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Sub-Inverse and g-inverse of an Intuitionistic Fuzzy Matrix Using Bi-implication Operator

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ABSTRACT

In this paper, we introduce a bi-implication operator $\leftrightarrow$ for intuitionistic fuzzy matrix and discuss several properties. Further, we obtain sub-inverses and g-inverses of an intuitionistic fuzzy matrix using bi-implication operator.

Keywords and Phrases: Intuitionistic Fuzzy Sets(IFSs); Intuitionistic Fuzzy Matrix(IFM); Intuitionistic Fuzzy relational equation and Bi-implication Operator.

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INTRODUCTION

After the introduction of fuzzy set theory by Zadah [13] in 1965, fuzzy concepts evolved in almost all fields. Thomason [11] conceived fuzzy matrix theory and Sanchez [12] used it in fuzzy relational equations. Hiroshi Hasimoto [5] used implication operator in fuzzy matrix theory and obtained results in sub-inverse of fuzzy matrix using fuzzy relational equation. Atanassov [3] generalized fuzzy set theory to intuitionistic fuzzy set theory and as a consequence Im et.al.[4] extended it to intuitionistic fuzzy matrix. Meenakshi and Gandhimathi [1,2] and Sriram and Murugadas [8,9,10] studied IFS and extended it to intuitionistic fuzzy matrix. The authors [6] have studied bi-implication operator for IFS and in [7] dual implication operator for IFM.

Definition 1.1. [3]

An Intuitionistic Fuzzy Set (IFS) A in E (universal set) is defined as an object of the following form A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\}$, where the functions: μ_A: E→ [0,1] and ν_A: E→ [0,1] define the membership and non-membership function of the element x ∈ E respectively and for every x ∈ E : $0 \le \mu_{A(x)} + \nu_{A(x)} \le 1$.

For simplicity, we consider the pair ⟨x,x'⟩ as membership and non-membership function of an IFS with x+x' ≤ 1.

An Intuitionistic Fuzzy Relation equation is an equation of the form Ax=b(xA=b), where A is an IFM and x and b are intuitionistic fuzzy vector of compatible size with unknown x.

Definition 1.2. [9]

An intuitionistic fuzzy matrix (IFM) is a matrix of pairs $A = (\langle a_{ij}, a'_{ij}\rangle)$ of non negative real numbers satisfying $a_{ij} + a'_{ij} \le 1$ for all i,j. For any two elements $A = (\langle a_{ij}, a'_{ij}\rangle)$, $B = (\langle b_{ij}, b'_{ij}\rangle) \in F_{nxm}$, define

$$A \vee B = (\langle a_{ij} \vee b_{ij}, a'_{ij} \wedge b'_{ij}\rangle),$$

$$A \wedge B = (\langle a_{ij} \wedge b_{ij}, a'_{ij} \vee b'_{ij}\rangle), \text{ for all } 1 \le i \le m \text{ and } 1 \le j \le n.$$

Further $A \le B \Rightarrow a_{ij} \le b_{ij}$ and $a'_{ij} \ge b'_{ij}$ for all i,j. Here F_{nxm} denotes the set of all intuitionistic fuzzy matrices of order nxm and F_{nxn} or F_n denotes the set of all IFM of order nxn.

Definition 1.3. [7]

Let ⟨a,a'⟩ , ⟨b,b'⟩ ∈ IFS define

$$\langle a,a'\rangle \to \langle b,b'\rangle = \begin{cases} \langle 1,0\rangle & \text{if } \langle a,a'\rangle \le \langle b,b'\rangle \\ \langle b,b'\rangle & \text{if } \langle a,a'\rangle > \langle b,b'\rangle \end{cases}$$

and ⟨a,a'⟩ ← ⟨b,b'⟩ = (⟨b,b'⟩ → ⟨a,a'⟩). Here ⟨a,a'⟩ > ⟨b,b'⟩ means a>b and a'< b'. Also ⟨a,a'⟩ ≠ ⟨b,b'⟩ means either a≠b or a'≠ b'.

Definition 1.4. [7]

Let $A = \langle a_{ij}, a'_{ij}\rangle \in F_{nxm}$, $B = \langle b_{ij}, b'_{ij}\rangle \in F_{mxn}$ define $A.B = \bigvee_{k=1}^{m} (\langle a_{ik}, a'_{ik}\rangle \wedge \langle b_{kj}, b'_{kj}\rangle)_{nxn}$ and for any $C \in F_{nxn}$.

Let $C^0 = I_n$, $C^1 = C$, $C^2 = C^1.C$, … $C^m = C^{m-1}.C$, if there exists a positive integer k such that $C^k = C^{k+1}$ then we say C is convergence of power.

Definition 1.5. [7]

Let $A \in F_{nxm}$ if $B \in F_{mxn}$ such that A.B.A = A then A is called regular, in this case B is called generalized inverse of A. If A.B.A. ≤ A, then B is called a sub-inverse of A.

Definition 1.6. [6]

Let ⟨a,a'⟩, ⟨b,b'⟩ ∈ IFS define

⟨a,a'⟩ ↔ ⟨b,b'⟩ = (⟨a,a'⟩ ← ⟨b,b'⟩) ∧ (⟨a,a'⟩ → ⟨b,b'⟩) that is

$$\langle a,a'\rangle \leftrightarrow \langle b,b'\rangle = \begin{cases} \langle b,b'\rangle & \text{if } \langle a,a'\rangle > \langle b,b'\rangle \\ \langle 1,0\rangle & \text{if } \langle a,a'\rangle = \langle b,b'\rangle \\ \langle a,a'\rangle & \text{if } \langle a,a'\rangle < \langle b,b'\rangle \end{cases}$$

Easily ⟨a,a'⟩ ↔ ⟨b,b'⟩ = ⟨b,b'⟩ ↔ ⟨a,a'⟩ , ⟨a,a'⟩= ⟨b,b'⟩ means a=b and a'= b'.

For $\langle a,a'\rangle, \langle b,b'\rangle, \langle c,c'\rangle \in$ IFS, the following Propositions and Lemmas hold.

Proposition 1.1. [6]

$(\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle) \leq (\langle a,a'\rangle \leftrightarrow \langle b,b'\rangle)$.

Proposition 1.2. [6]

Let $\langle a,a'\rangle, \langle b,b'\rangle \in$ IFS, then

$\langle a,a'\rangle \wedge (\langle a,a'\rangle \leftrightarrow \langle b,b'\rangle) \leq \langle b,b'\rangle)$.

Lemma 1.1. [6]

$(\langle a,a'\rangle \wedge \langle b,b'\rangle) \leftarrow \langle c,c'\rangle = (\langle a,a'\rangle \leftarrow \langle c,c'\rangle) \wedge \langle b,b'\rangle \leftarrow \langle c,c'\rangle)$.

Lemma 1.2. [6]

$(\langle a,a'\rangle \wedge \langle b,b'\rangle) \rightarrow \langle c,c'\rangle) = (\langle a,a'\rangle \rightarrow \langle c,c'\rangle) \vee (\langle b,b'\rangle \rightarrow \langle c,c'\rangle)$.

Lemma 1.3. [6]

$(\langle a,a'\rangle \vee \langle b,b'\rangle) \rightarrow \langle c,c'\rangle = (\langle a,a'\rangle \rightarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \rightarrow \langle c,c'\rangle)$.

Lemma 1.4.[7] For intuitionistic fuzzy matrix $A \in F_{nxm}$, $(A \leftarrow A^T)^T = A^T \rightarrow A$.

Proposition 1.3. [6]

(i) $(\langle a,a'\rangle \wedge \langle b,b'\rangle) \leftrightarrow \langle c,c'\rangle = [(\langle b,b'\rangle \leftarrow \langle c,c'\rangle) \wedge (\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle)] \vee [(\langle a,a'\rangle \leftarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle)]$

(ii) $(\langle a,a'\rangle \vee \langle b,b'\rangle) \leftrightarrow \langle c,c'\rangle = [(\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \rightarrow \langle c,c'\rangle)] \vee [(\langle a,a'\rangle \rightarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle)]$

Proposition 1.4. [6]

(i) $(\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle) \leq (\langle a,a'\rangle \wedge \langle b,b'\rangle) \leftrightarrow \langle c,c'\rangle \leq (\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \vee (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle)$

(ii) $(\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle) \leq (\langle a,a'\rangle \vee \langle b,b'\rangle) \leftrightarrow \langle c,c'\rangle \leq (\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \vee (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle)$

Proposition 1.5. [6]

If $\langle a,a'\rangle \wedge \langle b,b'\rangle \neq \langle c,c'\rangle$ then

(i) $(\langle a,a'\rangle \wedge \langle b,b'\rangle) \leftrightarrow \langle c,c'\rangle = (\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle)$

(ii) $(\langle a,a'\rangle \vee \langle b,b'\rangle) \leftrightarrow \langle c,c'\rangle = (\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \vee (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle)$

Proposition 1.6. [6]

For $\langle c,c'\rangle \neq \langle 1,0\rangle$, $(\langle a,a'\rangle \wedge \langle b,b'\rangle) \leftrightarrow \langle c,c'\rangle = (\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \wedge (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle)$ if and only if one of the following condition holds

(i) When $\langle a,a'\rangle > \langle b,b'\rangle$, $\langle b,b'\rangle \neq \langle c,c'\rangle$

(ii) When $\langle a,a'\rangle < \langle b,b'\rangle$, $\langle a,a'\rangle \neq \langle c,c'\rangle$

(iii) When $\langle a,a'\rangle = \langle b,b'\rangle$

Proposition 1.7. [6]

For $\langle c,c'\rangle \neq \langle 1,0\rangle$, $(\langle a,a'\rangle \vee \langle b,b'\rangle) \leftrightarrow \langle c,c'\rangle = (\langle a,a'\rangle \leftrightarrow \langle c,c'\rangle) \vee (\langle b,b'\rangle \leftrightarrow \langle c,c'\rangle)$ if and only if one of the following condition holds

(i) When $\langle a,a'\rangle > \langle b,b'\rangle$, $\langle b,b'\rangle \neq \langle c,c'\rangle$

(ii) When $\langle a,a'\rangle < \langle b,b'\rangle$, $\langle a,a'\rangle \neq \langle c,c'\rangle$

(iii) When $\langle a,a'\rangle = \langle b,b'\rangle$

Proposition 1.8. [7]

For any $A \in F_{nxm}$,

(i) $A \rightarrow A^T \geq I_n$

(ii) $A \leftarrow A^T \geq I_n$

(iii) $(A \rightarrow A^T) \cdot A \geq A$.

2. BI-IMPLICATION OPERATOR $\leftrightarrow$

Definition 2.1.

Let $B \in F_{mxn}$ be a symmetric square matrix, if there exists $A \in F_{mxn}$ such that $B=A.A^T$, then B is called realizable.

Definition 2.2.

Let $A = \langle a_{ij}, a'_{ij} \rangle \in F_n$, if $\langle a_{ii}, a'_{ii} \rangle \geq \langle a_{ik}, a'_{ik} \rangle \vee \langle a_{ki}, a'_{ki} \rangle$, $1 \leq i,k \leq n$, then A is called diagonally dominant matrix.

Definition 2.3.

Let $A = \langle a_{ij}, a'_{ij} \rangle \in F_{nxm}$, $B = \langle b_{ij}, b'_{ij} \rangle \in F_{mxn}$, define

$$A \leftarrow B = (\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik} \rangle \leftarrow \langle b_{kj}, b'_{kj} \rangle))_{nxn}$$

$$A \rightarrow B = (\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik} \rangle \rightarrow \langle b_{kj}, b'_{kj} \rangle))_{nxn}$$

$$A \leftrightarrow B = (A \leftarrow B) \wedge (A \rightarrow B)$$

According to the definition $A \leftrightarrow B = (\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik} \rangle \leftrightarrow \langle b_{kj}, b'_{kj} \rangle))_{nxn}$.

Remark 2.1.

Generally for IFMs of compatible order $A \leftrightarrow B \neq B \leftrightarrow A$.

This is illustrated through the following example.

Example 2.1.

For IFMs, A and B of compatible order

$$A = \begin{pmatrix} \langle 0.1, 0.2 \rangle & \langle 0.1, 0.3 \rangle \\ \langle 0.5, 0.1 \rangle & \langle 0.4, 0.2 \rangle \end{pmatrix}$$

$$B = \begin{pmatrix} \langle 0.3, 0.2 \rangle & \langle 0.4, 0.1 \rangle \\ \langle 0.1, 0.5 \rangle & \langle 0.1, 0.6 \rangle \end{pmatrix}$$

$$A \leftrightarrow B = \begin{pmatrix} \text{Min}(\langle 0.1, 0.2 \rangle, \langle 0.1, 0.5 \rangle) & \text{Min}(\langle 0.1, 0.2 \rangle, \langle 0.1, 0.6 \rangle) \\ \text{Min}(\langle 0.3, 0.2 \rangle, \langle 0.1, 0.5 \rangle) & \text{Min}(\langle 0.4, 0.1 \rangle, \langle 0.1, 0.6 \rangle) \end{pmatrix}$$

$$A \leftrightarrow B = \begin{pmatrix} \langle 0.1, 0.5 \rangle & \langle 0.1, 0.6 \rangle \\ \langle 0.1, 0.5 \rangle & \langle 0.1, 0.6 \rangle \end{pmatrix}$$

$$B \leftrightarrow A = \begin{pmatrix} \text{Min}(\langle 0.1, 0.2 \rangle, \langle 0.4, 0.1 \rangle) & \text{Min}(\langle 0.1, 0.3 \rangle, \langle 0.4, 0.2 \rangle) \\ \text{Min}(\langle 0.1, 0.5 \rangle, \langle 0.1, 0.6 \rangle) & \text{Min}(\langle 0.1, 0.5 \rangle, \langle 0.1, 0.6 \rangle) \end{pmatrix}$$

$$B \leftrightarrow A = \begin{pmatrix} \langle 0.1, 0.2 \rangle & \langle 0.1, 0.3 \rangle \\ \langle 0.1, 0.6 \rangle & \langle 0.1, 0.6 \rangle \end{pmatrix}$$ therefore $A \leftrightarrow B \neq B \leftrightarrow A$

Proposition 2.1.

Let $A \in F_{nxm}$, then

(i) $A \leftrightarrow A^T$ is a reflexive matrix ($A \leftrightarrow A^T \geq I_n$)

(ii) $A \leftrightarrow A^T$ is a symmetric matrix

(iii) $A \leftrightarrow A^T$ is a idempotent matrix

(iv) $A \leftrightarrow A^T$ is a power convergent matrix

(v) $A \leftrightarrow A^T$ is a diagonally dominant matrix

(vi) $A \leftrightarrow A^T$ is realizable

Proof:

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(i) From Proposition 2.1

$A \leftrightarrow A^T = (A \leftarrow A^T) \wedge (A \rightarrow A^T) \geq I_n \wedge I_n = I_n$

$A \leftrightarrow A^T \geq I_n$

(ii) $(A \leftrightarrow A^T)^T = [(A \rightarrow A^T) \wedge (A \leftarrow A^T]^T$

$A \leftrightarrow A^T = [(A \rightarrow A^T) \wedge (A \rightarrow A^T)]^T$

$= [(A \rightarrow A^T)^T \wedge (A \leftarrow A^T)^T]$

$= [(A^T \leftarrow A) \wedge (A^T \rightarrow A)]$ by Lemma 1.4.

$= A^T \leftrightarrow A$

$(A \leftrightarrow A^T)^T = A \leftrightarrow A^T$.

(iii) $(A \leftrightarrow A^T) . (A \leftrightarrow A^T)$

$$= (\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle))_{nxn} . (\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle))_{nxn}$$

$$= \vee_{f=1}^{n} [(\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle)) \wedge (\wedge_{k=1}^{m}(\langle a_{fk}, a'_{fk}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle))]_{nxn}$$

$$= \vee_{f=1}^{n} [(\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle) \wedge (\langle a_{jk}, a'_{jk}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle)]_{nxn}$$

$$= \vee_{f=1}^{n} [(\wedge_{k=1}^{m}((\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle) \wedge (\langle a_{jk}, a'_{jk}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle))]_{nxn}$$

By proposition (1.1)

$(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle) \wedge (\langle a_{jk}, a'_{jk}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle) \leq \langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle$

$1 \leq k \leq m,\ 1 \leq i, j, f \leq n$.

Therefore $(A \leftrightarrow A^T) . (A \leftrightarrow A^T)$

$$\leq (\vee_{f=1}^{n} [\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle)])_{nxn}$$

$$\leq (\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle))_{nxn}$$

$(A \leftrightarrow A^T)^2 \leq A \leftrightarrow A^T$.

On the other hand

$$(A \leftrightarrow A^T) . (A \leftrightarrow A^T) = \vee_{f=1}^{n} [\wedge_{k=1}^{m}((\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle) \wedge (\langle a_{jk}, a'_{jk}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle))]_{nxn} \geq$$

$$\wedge_{k=1}^{m}[(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle) \wedge (\langle a_{jk}, a'_{jk}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle)]_{nxn} \geq (\wedge_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle))_{nxn} \geq A \leftrightarrow A^T$$

$(A \leftrightarrow A^T)^2 \geq A \leftrightarrow A^T$

$(A \leftrightarrow A^T)^2 = A \leftrightarrow A^T$.

(iv) Can be got at once by (iii)

(v) Can be obtained at once by (i)

(vi) Can be verified immediately by (ii) and (iii).

Remark 2.2.

In general, for any $A \in F_{nxn}$, $A \leftrightarrow A \neq A$.

It is evident from the following example.

Example 2.2.

$$A = \begin{pmatrix} \langle 0.1, 0.2\rangle & \langle 0.1, 0.3\rangle \\ \langle 0.5, 0.1\rangle & \langle 0.4, 0.2\rangle \end{pmatrix}$$

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$$A\leftrightarrow A = \begin{pmatrix} Min(\langle 1,0\rangle,\langle 0.1,0.3\rangle) & Min(\langle 0.1,0.3\rangle,\langle 0.1,0.3\rangle) \\ Min(\langle 0.1,0.2\rangle,\langle 0.4,0.2\rangle) & Min(\langle 0.1,0.3\rangle,\langle 1,0\rangle) \end{pmatrix}$$

$$A\leftrightarrow A = \begin{pmatrix} \langle 0.1,0.3\rangle & \langle 0.1,0.3\rangle \\ \langle 0.1,0.2\rangle & \langle 0.1,0.3\rangle \end{pmatrix}$$

Easily to see $A \leftrightarrow A \neq A$. but we have conclusions of the following.

Proposition 2.2.

If $A = \langle a_{ij},a'_{ij}\rangle \in F_{nxn}$ is a weakly reflexive matrix, that is for $1 \leq i, j \leq n$, $\langle a_{ij},a'_{ij}\rangle \leq \langle a_{ii},a'_{ii}\rangle$

for $i \neq f$ or $j \neq k$, $\langle a_{ij},a'_{ij}\rangle \neq \langle a_{fk},a'_{fk}\rangle$ then $A \leftrightarrow A \leq A$.

Proof:

Since $A \leftrightarrow A = (\wedge_{k=1}^{n}(\langle a_{ik},a'_{ik}\rangle \leftrightarrow \langle a_{kj},a'_{kj}\rangle))_{nxn}$ hence $\wedge_{k=1}^{n}(\langle a_{ik},a'_{ik}\rangle \leftrightarrow \langle a_{kj},a'_{kj}\rangle) \leq \langle a_{ii},a'_{ii}\rangle \leftrightarrow \langle a_{ij},a'_{ij}\rangle$

$= \langle a_{ij},a'_{ij}\rangle = A$, $1 \leq I, j \leq n$.

Thus $A \leftrightarrow A \leq A$.

Proposition 2.3.

For any $A\in F_{nxn}$, if A is a reflexive matrix then

(i)) $A^T \leftrightarrow A \leq A$

(ii) $A \leftrightarrow A^T \leq A$

Proof:

(i) Since $A \geq I_n$, $\langle a_{ii},a'_{ii}\rangle = \langle 1,0\rangle, 1 \leq i \leq n$ $A^T \leftrightarrow A = \wedge_{k=1}^{n}(\langle a_{ki},a'_{ki}\rangle \leftrightarrow \langle a_{kj},a'_{kj}\rangle)_{nxn} \leq (\langle a_{ii},a'_{ii}\rangle \leftrightarrow \langle a_{ij},a'_{ij}\rangle) =$

$(\langle a_{ij},a'_{ij}\rangle) = A$

Therefore $A^T \leftrightarrow A \leq A$.

(ii) $A \leftrightarrow A^T =(\wedge_{k=1}^{n}(\langle a_{ik},a'_{ik}\rangle \leftrightarrow \langle a_{jk},a'_{jk}\rangle))_{nxn} \leq \langle a_{ij},a'_{ij}\rangle \leftrightarrow \langle a_{jj},a'_{jj}\rangle , \leq \langle a_{ij},a'_{ij}\rangle$

Therefore $A \leftrightarrow A^T \leq A$.

Proposition 2.4.

For any $A\in F_{nxm}$, if $\langle a_{ij},a'_{ij}\rangle \neq \langle 0,1\rangle$, $1 \leq i \leq n$, $1 \leq j \leq m$, then $A \leftrightarrow I_m = (\langle 0,1\rangle)_{nxm}$ (the nxm zero matrix).

Proof:

Since $A\leftrightarrow I_m = (\wedge_{k=1}^{m}(\langle a_{ik},a'_{ik}\rangle \leftrightarrow \langle \delta_{kj},\delta'_{kj}\rangle))_{nxm}$

where $\langle \delta_{kj},\delta'_{kj}\rangle = \begin{cases} \langle 1,0\rangle & \text{if } \langle i,i'\rangle = \langle j,j'\rangle \\ \langle 0,1\rangle & \text{otherwise} \end{cases}$

Hence $\wedge_{k=1}^{m}(\langle a_{ik},a'_{ik}\rangle \leftrightarrow \langle \delta_{kj},\delta'_{kj}\rangle)$

$(\langle a_{i1},a'_{i1}\rangle \leftrightarrow \langle \delta_{1j},\delta'_{1j}\rangle) \wedge (\langle a_{i2},a'_{i2}\rangle \leftrightarrow \langle \delta_{2j},\delta'_{2j}\rangle) \ldots \wedge (\langle a_{ij},a'_{ij}\rangle \leftrightarrow \langle \delta_{jj},\delta'_{jj}\rangle) \wedge \ldots \wedge$

$(\langle a_{im},a'_{im}\rangle \leftrightarrow \langle \delta_{mj},\delta'_{mj}\rangle) = \langle 0,1\rangle \wedge \langle 0,1\rangle \ldots (\langle a_{ij},a'_{ij}\rangle \leftrightarrow \langle \delta_{jj},\delta'_{jj}\rangle) \wedge \ldots \wedge \langle 0,1\rangle = \langle 0,1\rangle$

$A \leftrightarrow I_m = \langle 0,1\rangle$

Proposition 2.5.

For any $A\in F_{nxm}$, then

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(i) $(A \leftrightarrow A^T) A = A$

(ii) $A(A^T \leftrightarrow A) = A$.

Proof:

Let $(A \leftrightarrow A^T) A = (\langle r_{ij}, r'_{ij}\rangle)$, then

$$(\langle r_{ij}, r'_{ij}\rangle) = (\mathop{\wedge}_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{jk}, a'_{jk}\rangle))(\langle a_{ij}, a'_{ij}\rangle)$$

$$= \mathop{V}_{f=1}^{n}(\mathop{\wedge}_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{fk}, a'_{fk}\rangle) \wedge \langle a_{fj}, a'_{fj}\rangle).$$

Thus

$(\langle r_{ij}, r'_{ij}\rangle) = (\mathop{\wedge}_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{1k}, a'_{1k}\rangle) \wedge \langle a_{1j}, a'_{1j}\rangle) \vee (\mathop{\wedge}_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{2k}, a'_{2k}\rangle) \wedge \langle a_{2j}, a'_{2j}\rangle) \vee \ldots \vee (\mathop{\wedge}_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{nk}, a'_{nk}\rangle) \wedge \langle a_{nj}, a'_{nj}\rangle)$.

By Proposition (1.2) for $h \neq i$, $\mathop{\wedge}_{k=1}^{m}(\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{hk}, a'_{hk}\rangle) \wedge \langle a_{hj}, a'_{hj}\rangle \leq (\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{hj}, a'_{hj}\rangle) \wedge \langle a_{hj}, a'_{hj}\rangle \leq \langle a_{ij}, a'_{ij}\rangle$, yet $\mathop{\wedge}_{k=1}^{m}((\langle a_{ik}, a'_{ik}\rangle \leftrightarrow \langle a_{hk}, a'_{hk}\rangle) \wedge \langle a_{hj}, a'_{hj}\rangle) = \mathop{\wedge}_{k=1}^{m}(\langle a_{ij}, a'_{ij}\rangle) = \langle a_{ij}, a'_{ij}\rangle$. Thus (i) holds. Similarly, we can prove (ii).

APPLICATION ON INTUITIONISTIC FUZZY RELATION EQUATION

In this section, we study sub-inverse and g-inverse of IFM using bi-implication operator.

Proposition 3.1.

For any $A \in F_{mn}$, $XA \geq A$ always has a solution $A \rightarrow A^T$.

Proof:

The proof is straightforward from proposition (1.8(iii)).

Proposition 3.2.

If $A \in F_n$ is an idempotent, then $AX \leq A(XA \leq A)$ always has a solution $A \leftrightarrow A$.

Proof:

Since A is idempotent $A^2 = A$, $\left(\mathop{V}_{f=1}^{n}(\langle a_{ik}, a'_{ik}\rangle \wedge \langle a_{kj}, a'_{kj}\rangle)\right) = (\langle a_{ij}, a'_{ij}\rangle)$, hence $\langle a_{ij}, a'_{ij}\rangle = \langle a_{ii}, a'_{ii}\rangle \wedge \langle a_{ij}, a'_{ij}\rangle \leq \langle a_{ii}, a'_{ii}\rangle$, that is, A is weakly reflexive. By Proposition (2.2), $A \leftrightarrow A \leq A$. So $A(A\leftrightarrow A) \leq AA = A^2 = A$. Consequently, $AX \leq A$, always has a solution $A\leftrightarrow A$. Similarly, we can prove $XA \leq A$ always has a solution $A \leftrightarrow A$ also.

Proposition 3.3.

If $B \in F_n$ is weakly reflexive, then $B \leq B^2$.

Proof:

$$B^2 = \left(\mathop{V}_{k=1}^{n}(\langle b_{ik}, b'_{ik}\rangle \wedge \langle b_{kj}, b'_{kj}\rangle)\right) \geq (\langle b_{ii}, b'_{ii}\rangle \wedge \langle b_{ij}, b'_{ij}\rangle) = (\langle b_{ij}, b'_{ij}\rangle) = B.$$

The following corollary is evident from the above proposition (3.3).

Corollary 3.1.

The necessary and sufficient condition for $A \in F_n$ is weakly reflexive and transitive.

Proposition 3.4.

For any $A \in F_{nm}$, $AX = A$, $XA = A$ has a solution $A^T \leftrightarrow A$, $A \leftrightarrow A^T$ respectively.

Proof:

Trivial by Proposition (2.5).

Proposition 3.5.

For any A $\in F_n$, the following are equivalent.

i) A is a transitive matrix.

ii) $A(A \leftrightarrow A^T) A \leq A$.

iii) $A(AT \leftrightarrow A) A \leq A$.

Proof:

(i) $\Rightarrow$ (ii).

If $A^2 \leq A$, by Proposition (2.5)

$A(A\leftrightarrow A^T)A = A((A\leftrightarrow A^T)A) = AA = A^2 \leq A$. Similarly, we can prove the others. The proposition shows that $(A \leftrightarrow A^T)$, $(A^T\leftrightarrow A)$ are sub-inverses of A.

Proposition 3.6.

If $A\in F_n$ is idempotent then both $(A \leftrightarrow A^T)$, $(A^T \leftrightarrow A)$ are g-inverse of A.

Proof:

By Proposition (2.5), $(A \leftrightarrow A^T)A = A$, $A(A \leftrightarrow A^T)A = AA = A$. Thus, $(A \leftrightarrow A^T)$ is a g-inverse of A. Dually we can prove the other.

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