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Some Universal Constructions For I- Fuzzy Topological Spaces

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ABSTRACT

Geetha S. [J. Math. Anal. Appl. 174 (1993), 147-152] has introduced the concept of I-fuzzy topological spaces (X,μ,F) where X is an ordinary set, μ is a fuzzy set in X and F is a family of fuzzy sets in X satisfying some axioms. In this paper we introduce universal constructions, namely, fuzzy products, fuzzy equalizers and fuzzy pullbacks for I -fuzzy topological spaces. Also we discuss some results concerning all such universal objects.

Keywords:

Fuzzy sets; I -fuzzy topological spaces; Fuzzy equalizers; Fuzzy pullbacks; Fuzzy products.

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1. INTRODUCTION

Zadeh [12] introduced the notion of a fuzzy set as a function from the given set to the unit interval. The first categorical definition of fuzzy sets was introduced by J. A. Goguen [5]. In the case of fuzzy topology, there are various interesting categories of fuzzy topological spaces. The collection of all fuzzy topological spaces and fuzzy continuous functions form a category. Since C. Chang, R. Lowen and J. Goguen have defined fuzzy topology in different ways, each of them defines a different category of fuzzy topological spaces [8,10]. Geetha S. [4,5,6] introduced a new category **FTOP**, the object is I - fuzzy topological spaces (X, μ, F) where X is an ordinary set, μ is a fuzzy set in X and F is a family of fuzzy sets in X satisfying some axioms. Some applications of category theory in fuzzy topology are presented in [2, 7,11]. Behera [2] introduced the concepts of fuzzy equalizers, fuzzy pullbacks and their duals for fuzzy topological spaces in the sense of Chang. In this paper we introduce the universal constructions, namely, fuzzy products, fuzzy equalizers and fuzzy pullbacks for I -fuzzy topological spaces. Also we discuss some results concerning all such universal objects.

2. PRELIMINARIES

As usual I denotes the closed unit interval [0,1]. A fuzzy set *A* in a set *X* is a function on *X* into the closed unit interval [0, 1] of the real line. The fuzzy sets in *X* taking on respectively the constant values 0 and 1 are denoted by 0_X and 1_X [11] respectively. For two fuzzy sets *A, B* in *X*, we write $A \leq B$ if $A(x) \leq B(x)$ for each $x \in X$. For a collection of fuzzy sets $\{A_i : i \in J\}$, the union $C = \bigcup_{i \in J} A_i$ and the intersection $D = \bigcap_{i \in J} A_i$ are defined by

$$C(x) = \bigvee_{i \in J} A_i(x), \quad \text{for all } x \in X,$$

$$D(x) = \bigwedge_{i \in J} A_i(x), \quad \text{for all } x \in X.$$

If $f: X \to Y$ is a function, and A, B are fuzzy sets in X, Y respectively, then the fuzzy set $f^{-1}(B)$ in X is defined by $f^{-1}(B) = B \circ f$, and $f(A): Y \to I$ is defined as follows [3]:

$$f(A)(y) = \begin{cases} \bigvee \{A(x) : x \in f^{-1}(y)\}, & if\ f^{-1}(y) \neq \emptyset; \\ 0 & if\ f^{-1}(y) = \emptyset. \end{cases}$$

Definition 2.1 [5] *Let X be a set , $\mu: X \to I$ be a fuzzy set in X and F be a family of fuzzy sets in X satisfying the following conditions:*

(1) $A \in F$ implies that $A(x) \leq \mu(x)$ for all $x \in X$,

(2) If $A_i \in F$, $i \in J$, then $\bigcup_{i \in J} A_i \in F$,

(3) If $A, B \in F$, then $A \cap B \in F$,

(4) $0_X, \mu \in F$.

The triple (X, μ, F) is called an $I -$ fuzzy topological space or I - fts. The members of F are called $I -$ fuzzy open sets and their complements are called $I -$ fuzzy closed sets.

Remark 2.2 when $\mu = 1_X$, an $I -$ *fuzzy topological space is nothing but a fuzzy topology in the sense of Chang[3].*

Definition 2.3 Let (X_1, μ_1, F_1) and (X_2, μ_2, F_2) be two $I -$ *fuzzy topological spaces. A function* $f: (X_1, \mu_1, F_1) \to (X_2, \mu_2, F_2)$ *is fuzzy continuous if:*

i. $\mu_1(x) \leq \mu_2(f(x)), \ \forall\, x \in X,$

ii. $\mu_1 \cap f^{-1}(U) \in F_1, \ \forall\, U \in F_2.$

The notion **FTOP** will denote the category of I – fuzzy topological spaces and fuzzy continuous functions. We shall use the categorical terminology of [1]. For more information about the category **FTOP**, the reader could consult [4].

3. UNIVERSAL CONSTRUCTIONS IN FTOP

In this section we discuss fuzzy products, fuzzy equalizers and fuzzy pullbacks of $I -$ fuzzy topological spaces. By remark 2.2, some results in [2] are considered as a special case of the results below. The word "map" will always mean a continuous function, but the word "function" does not imply continuity.

The concept of fuzzy product has introduced in [4,5,8].The following theorem emphasizes the universal property of fuzzy product in **FTOP**. From now on, J is referred to as the index set and the word fuzzy spaces means $I -$ fuzzy topological spaces.

Theorem 3.1 *For a given fuzzy spaces $(X_i, \mu_i, F_i), i \in J$, the following hold:*

(1) There exists a fuzzy space (P, μ, F) and a fuzzy maps $p_i: (P, \mu, F) \to (X_i, \mu_i, F_i)$ for each $i \in J$.

(2) *For any fuzzy space* (X,γ,H) *with fuzzy maps* $\varphi_i:(X,\gamma,H)\to(X_i,\mu_i,F_i)$*, there is a unique fuzzy map* $\theta:(X,\gamma,H)\to:(P,\mu,F)$ *such that* $p_i\circ\theta=\varphi_i$ *for each* $i\in J$.

Proof. (1) For the given fuzzy spaces $(\boldsymbol{X_i},\boldsymbol{\mu_i},\boldsymbol{F_i})$, we consider their product $\prod_{i\in J}(X_i,\mu_i,F_i)$ to be the fuzzy space (P,μ,F), where $P=\prod_{i\in J}X_i$ is the usual set product, μ the product fuzzy set in P defined by $\mu(x)=\wedge_{i\in J}\{\mu_i(x_i):x=(x_i)_{i\in J}\in P\}$ and F is generated by the subbase $\beta=\{p_i^{-1}(U_i)\cap\mu:U_i\in F_i\}$ for each $i\in J$ [4]. The fuzzy topology on $P=\prod_{i\in J}X_i$ is the coarsest fuzzy topology so that the projections $p_i:(P,\mu,F)\to(X_i,\mu_i,F_i)$ are fuzzy maps for each $i\in J$ [4].

(2) Define $\theta:(X,\gamma,H)\to:(P,\mu,F)$ by $\theta(x)=(\varphi_i(x))_{i\in J}$ for all $x\in X,i\in J$. With the definition of θ, we have $p_i\circ\theta=\varphi_i$. Since φ_i is a fuzzy map for each $i\in J$, then continuity of θ comes directly by the fact: θ is a fuzzy map if and only if $p_i\circ\theta$ is a fuzzy map [phd].This shows the existence of the universal property. Let $\theta:(X,\gamma,H)\acute{\to}:(P,\mu,F)$ be any fuzzy map with the property that $p_i\circ\acute{\theta}=\varphi_i$ for each $i\in J$. Then the *i-th* coordinate of $\acute{\theta}(x)$ is $\varphi_i(x)$ for each $x\in P$. Thus $\acute{\theta}=\theta$ defined above. This shows the uniqueness of the universal property. □

Proposition 3.2 *Let* $(X_i,\mu_i,F_i),i\in J$*, be a collection of fuzzy spaces, and give* $\prod_{i\in J}X_i$ *the fuzzy product topology. Then the fuzzy product is unique up to fuzzy homeomorphism*.

Proof. Let $P=\prod_{i\in J}X_i$ and $Q=\prod_{i\in J}X_i$ be two fuzzy product of spaces (X_i,μ_i,F_i) with projections $p_i:(P,\mu,F)\to(X_i,\mu_i,F_i)$ and $q_i:(Q,\gamma,G)\to(X_i,\mu_i,F_i)$ respectively. Then the universal property of the fuzzy product P implies that there is a unique fuzzy map $\theta:\ :(Q,\gamma,G)\to(P,\mu,F)$ such that $p_i\circ\theta=q_i$ for each $i\in J$. In similar way, there exist a unique fuzzy map $\varphi:(P,\mu,F)\to Q,\gamma,G)$ such that $q_i\circ\varphi=p_i$ for each $i\in J$. Thus $p_i=q_i\circ\varphi=p_i\circ\theta\circ\varphi=p_i\circ id_P$ for each $i\in J$. Form the uniqueness condition of theorem 3.1, it follows that $\theta\circ\varphi=id_P$. Similarly, we have $\varphi\circ\theta=id_Q$. Therefore, P and Q are fuzzy homeomorphic [3]. □

For the sake of simplicity, we shall use the symbol (μ_X,F_X) for the $I-$fuzzy topological space (X,μ,F). The following theorem states the definition [1]and proves the existence of fuzzy equalizers in **FTOP**.

Theorem 3.3 Let $f,g:(\mu_X,F_X)\to(\mu_Y,F_Y)$ be fuzzy maps, then

(1) There exists a fuzzy space (μ_E,F_E) and a fuzzy map $e:(\mu_E,F_E)\to(\mu_X,F_X)$ such that $f\circ e=g\circ e$.

(2) For any fuzzy space (μ_A,F_A) with a fuzzy map $\varphi:(\mu_A,F_A)\to(\mu_X,F_X)$ satisfying $f\circ\varphi=g\circ\varphi$, there exists a unique fuzzy map $h:(\mu_A,F_A)\to(\mu_E,F_E)$ such that $\varphi=e\circ h$.

Proof. (1) We consider (μ_E,F_E) as a subspace of (μ_X,F_X) where $E=\{x\in X:f(x)=g(x)\}$, μ_E is the restriction $\mu_X|_E$, that is $\mu_X\cap 1_E$, and $F_E=\{V|_E:V\in F_X\}$[4]. Define $e:(\mu_E,F_E)\to(\mu_X,F_X)$ by $e(x)=x,\ x\in E$; it is clear that $f\circ e=g\circ e$ and e is a fuzzy map since it equals the identity map id_X restricted to E [4].

(2)We then have to verify the universal property. For any fuzzy space (μ_A,F_A), we define $h:(\mu_A,F_A)\to(\mu_E,F_E)$ by $h(a)=\varphi(a),a\in A$. Since $f(\varphi(a))=g(\varphi(a))$, we have $\varphi(a)\in E$. Therefore, we get $\varphi=e\circ h$ since $e(h(a))=h(a)=\varphi(a),\ a\in A$. Now, we have to show that h is a fuzzy map, that is $\mu_A(a)\le\mu_E(h(a)),a\in A$ and $\mu_A\cap h^{-1}(U)\in F_A,\ \forall U\in F_E$. Since φ is a fuzzy map, then

$$\mu_A(a)\le\mu_X(\varphi(a))=\mu_E(\varphi(a))=\mu_E(e(h(a))=\mu_E(h(a)).$$

With the definition of F_E, we set $U=V|_E,V\in F_X$. Hence we get

$$(\mu_A\cap h^{-1}(U))(a)=\mu_A(a)\wedge h^{-1}(U)(a)=\mu_A(a)\wedge U(h(a))=\mu_A(a)\wedge V(h(a))=(\mu_A\cap V(\varphi(a)))(a)$$
$$=(\mu_A\cap\varphi^{-1}(V))(a).$$

Thus $(\mu_A\cap h^{-1}(U)=(\mu_A\cap\varphi^{-1}(V)\in F_A$, as desired. The uniqueness of h comes directly from the definition. □

We now define ***fuzzy pullbacks***. Suppose we have fuzzy maps $f:(\mu_X,F_X)\to(\mu_Z,F_Z)$ and $g:(\mu_Y,F_Y)\to(\mu_Z,F_Z)$. Then the fuzzy pullback of these fuzzy maps is an $I-$fts (μ_B,F_B) together with fuzzy maps $\alpha:(\mu_B,F_B)\to(\mu_X,F_X)$ and $\beta:(\mu_B,F_B)\to(\mu_Y,F_Y)$ such that $f\circ\alpha=g\circ\beta$, and such that the following *universal property* holds: Suppose that (μ_C,F_C) is an $I-$fts and that $\acute{\alpha}:(\mu_C,F_C)\to(\mu_X,F_X)$ and $\acute{\beta}:(\mu_C,F_C)\to(\mu_Z,F_Z)$ are fuzzy maps with $f\circ\acute{\alpha}=g\circ\acute{\beta}$. Then there is a unique fuzzy map $\varphi:(\mu_C,F_C)\to(\mu_B,F_B)$ with $\alpha\circ\varphi=\acute{\alpha}$ and $\beta\circ\varphi=\acute{\beta}$. We then call (μ_B,F_B) a fuzzy pullback of f and g.

In the following theorem, we show that fuzzy pullbacks exist in the category **FTOP** by constructing them as fuzzy products.

Theorem 3.4 Let $f:(\mu_X,F_X)\to(\mu_Z,F_Z)$ and $g:(\mu_Y,F_Y)\to(\mu_Z,F_Z)$ be any fuzzy maps. Then there exists a fuzzy pullback (μ_B,F_B).

Proof. We consider $B=\{(x,y)\in X\times Y:f(x)=g(y)\}$ as a subset of $X\times Y$, $\mu_B=(\mu_X\cap\mu_Y)|_BandF_B$is generated by the base $\beta=\{((U_1\times U_2)\cap\mu_{X\times Y})|_B:U_1\in F_X,U_2\in F_Y\}$. Define the projection maps $\alpha:(\mu_B,F_B)\to(\mu_X,F_X)$ and $\beta:(\mu_B,F_B)\to(\mu_Y,F_Y)$ by $\alpha(x,y)=x$ and $\beta(x,y)=y$ for all $(x,y)\in B$, clearly α and β are fuzzy maps [4]. It is clear from the definition of B that $f\circ\alpha=g\circ\beta$. To show that B satisfies the universal property, suppose that (μ_C,F_C) is a fuzzy space with fuzzy maps $\acute{\alpha}:(\mu_C,F_C)\to(\mu_X,F_X)$ and $\acute{\beta}:(\mu_C,F_C)\to(\mu_Y,F_Y)$ with $f\circ\acute{\alpha}=g\circ\acute{\beta}$. Define $\varphi:(\mu_C,F_C)\to(\mu_B,F_B)$ by $\varphi(c)=(\acute{\alpha}(c),\acute{\beta}(c))$. Then φ maps C into B since $f(\acute{\alpha}(c))=g(\acute{\beta}(c))$ by the equation $f\circ\acute{\alpha}=g\circ\acute{\beta}$. Also, it is clear that $\alpha\circ\varphi=\acute{\alpha}$ and $\beta\circ\varphi=\acute{\beta}$.

By the uniqueness property of the fuzzy projection maps (theorem 3.1) , φ is unique. Finally, we show that φ is fuzzy continuous. Since $\acute{\alpha}$ and $\acute{\beta}$ are fuzzy maps, then $\mu_C(c) \leq \mu_X(\acute{\alpha}(c))$ and $\mu_C(c) \leq \mu_Y(\acute{\beta}(c))$. Hence,

$$\mu_B(\varphi(c)) = \mu_B\left(\acute{\alpha}(c), \acute{\beta}(c)\right) = \mu_X(\acute{\alpha}(c)) \wedge \mu_Y(\acute{\beta}(c)) \geq \mu_C(c).$$

With the definition of F_B, let U be a fuzzy open set in F_B,that is $U = ((U_1 \times U_2) \cap \mu_{X\times Y}))|_B$ for all $U_1 \in F_X, U_2 \in F_Y$, then

$$(\mu_C \cap \varphi^{-1}(U))(c) = \mu_C(c) \wedge U(\varphi(c)) = \mu_C(c) \wedge U\left(\acute{\alpha}(c), \acute{\beta}(c)\right)$$

$$= \mu_C(c) \wedge \left[(U_1 \times U_2)\left(\acute{\alpha}(c), \acute{\beta}(c)\right) \wedge \mu_{X\times Y}\left(\acute{\alpha}(c), \acute{\beta}(c)\right)\right]$$

$$= \mu_C(c) \wedge \left[U_1(\acute{\alpha}(c)) \wedge U_2\left(\acute{\beta}(c)\right) \wedge \left(\mu_X(\acute{\alpha}(c)) \wedge \mu_Y\left(\acute{\beta}(c)\right)\right)\right]$$

$$= \mu_C(c) \wedge \left[\acute{\alpha}^{-1}(U_1)(c) \wedge \acute{\beta}^{-1}(U_2)(c) \wedge \left(\mu_X(\acute{\alpha}(c)) \wedge \mu_Y\left(\acute{\beta}(c)\right)\right)\right].$$

Since $\mu_X(\acute{\alpha}(c)) \wedge \mu_Y(\acute{\beta}(c)) \geq \mu_C(c)$, then

$$(\mu_C \cap \varphi^{-1}(U))(c) = [\mu_C(c) \wedge \acute{\alpha}^{-1}(U_1)(c)] \wedge [[\mu_C(c) \wedge \acute{\beta}^{-1}(U_2)(c)]$$

$$= (\mu_C \cap \acute{\alpha}^{-1}(U_1))(c) \wedge (\mu_C \cap \acute{\beta}^{-1}(U_2))(c).$$

Thus $\mu_C \cap \varphi^{-1}(U) = (\mu_C \cap \acute{\alpha}^{-1}(U_1) \cap (\mu_C \cap \acute{\beta}^{-1}(U_2)$ belongs to F_C. Therefore, φ is indeed a fuzzy map. □

Example 3.5 Given the diagram of fuzzy maps

$$(\mu_X, F_X) \rightarrow (\mu_{\{x\}}, F_{\{x\}}) \leftarrow (\mu_Y, F_Y),$$

where $(\mu_{\{x\}}, F_{\{x\}})$ is a terminal object in $\mathbb{FTOP}$[4]. Then the fuzzy pullback (μ_B, F_B) is the fuzzy space $(\mu_{X\times Y}, F_{X\times Y})$.

Proposition 3.6 Given any fuzzy maps $f: (\mu_X, F_X) \rightarrow (\mu_Z, F_Z)$ and $g: (\mu_Y, F_Y) \rightarrow (\mu_Z, F_Z)$, the fuzzy pullback (μ_B, F_B) is unique up to fuzzy homeomorphism.

Proof. Suppose $(\mu_{B'}, F_{B'})$, with fuzzy maps $f': (\mu_{B'}, F_{B'}) \rightarrow (\mu_X, F_X)$ and $f': (\mu_{B'}, F_{B'}) \rightarrow (\mu_Y, F_Y)$, is another fuzzy pullback. Take $(\mu_C, F_C) = (\mu_{B'}, F_{B'})$; we find a fuzzy map $\varphi: (\mu_{B'}, F_{B'}) \rightarrow (\mu_B, F_B)$ such that $\alpha \circ \varphi = \acute{f}$ and $\beta \circ \varphi = \acute{g}$. By reversing the roles of (μ_B, F_B) and $(\mu_{B'}, F_{B'})$, we find a fuzzy map $\varphi': (\mu_B, F_B) \rightarrow (\mu_{B'}, F_{B'})$ such that $\acute{f} \circ \varphi' = \alpha$ and $\acute{g} \circ \varphi' = \beta$. Then $\alpha \circ \varphi \circ \varphi' = \alpha$, and similarly $\beta \circ \varphi \circ \varphi' = \beta$. Now take $(\mu_C, F_C) = (\mu_B, F_B), \alpha = \alpha'$ and $\beta = \beta'$. We have two fuzzy maps $\varphi \circ \varphi': (\mu_B, F_B) \rightarrow (\mu_B, F_B)$ and id_B that satisfy the conditions of fuzzy pullback; by the uniqueness, $\varphi \circ \varphi' = id_B$. Similarly, $\varphi' \circ \varphi = id_{B'}$, so that φ and φ' are inverse fuzzy homeomorphisms. □

We close this section by investigation the relationship among the universal constructions mentioned above .

Lemma 3.7 In **FTOP**, fuzzy pullbacks exist if and only if fuzzy equalizers exist.

Proof. Suppose that fuzzy pullbacks exist. Given any two fuzzy maps $f, g: (\mu_X, F_X) \rightarrow (\mu_Y, F_Y)$, we write $(f, g): (\mu_X, F_X) \rightarrow (\mu_{Y\times Y}, F_{Y\times Y})$, $x \mapsto (f(x), g(x))$ and $\Delta: (\mu_Y, F_Y) \rightarrow (\mu_{Y\times Y}, F_{Y\times Y})$, $y \mapsto (y, y)$. Now we show that (f, g) and Δ are fuzzy maps. Sine f and g are fuzzy maps, then $\mu_X(x) \leq \mu_{Y\times Y}((f, g)(x))$. Let U be a fuzzy open set in $F_{Y\times Y}$,that is $U = (U_1 \times U_2) \cap \mu_{Y\times Y}$ for all $U_1, U_2 \in F_Y$, then $(\mu_X \cap (f, g)^{-1}(U))(x) = \mu_X(x) \wedge U((f, g)(x))$

$$= \mu_X(x) \wedge [U_1(f(x)) \wedge U_2(g(x)) \wedge \mu_{Y\times Y}((f, g)(x)).$$

Since $\mu_X(x) \leq \mu_{Y\times Y}((f, g)(x))$, then

$$\mu_X \cap (f, g)^{-1}(U) = [\mu_X \cap f^{-1}(U_1)] \cap [\mu_X \cap g^{-1}(U_2)] \in F_X.$$

Therefore, (f, g) is a fuzzy map. In similar argument, Δ is a fuzzy map. Let (μ_B, F_B) be the fuzzy pullback of (f, g) and Δ so that $\alpha: (\mu_B, F_B) \rightarrow (\mu_X, F_X)$ and $\beta: (\mu_B, F_B) \rightarrow (\mu_Y, F_Y)$ are fuzzy maps with $(f, g) \circ \alpha = \Delta \circ \beta$. Then

$$(\mu_B, F_B) \overset{\alpha}{\rightarrow} (\mu_X, F_X) \underset{g}{\overset{f}{\rightrightarrows}} (\mu_Y, F_Y)$$

is a fuzzy equalizer diagram. It is obvious that $p_1 \circ (f, g) = f$, $p_2 \circ (f, g) = g$ and $p_1 \circ \Delta = id_Y$, $p_2 \circ \Delta = id_Y$ where $p_i: Y \times Y \rightarrow Y$ are the fuzzy projection maps, $i = 1,2$. Thus $f \circ \alpha = p_1 \circ (f, g) \circ \alpha = p_1 \circ \Delta \circ \beta = id_Y \circ \beta = \beta$, $g \circ \alpha = p_2 \circ (f, g) \circ \alpha = p_2 \circ \Delta \circ \beta = id_Y \circ \beta = \beta$ and hence $f \circ \alpha = g \circ \alpha$. For any fuzzy map (μ_C, F_C) and $\theta: (\mu_C, F_C) \rightarrow (\mu_X, F_X)$ with $f \circ \theta = g \circ \theta$. Then $p_1 \circ (f, g) \circ \theta = f \circ \theta = g \circ \theta = id_Y \circ g \circ \theta = p_1 \circ \Delta \circ g \circ \theta$, $p_2 \circ (f, g) \circ \theta = g \circ \theta = id_Y \circ g \circ \theta = p_2 \circ \Delta \circ g \circ \theta$. Theorem 3.1 implies that $(f, g) \circ \theta = \Delta \circ g \circ \theta$. Since (μ_B, F_B) is the fuzzy pullback of (f, g) and Δ, then there exists a fuzzy map $\varphi: (\mu_C, F_C) \rightarrow (\mu_B, F_B)$ such that $\theta = \alpha \circ \varphi$ and thus the universal property is satisfied.

Consider the arbitrary fuzzy maps $f: (\mu_X, F_X) \rightarrow (\mu_Z, F_Z)$ and $g: (\mu_Y, F_Y) \rightarrow (\mu_Z, F_Z)$. Let (μ_E, F_E) together with a fuzzy map $e: (\mu_E, F_E) \rightarrow (\mu_{X\times Y}, F_{X\times Y})$ be the fuzzy equalizer of fuzzy maps $f \circ p_1$, $g \circ p_2: (\mu_{X\times Y}, F_{X\times Y}) \rightarrow (\mu_Z, F_Z)$ such that $(f \circ p_1) \circ e = (g \circ p_2) \circ e$. Now we prove that (μ_E, F_E) together with fuzzy maps $p_1 \circ e: (\mu_E, F_E) \rightarrow (\mu_X, F_X)$ and $p_2 \circ e: (\mu_E, F_E) \rightarrow$

(μ_Y, F_Y) is the fuzzy pullback of the fuzzy maps f and g. Let (μ_C, F_C) be a fuzzy space with fuzzy maps $\alpha: (\mu_C, F_C) \rightarrow (\mu_X, F_X)$ and $\beta: (\mu_C, F_C) \rightarrow (\mu_Y, F_Y)$ so that $f \circ \alpha = g \circ \beta$. By theorem 3.1, there exists a unique fuzzy map $\theta: (\mu_C, F_C) \rightarrow (\mu_{X\times Y}, F_{X\times Y})$ such that $p_1 \circ \theta = \alpha$, $p_2 \circ \theta = \beta$. Since (μ_E, F_E) is the fuzzy equalizer of fuzzy maps $f \circ p_1$, $g \circ p_2: (\mu_{X\times Y}, F_{X\times Y}) \rightarrow (\mu_Z, F_Z)$ and $(f \circ p_1) \circ \theta = f \circ (p_1 \circ \theta) = f \circ \alpha = g \circ \beta = g \circ (p_2 \circ \theta) = (g \circ p_2) \circ \theta$, then theorem 3.1 implies that there exists a unique fuzzy map $\varphi: (\mu_C, F_C) \rightarrow (\mu_E, F_E)$ such that $e \circ \varphi = \theta$. Hence $(p_1 \circ e) \circ \varphi = p_1 \circ \theta = \alpha$, $(p_2 \circ e) \circ \varphi = p_2 \circ \theta = \beta$. Thus the desired fuzzy pullback is obtained. □

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