuous.

Conversely, let $x \in X$ and let F be any regular closed set of Y containing $f(x)$. Since f is semi-B_c-continuous function, then there exists a B_c-open set U of X containing x such that $f(U) \subset Cl(F) = F$.

Theorem 3.3. A function $f : X \rightarrow Y$ is semi-B_c-continuous if and only if for each $x \in X$ and each θ-semi-open set V of Y containing $f(x)$, there exists a B_c-open set U of X containing x such that $f(U) \subset V$.

Proof. Let $x \in X$ and V be any semi-open set of Y containing $f(x)$. So Cl(V) is a θ-semi-open set of Y containing $f(x)$. By hypothesis there exists a B_c-open set U of X containing x such that $f(U) \subset Cl(V)$. This shows that f is semi-B_c-continuous.

Conversely, let V be any θ-semi-open set of Y containing $f(x)$, then there exists a semi-open set G of Y such that $f(x) \in G \subset Cl(G) \subset V$. Since f is semi-B_c-continuous, then there exists a B_c-open set U of X containing x such that $f(U) \subset G \subset V$.

Theorem 3.4. For a function $f : X \rightarrow Y$, the following statements are equivalent.

1. f is semi-B_c-continuous.
2. For each $x \in X$ and each semi-open set V of Y containing $f(x)$, there exists a B_c-open set U in X containing x such that $f(U) \subset pCl(V)$.
3. For each $x \in X$ and each regular closed set F of Y containing $f(x)$, there exists a B_c-open set U in X containing x such that $f(U) \subset F$.
4. For each $x \in X$ and each θ-semi-open set V of Y containing $f(x)$, there exists a B_c-open set U in X containing x such that $f(U) \subset V$.

Proof.

(1) ⇒ (2). Since $Cl(V) = pCl(V)$ for every $V \in SO(Y, f(x))$.

(2) ⇒ (3). Obvious.

(3) ⇒ (4). Let $x \in X$ and let V be any θ-semi-open set of Y containing $f(x)$. Then for each $f(x) \in V$, there exists a regular closed set F containing $f(x)$ such that $F \subset V$. By (3), there exists a B_c-open set U in X containing x such that $f(U) \subset F \subset V$. This completes the proof.

(4) ⇒ (1). It is already proved in Theorem 3.3.

Theorem 3.5. If a function $f : X \rightarrow Y$ is b-irresolute, then we have the following statements are equivalent.

1. f is semi-B_c-continuous.
2. For each $x \in X$ and each semi-open set V of Y containing $f(x)$, there exists a closed set F in X containing x such that $f(F) \subset Cl(V)$.
3. For each $x \in X$ and each semi-open set V of Y containing $f(x)$, there exists a closed set F in X containing x such that $f(F) \subset pCl(V)$.
4. For each $x \in X$ and each regular closed set E of Y containing $f(x)$, there exists a closed set F in X containing x such that $f(F) \subset E$.
5. For each $x \in X$ and each θ-semi-open set V of Y containing $f(x)$, there exists a closed set F in X containing x such that $f(F) \subset V$.

Proof.

(1) ⇒ (2). Let $x \in X$ and let V be any semi-open set of Y containing $f(x)$. By (1), there exists a B_c-open set U of X containing x such that $f(U) \subset Cl(V)$. Since U is B_c-open set. Then for each $x \in U$, there exists a closed set F of X such that $x \in F \subset U$. Therefore, we have $f(F) \subset Cl(V)$.

(2) ⇒ (3). Obvious.

(3) ⇒ (4). Let $x \in X$ and let E be any regular closed set of Y containing $f(x)$. Then E is a semi-open set of Y containing $f(x)$. By (3), there exists a closed set F in X containing x such that $f(F) \subset pCl(E) \subset Cl(E) = E$.

(4) ⇒ (5). Let $x \in X$ and let V be any θ-semi-open set of Y containing $f(x)$. Then for each $f(x) \in V$, there exists a regular closed set E containing $f(x)$ such that $E \subset V$. By (4), there exists a closed set F in X containing x such that $f(F) \subset E \subset V$.

(5) ⇒ (1). Let V be any θ-semi-open set of Y. We have to show that $f^{-1}(V)$ is B_c-open set in X. Since f is b-irresolute, then $f^{-1}(V)$ is b-open set in X. Let $x \in f^{-1}(V)$. Then $f(x) \in V$. By hypothesis, there exists a closed set F of X containing x such that $f(F) \subset V$. Which implies that $x \in F \subset f^{-1}(V)$. Therefore, $f^{-1}(V)$ is B_c-open set in X. Hence clearly $f(f^{-1}(V)) \subset V$ and by Theorem 3.3, f is semi-B_c-continuous.

Theorem 3.6. For a function $f : X \rightarrow Y$, the following statements are equivalent.

1. f is semi-B_c-continuous.

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2. $f^{-1}(Cl(V))$ is B_c-open set in X, for each semi-open set V in Y.
3. $f^{-1}(Int(F))$ is B_c-closed set in X, for each semi-closed set F in Y.
4. $f^{-1}(V)$ is B_c-closed set in X, for each regular open set V of Y.
5. $f^{-1}(F)$ is B_c-open set in X, for each regular closed set F of Y.
6. $f^{-1}(V)$ is B_c-open set in X, for each θ-semi-open set V of Y.
7. $f^{-1}(F)$ is B_c-closed set in X, for each θ-semi-closed set F of Y.
8. $f(B_cCl(A)) \subset \theta sCl(f(A))$, for each subset A of X.
9. $B_cCl(f^{-1}(B)) \subset f^{-1}(\theta sCl(B))$, for each subset B of Y.
10. $f^{-1}(\theta sInt(B)) \subset B_cInt(f^{-1}(B))$, for each subset B of Y.
11. $\theta sInt(f(A)) \subset f(B_cInt(A))$, for each subset A of X.

Proof.

(1) ⇒ (2). Let V be any semi-open set in Y. We have to show that $f^{-1}(Cl(V))$ is B_c-open set in X. Let $x \in f^{-1}(Cl(V))$. Then $f(x) \in Cl(V)$ and Cl(V) is a regular closed set in Y. Since f is semi-B_c-continuous. Then by Theorem 3.2, there exists a B_c-open set U of X containing x such that $f(U) \subset Cl(V)$. Which implies that $x \in U \subset f^{-1}(Cl(V))$. Therefore, $f^{-1}(Cl(V))$ is B_c-open set in X.

(2) ⇒ (3). Let F be any semi-closed set of Y. Then Y\F is a semi-open set of Y. By (2), $f^{-1}(Cl(Y\setminus F))$ is B_c-open set in X and $f^{-1}(Cl(Y\setminus F)) = f^{-1}(Y\setminus Int(F)) = X\setminus f^{-1}(Int(F))$ is B_c-open set in X and hence $f^{-1}(Int(F))$ is B_c-closed set in X.

(3) ⇒ (4). Let V be any regular open in Y. Then V is semi-closed in Y and Int(V) = V. By (3) $f^{-1}(Int(V)) = f^{-1}(V)$ is B_c-closed set in X.

(4) ⇒ (5). Let F be any regular closed set of Y. Then Y\F is regular open set of Y. By (4) $f^{-1}(Y\setminus F)$ is B_c-closed set in X and $f^{-1}(Y\setminus F) = X\setminus f^{-1}(F)$. Therefore $f^{-1}(F)$ is B_c-open set in X.

(5) ⇒ (6). This follows from the fact that any θ-semi-open set is a union of regular closed sets.

(6) ⇒ (7). It is entirely analogous to part (4) ⇒ (5) and is thus omitted.

(7) ⇒ (8). Let A be any subset of X and $y \notin \theta sCl(f(A))$. Then, there exists $V \in SO(Y, y)$ such that $f(A) \cap Cl(V) = \varphi$. Since Cl(V) is θ-semi-open, $f^{-1}(Cl(V))$ is B_c-open in X and $A \cap f^{-1}(Cl(V)) = \varphi$. Therefore, $B_cCl(A) \cap f^{-1}(Cl(V)) = \varphi$ and $f(B_cCl(A)) \cap Cl(V) = \varphi$. Consequently, we obtain $y \notin f(B_cCl(A))$ and hence $f(B_cCl(A)) \subset \theta sCl(f(A))$.

(8) ⇒ (9). Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X, by (8) $f(B_cCl(f^{-1}(B))) \subset \theta sCl(f(f^{-1}(B))) = \theta sCl(B)$. Therefore we obtain $B_cCl(f^{-1}(B) \subset f^{-1}(\theta sCl(B)$.

(9) ⇒ (10). Let B be any subset of Y. Then X\B is also subset of Y. Applying (9) we obtain $B_cCl(f^{-1}(Y\setminus B)) \subset f^{-1}(\theta sCl(Y\setminus B))$ ⇔ $B_cCl(X\setminus f^{-1}(B)) \subset f^{-1}(Y\setminus \theta sInt(B))$ ⇔ $X\setminus B_cInt(f^{-1}(B)) \subset X\setminus f^{-1}(\theta sInt(B))$ ⇔ $f^{-1}(\theta sInt(B)) \subset B_cInt(f^{-1}(B))$. Therefore, $f^{-1}(\theta sInt(B)) \subset B_cInt(f^{-1}(B))$.

(10) ⇒ (11). Let A be any subset of X. Then $f(A)$ is a subset of Y. By (10), we have $f^{-1}(\theta sInt(f(A))) \subset B_cInt(f^{-1}(f(A))) = B_cInt(A)$. Therefore, $\theta sInt(f(A)) \subset f(B_cInt(A))$.

(11) ⇒ (1). Let $x \in X$ and let V be any semi-open set of Y containing $f(x)$. Then $x \in f^{-1}(Cl(V))$ and $f^{-1}(Cl(V))$ is a subset of X. By (11), we have $\theta sInt(f(f^{-1}(Cl(V))) \subset f(B_cInt(f^{-1}(Cl(V)))$. Then $\theta sInt(Cl(V)) = Cl(V) \subset f(B_cInt(f^{-1}(Cl(V))))$. Then $Cl(V) \subset f(B_cInt(f^{-1}(Cl(V)))$ implies that $f^{-1}(Cl(V)) \subset B_cInt(f^{-1}(Cl(V)))$. Therefore, $f^{-1}(Cl(V))$ is B_c-open set in X which contains x and clearly $f(f^{-1}(Cl(V))) \subset Cl(V)$. Hence f is semi-B_c-continuous.

Theorem 3.7. For a function $f : X \to Y$, the following statements are equivalent.

1. f is semi-B_c-continuous.
2. $B_cCl(f^{-1}(V)) \subset f^{-1}(Int(Cl(V))$, for each pre-open set V of Y.
3. $f^{-1}(Cl(Int(F))) \subset B_cInt(f^{-1}(F))$, for each pre-closed set F of Y.

Proof.

(1) ⇒ (2). Let V be any pre-open set of Y. Then $V \subset Int(Cl(V))$ and Int(Cl(V)) is regular open set in Y. Since f is semi-B_c-continuous, by Theorem 3.6,(4), $f^{-1}(Int(Cl(V)))$ is B_c-closed set in X and hence we obtain that $B_cCl(f^{-1}(V)) \subset f^{-1}(Int(Cl(V))$.

(2) ⇔ (3). Let F be any pre-closed set of Y. Then Y\F is pre-open set of Y and by (2), we have $B_cCl(f^{-1}(Y\setminus F)) \subset f^{-1}(Int(Cl(Y\setminus F))$ ⇔ $X\setminus B_cInt(f^{-1}(F) \subset f^{-1}(Y\setminus Cl(Int(F)))$ ⇔ $X\setminus B_cInt(f^{-1}(F)) \subset X\setminus f^{-1}(Cl(Int(F)))$. Therefore, $f^{-1}(Cl(Int(F))) \subset B_cInt(f^{-1}(F))$.

(2) ⇒ (1). Let V be any regular open set of Y. Then V is preopen set of Y. By hypothesis, we have $B_cCl(f^{-1}(V)) \subset f^{-1}(Int(Cl(V))) = f^{-1}(V)$. Therefore, $f^{-1}(V)$ is B_c-closed set in X and hence by Theorem 3.6,(4), f is semi-B_c-continuous.

Corollary 3.8. For a function $f : X \to Y$, the following statements are equivalent.

1. f is semi-B_c-continuous.
2. $B_cCl(f^{-1}(V)) \subset f^{-1}(sCl(V)$, for each pre-open set V of Y.
3. $f^{-1}(sInt(F)) \subset B_cInt(f^{-1}(F))$, for each pre-closed set F of Y.

Proof. Follows from the above Theorem and the fact that $sCl(V) = Int(Cl(V))$ for each $V \in PO(Y)$.

Theorem 3.9. A function $f : X \to Y$ is semi-B_c-continuous if and only if $f^{-1}(V) \subset B_cInt(f^{-1}(Cl(V))$ for each semi-open set V of Y.

Proof. Let V be any semi-open set of Y. Then $V \subset Cl(V)$ and $Cl(V)$ is regular closed set in Y. Since f is semi-B_c-continuous, by Theorem 3.6,(5), $f^{-1}(Cl(V))$ is B_c-open set in X and hence we obtain that $f^{-1}(V) \subset f^{-1}(Cl(V)) = B_cInt(f^{-1}(Cl(V)))$.

Conversely, let V be any regular closed set of Y. Then V is semi-open set of Y. By hypothesis, we have $f^{-1}(V) \subset B_cInt(f^{-1}(Cl(V)) = B_cInt(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is B_c-open set in X and hence by Theorem 3.6, f is semi-B_c-continuous.

Corollary 3.10. A function $f : X \to Y$ is semi-B_c-continuous if and only if $f^{-1}(V) \subset B_cInt(f^{-1}(pCl(V))$ for each semi-open set V of Y.

Corollary 3.11. A function $f : X \to Y$ is semi-B_c-continuous if and only if $B_cCl(f^{-1}(Int(F)) \subset f^{-1}(F)$ for each semi-closed set F of Y.

Corollary 3.12. A function $f : X \to Y$ is semi-B_c-continuous if and only if $B_cCl(f^{-1}(pInt(F)))) \subset f^{-1}(F)$ for each semi-closed set F of Y.

Theorem 3.13. If a function $f : X \to Y$ is semi-B_c-continuous, then $f^{-1}(V) \subset B_cCl(f^{-1}(Int(Cl(V))))$ for each pre-open set V of Y.

Proof. Let V be any pre-open set of Y. Then $V \subset Int(Cl(V))$ and $Int(Cl(V))$ is regular open set in Y. Since f is semi-B_c-continuous, by Theorem 3.6, $f^{-1}(Int(Cl(V)))$ is B_c-closed set in X and hence we obtain that $f^{-1}(V) \subset f^{-1}(Int(Cl(V))) = B_cCl(f^{-1}(Int(Cl(V))))$.

Corollary 3.14. If a function $f : X \to Y$ is semi-B_c-continuous, then $f^{-1}(V) \subset B_cCl(f^{-1}(sCl(V)))$ for each pre-open set V of Y.

Corollary 3.15. If a function $f : X \to Y$ is semi-B_c-continuous, then $B_cInt(f^{-1}(Cl(Int(F))) \subset f^{-1}(F)$ for each pre-closed set F of Y.

Corollary 3.16. If a function $f : X \to Y$ is semi-B_c-continuous, then $B_cInt(f^{-1}(sInt(F))) \subset f^{-1}(F)$ for each pre-closed set F of Y.

4 Comparison and Further Properties

In this section we study further properties of semi-B_c-continuous and investigate their relationships to other types of functions between topological spaces.

Remark 4.1. Every semi-B_c-continuous function is almost contra-*b*-continuous.

Remark 4.2. Every semi-B_c-continuous function is (b, s)-continuous.

Theorem 4.3. The following properties are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$, whenever X is T_1-space.

1. f is almost contra-*b*-continuous.
2. f is semi-B_c-continuous.

Proof. This follows from Proposition 2.25 [7].

Theorem 4.4. The following properties are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$, whenever X is T_1-space.

1. f is (b, s)-continuous.
2. f is semi-B_c-continuous.

Proof. This follows from Proposition 2.9 [7].

Corollary 4.5. If a function $f : (X, \tau) \to (Y, \sigma)$ is weakly θ-irresolute and X is locally indiscrete space, then it is semi-B_c-continuous.

Proof. Follows directly from Proposition 2.14 [7].

Remark 4.6. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is α-quasi-irresolute and X is locally indiscrete space, then it is semi-B_c-continuous.

Remark 4.7. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is (p, s)-continuous and X is T_1-space, then it is semi-B_c-continuous.

Remark 4.8. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is (θ, s)-continuous and X is locally indiscrete or regular or T_1-space, then it is semi-B_c-continuous.

We recall that a space X is called extremally disconnected [5] if the closure of each open set of X is open in X, equivalently if every semi-open set is α-open. The space X is called submaximal [5] if every dense subset of X is open in X, equivalently if every preopen set is open.

Lemma 4.9. If (X, τ) is a submaximal extremally disconnected space, then $\tau = \tau^\alpha = SO(X, \tau) = PO(X, \tau) = BO(X, \tau) = \beta O(X, \tau)$

Theorem 4.10. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is semi-B_c-continuous, and X and Y are a submaximal extremally disconnected space, then it is quasi-irresolute.

Proof. Let V be any semi-open in Y containing $f(x)$. Since Y is submaximal extremally disconnected, Cl(V) = sCl(V) for every $V \in SO(Y)$. Since f is semi-B_c-continuous, then there exists a B_c-open set U in X containing x such that $f(U) \subset Cl(V)$. But X is submaximal extremally disconnected, so by Lemma 4.9, U is semi-open. Therefore $f(U) \subset Cl(V) = sCl(V)$. This shows that f is quasi-irresolute.

Theorem 4.11. If (X, τ) is a submaximal extremally disconnected T_1-space, then the following are equivalent for a function $f : (X, \tau) \rightarrow (Y, \sigma)$:

1. f is (b, s)-continuous.
2. f is semi-B_c-continuous.
3. f is (θ, s)-continuous.
4. f is (p, s)-continuous.
5. f is α-quasi-irresolute.
6. f is weakly θ-irresolute.
7. f is β-quasi-irresolute.

Proof. It follows from Lemma 4.9 and Theorem 4.4.

Definition 4.12. A space X is said to be:

1. s-Urysohn [4] if for each pair of distinct points x and y in X, there exist $U \in SO(X, x)$ and $V \in SO(X, y)$ such that $Cl(U) \cap Cl(V) = \emptyset$.
2. b-T_2 [6] if for each pair of distinct points x and y in X, there exist $U \in BO(X, x)$ and $V \in BO(X, y)$ such that $U \cap V = \emptyset$.

Theorem 4.13. If $f : X \rightarrow Y$ is a semi-B_c-continuous injection and Y is s-Uryson, then X is b-T_2 space.

Proof. Since f is injective, it follows that $f(x_1) \neq f(x_2)$ for any two distinct points x_1 and x_2. Since Y is s-Uryson, there exist semi-open sets V_1 and V_2 of Y such that $f(x_1) \in V_1$, $f(x_2) \in V_2$, and $Cl(V_1) \cap Cl(V_2) = \varphi$. Since f is semi-B_c-continuous, there exist B_c-open sets U_1 and U_2 of X containing x_1 and x_2 respectively such that $f(U_1) \subset Cl(V_1)$ and $f(U_2) \subset Cl(V_2)$. Hence $U_1 \cap U_2 = \varphi$. Since U_1 and U_2 are B_c-open, then clearly U_1 and U_2 are b-open. This shows that X is b-T_2.

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